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Observation of Gait Patterns and Orientation Angles for Development of an Active Ankle-Foot Prosthesis

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Abstract: Recent studies on ankle-foot prostheses which are commonly used for transtibial amputees have focused on adaptation of the ankle angle of the prosthesis according to ground conditions in order to reduce the difficulties which the patients experience while walking on stairs or a ramp. For adaptation to the various ground conditions (e.g., incline, decline, step, etc.), the ankle-foot prostheses should first recognize the ground conditions as well as the current human motion pattern. For this purpose, the ground reaction forces and orientation angle of the prosthesis provide fundamental information. The measurement of the orientation angle, however, creates a challenge in practice. Although various sensors, such as accelerometers and gyroscopes, can be utilized to measure the orientation angles of the prosthesis, none of these sensors can be used as a sole sensing mechanism due to their intrinsic drawbacks. A number of sensor-fusion methods have been proposed to address this issue. In this paper, a time-varying complementary filtering (TVCF) method is proposed to incorporate the measurements from an accelerometer and a gyroscope to obtain a precise orientation angle. The cut-off frequency of TVCF is adaptively determined according to the human motion phase. The performance of the proposed method is verified by experiments.

Keywords: Sensor Fusion, Signal processing, Linear-time-variant filters, Posture estimation

1. INTRODUCTION

Proprio-Foot developed by Ossur (2013) is one of the most popular active ankle-foot prostheses (AAFP) that adjust the ankle angles according to the ground conditions. The ankle motions and gait patterns of the device are detected continuously by sensors measuring the orientation angles and ground reaction forces of the device. Once these data are obtained, an intelligent control algorithm, called 'Terrain Logic,' identifies the ground conditions. Then, the ankle motion is generated by a stepping motor so that it follows the desired angle trajectory determined by the Terrain Logic.

As in the control strategy of the Proprio-Foot, it is in general necessary to detect both the human motions and ground conditions when developing the AAFP which adapts its ankle angle according to the ground condition. There have been many researches on human motion detection while walking. Kong and Tomizuka (2009) developed a gait monitoring system based on air pressure sensors, which detects gait phases by a fuzzy logic algorithm. Pappas et al. (2001) studied a gait phase detection system which has the success rate of detection above 96% for subjects with impaired gait.

Meanwhile, the ground conditions (e.g., incline, decline, step, etc.) should also be identified from ground reaction forces applied to the prosthetic foot and the orientation angle of the prosthesis for estimating the slope of the ground. Therefore, it is important to accurately measure the orientation angle of the prosthesis in order to identify the ground conditions. There are many available sensors or sensor modules to measure the orientation angle, such as inertial measurement units (IMUs), ultrasonic sensors, and vision sensors [Bachmann et al. (2003); Robertson et al. (1998); Rencken (1993)]. However, it is difficult to obtain the accurate orientation angle of a prosthesis by these sensors due to their inherent drawbacks. Assuming that the majority of the foot motions are on the sagittal plane, the orientation angle of an AAFP can be estimated by accelerometers and rate gyroscopes. Accelerometers are utilized to estimate the direction of gravity in a stationary

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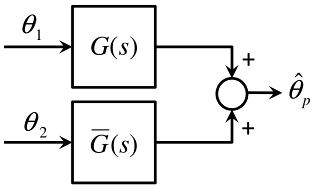

Fig. 1. Block diagram of general two input complementary filtering.

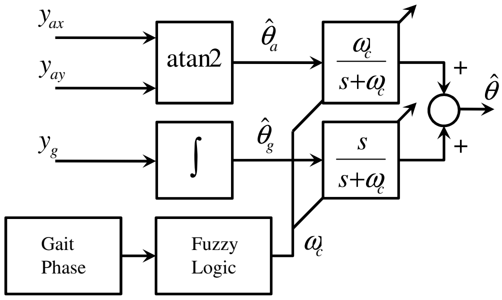

Fig. 2. Block diagram of proposed TVCF method with gyroscope and accelerometer signals.

condition. On the other hand, gyroscopes measure body angular rates and make it possible to estimate the attitude by integrating the signal. From these two sensing methods, the same physical quantity, pitch or roll, can be estimated. Since each sensor provides different frequency domain characteristics, an improved result is expected if the two methods can be fused intelligently.

Complementary filtering (CF), shown in Fig. 1, is an alternative signal processing method that can be used for the same purpose [Baerveldt and Klang (1997); Hadri and Benallegue (2009); Brown (1983)]. The conventional CF method utilizes linear-time-invariant (LTI) filters with different frequency characteristics, such that only the reliable components are selectively extracted in the frequency domain. Since integration is required to estimate the pitch or roll attitude from a rate gyroscope, where the integration amplifies uncertainty at low frequencies, the reliability of the estimate by gyroscope is poor at low frequencies (e.g., drift). The limitation in estimating attitude from accelerometers is evident due to the difficulty in decoupling the gravity vector from the higher frequency motion accelerations, and thus its reliability is poor at high frequencies. Therefore, the CF method can be applied to this problem, where a lowpass filter and a highpass filter are used as the filters, $G(s)$ and $\bar{G}(s)$, in Fig. 1. However, the challenge of the CF method is to design the LTI filters according to the frequency dependent reliabilities. When the frequency characteristics are time-invariant and clearly distinguished, the design of the filters require minor tuning. However, in many cases, the frequency characteristics are not fixed and the simple LTI filters may not produce the best result.

In this paper, a time-varying complementary filtering (TVCF) method is proposed as shown in Fig. 2 to estimate the orientation angle of the AAFP. The proposed method utilizes linear time-varying (LTV) filters for estimating a physical quantity from multiple sensors whose reliability in the frequency domain is time-varying. The advantage of

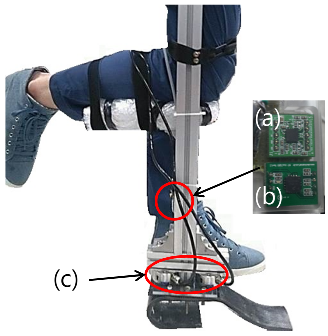

Fig. 3. Experimental device for detection of a gait pattern and an orientation angle of the device; (a) gyroscope, (b) accelerometer, and (c) load cells.

this method is that it improves the performance of the CF method with more degrees of freedom in the design of the filters. The main issue is determining the best conditions with which to adapt the LTV filters. A fuzzy logic scheme is proposed to formalize the qualitative conditions for adaptation. The performance is verified by comparing the estimation results with those of other sensor such as vision sensor.

2. PROPOSED ORIENTATION ANGLE MEASUREMENT

2.1 Application of CF Method to Angle Measurement

The main idea of the CF method, represented in Fig. 1, is to fuse two measurements, $\theta_1(t)$ and $\theta_2(t)$, of the same physical quantity, $\theta_p(t)$. $\bar{G}(s)$ is often designed as $1-G(s)$, such that $G(s)+\bar{G}(s) = 1$. In this case, the estimate $\hat{\theta}_p(t)$ is exactly the physical quantity if the measurements are without noise and error.

Suppose that $\theta_1(t)$ and $\theta_2(t)$ are reliable at low and high frequencies respectively. In a first-order implementation, $G(s)$ can be set as a low-pass filter with a fixed cut-off frequency, ω_c, i.e.

$$G(s) = \frac{\omega_c}{s+\omega_c} \tag{1}$$

The complement of $G(s)$, i.e., $\bar{G}(s) = 1 - G(s)$, is then

$$\bar{G}(s) = \frac{s}{s+\omega_c} \tag{2}$$

which becomes a high-pass filter. Note that $G(s)+\bar{G}(s) = 1$ for all s and consequently there is no phase delay in the estimate, $\hat{\theta}_p(t)$.

If $\theta_1(t) = \theta_p(t) + \eta_1(t)$ and $\theta_2(t) = \theta_p(t) + \eta_2(t)$, where η_1 is high frequency noise and η_2 is low frequency noise both relative to ω_c, then by (1), and (2) the estimate by CF is

$$\hat{\theta}_p = G(s)[\theta_p + \eta_1] + \bar{G}(s)[\theta_p + \eta_2] \tag{3}$$

$$= \theta_p + G(s)[\eta_1] + \bar{G}(s)[\eta_2]. \tag{4}$$

where the signals are in the Laplace domain. If the cut-off frequency, ω_c, is properly selected, the magnitude of $G(s)[\eta_1] + \bar{G}(s)[\eta_2]$ in the time domain is small compared to θ_p due to the characteristics of the noise and LTI filters.

With a known initial condition, θ_o, the single axis gyroscope can be integrated to achieve an attitude estimate,

Table 1. Fuzzy rule basis for gait phase detection

Heel	Toe	Fuzzy membership value
high	*low*	$\mu_{Initialcontact} \to 1$
high	*high*	$\mu_{Midstance} \to 1$
low	*high*	$\mu_{Terminalstance} \to 1$
low	*low*	$\mu_{Swing} \to 1$

$\hat{\theta}_g(t) = \int_0^t y_g(\tau)d\tau + \theta_o$. On the other hand, in the near static or constant velocity condition, the motion accelerations are near zero and the orthogonal components of the dual axis accelerometer can estimate the attitude from the gravity vector by $\hat{\theta}_a(t) = \arctan 2(y_{ax}(t), y_{ay}(t))$. The $\arctan 2()$ function is chosen as opposed to the $\arctan()$ function to estimate $\theta(t)$, since the range is $[-\pi, \pi)$ (rad) and not just $(-\frac{\pi}{2}, \frac{\pi}{2})$ (rad).

Individually, the gyroscope gives a decent estimate of angle at higher frequencies, but this estimate tends to drift due to the bias and noise. The accelerometer gives an accurate static estimate, but this estimate is predominately corrupted by motion accelerations. Therefore, the attitude estimates from the gyroscope and accelerometer can be modeled as

$$\hat{\theta}_g(t) = \theta(t) + \eta_d(t) \tag{5}$$

$$\hat{\theta}_a(t) = \theta(t) + \eta_m(t) \tag{6}$$

where $\theta(t)$ is the true attitude angle to be estimated by CF, and $\eta_d(t)$ and $\eta_m(t)$ are the measurement errors corresponding to the drift and motion acceleration terms respectively. In order to apply the CF method, $\eta_d(t)$ and $\eta_m(t)$ are treated as noise; $\eta_d(t)$ is low frequency and $\eta_m(t)$ is high frequency. Using this method a more accurate estimate of $\theta(t)$ compared to the individual sensors can be produced. Choosing a larger ω_c places more trust on the accelerometer estimate, $\hat{\theta}_a(t)$, while choosing a smaller ω_c places more trust on the gyroscope estimate, $\hat{\theta}_g(t)$.

2.2 Time Varying Complementary Filtering Based on Gait Phase

In general, conventional CF does not always result in the best estimate of the angle, i.e., $\hat{\theta}_p$ in (4), since the noise characteristics ($\eta_d(t)$ and $\eta_m(t)$) are time varying. To improve the estimation performance of the CF, an intuitive idea is to utilize a strategy in that the cut-off frequency is adjusted according to the time varying noise characteristics. For example, *when* the motion accelerations occur, then the cut-off frequency, ω_c in (1) and (2), can be adjusted to a low value, ω_{low}, and effectively filter out the high frequency noise, $\eta_m(t)$, in (6). Conversely, when the rigid body is not accelerating, the cutoff frequency can be increased to a high value, ω_{high}, to update the estimate by the accelerometer measurement. Based on these considerations, it is proposed to use the Time Varying Complementary Filter (TVCF) approach.

Fuzzy Logic In order to detect precise motion acceleration, the gait phase which represents a unique pattern presented during walking [Perry (1992)] is utilized in this paper. For example, the swing phase and the stance phase (which are the most fundamental gait phases) exhibit different motion acceleration profiles; there is large motion acceleration in the swing phase while less motion acceleration is presented in the stance phase. Also a large peak in the acceleration tends to be measured when the phase changes from the stance phase to the swing phase and also from the swing phase to the stance phase due to the impact caused by ground contact.

Table 2. Fuzzy rule basis for gait transition detection

Swing	Initial contact	Terminal stance	Fuzzy membership value
high	*high*	*low*	$\nu_{Swing\ to\ Ic} \to 1$
high	*low*	*high*	$\nu_{Ts\ to\ Swing} \to 1$

TVCF should be able to take these acceleration characteristics into consideration to render the cut-off frequency appropriately and to provide the accurate orientation angle. In this paper the gait phase is divided into four phases, i.e., initial contact, mid stance, terminal stance and swing, and it is detected to derive precise motion acceleration. Then the TVCF changes the cut-off frequency based on the detected gait phase to adjust the reliability of each sensor and estimate the precise orientation angle as the attitude of the ankle-foot prosthesis.

A ground reaction force (GRF) is measured to detect the gait phase, and fuzzy logic is utilized to infer the precise gait phase based on the measured GRFs. Figure 3 shows the experimental device used in this research, where two load cells are installed to measure the GRFs applied to the heel and the toe. Since AAFPs should be designed to endure repetitive loading for many cycles, the sensors to measure GRFs were installed not under the foot but near the ankle.

Table 1 shows a fuzzy rule basis for the gait phase detection. A membership function in (7) is used to determine the *high* and *low* of the load cells.

$$f(x) = 0.5\,[tanh(s(x - x_0)) + 1] \in [0, 1] \tag{7}$$

where x, x_0, and s are the load cell value, the threshold value, and the sensitivity coefficient, respectively [Kong and Tomizuka (2009)]. It should be noted that the smooth phase shift is detected by adjusting the sensitivity coefficient s to the lower value.

The rule to distinguish the gait phase which is presented in Table 1 is calculated as (8) by the Larsen product implication [Larsen (1980)], which is the logical product of the output of the fuzzy membership functions (f_{toe}, f_{heel}). When the fuzzy membership value (μ) of each phase is close to one, the gait is likely to be in that phase.

$$\mu(x) = f_{toe}(x) \times f_{heel}(x) \in [0, 1] \tag{8}$$

In addition, the phase change between the swing phase and the stance phase should be detected, and another fuzzy rule is proposed in Table 2 to this end. The fuzzy membership value (ν) for the detection of the phase transition consists of $g(x)$ which is another membership function of gait phases.

$$g(x) = 0.5\left[\frac{\mu(x) - \mu_0}{|\mu(x) - \mu_0|} + 1\right] \in [0, 1] \tag{9}$$

$$\nu(x) = g_S(x) \times [g_{Ic}(x) + g_{Ts}(x)] \in [0, 1] \tag{10}$$

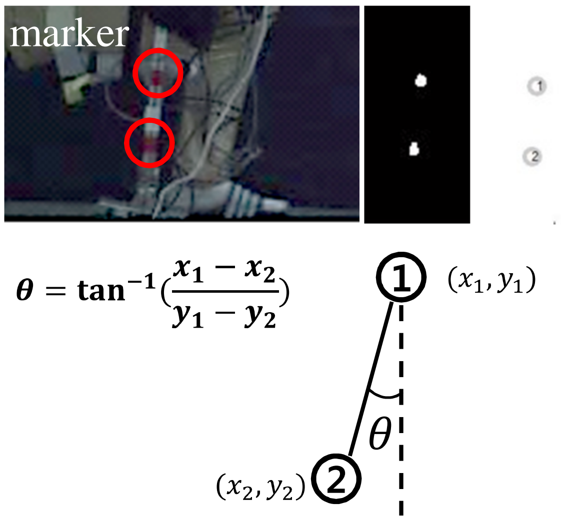

Fig. 4. Estimation of orientation angle by vision system; (a) camera image, (b) detected red markers, (c) positions of the markers in pixel, and (d) angle estimation

$g(x)$ is defined as (9) which lowers the threshold of μ to detect the phase transition not the phase itself, and this can reduce the calculation effort compared to (7). The subscripts S, Ic and Ts represent $Swing$, $Initial\ contact$ and $Terminal\ stance$, respectively.

The overall fuzzy rule basis is presented in Table 3, and the cut-off frequency of the TVCF is determined based on this fuzzy rule. The proposed fuzzy rule basis calculates ξ which is given in the last column of Table 3 as the parameter which decides the cut-off frequency of the TVCF as follows

$$\omega_c(t) = \xi(t)\omega_{high} + (1 - \xi(t))\omega_{low} \tag{11}$$

Notice that if $\xi = 1$ then $\omega_c = \omega_{high}$ thus signifying when the estimate from the accelerometer is more trustworthy.

(18) is the inference function which decides ξ as a function of the input variables x_1 to x_6 which present all the membership values of the phases as (12) to (17).

$$x_1(t) = \mu_{Swing}(t) \tag{12}$$
$$x_2(t) = \nu_{Swing\ to\ Initial\ contact}(t) \tag{13}$$
$$x_3(t) = \mu_{Initial\ contact}(t) \tag{14}$$
$$x_4(t) = \mu_{Mid\ stance}(t) \tag{15}$$
$$x_5(t) = \mu_{Terminal\ stance}(t) \tag{16}$$
$$x_6(t) = \nu_{Terminal\ stance\ to\ Swing}(t) \tag{17}$$

$$\xi(t) = x_1(t) \times x_2(t) \times x_6(t) + x_3(t) + x_4(t) + x_5(t) \in [0, 1] \tag{18}$$

This definition of ξ gives zero value only when the gait is in the swing phase or the gait transition.

3. EXPERIMENTAL RESULTS

3.1 Experimental Apparatus

An experimental device was devised to verify the effectiveness of the proposed TVCF to measure the attitude of the ankle-foot prosthesis. A human subject wore the device and walked on a flat surface, and the orientation angle of the device was estimated using the measurements of an accelerometer (NT-ACC7260, $\pm 1.7\ g$) and a gyroscope (P0-GRA-12-01, $\pm 2000\ deg/s$) attached to the device.

At the same time, the orientation angle of the device was also estimated by a vision system (High-speed camera TS3, Fastec Imaging) to validate the output of the proposed algorithm. Two red markers were attached to the device to estimate the orientation angle of the device by using the vision system, and the orientation angle was obtained by tracking the positions of the red markers using MATLAB as shown in Fig. 4. The orientation angle estimated by the vision system was sufficiently accurate so that it was considered as a reference value to evaluate the accuracy of the proposed algorithm.

For the detection of the gait phase, two load cells were installed on the ankle part of the device. The proposed algorithm and data acquisition were implemented on National Instrument's Single Board RIO 9636 and LabVIEW with a fixed sampling period of 0.012 (s).

3.2 Selection of Parameters

There are parameters to be determined to utilize the TVCF, which are s, x_0, and μ_0. Those values were selected as 0.03, $150N$, and 0.1, respectively. If s value is large, then fuzzy membership value μ will be changed rapidly, which makes the gait transition shown in a very short period. Therefore, s was tuned such that the gait transition is detected enough. Also, threshold values were selected for the gait phase to be well detected.

Meanwhile, the most important parameters to be determined are ω_{low} and ω_{high}. In this paper, optimization method was used to set them. ω_{low} was selected to minimize the 2-norm of the error between the estimated orientation angles by a vision system and the CF method during the period where the accelerometer has a low reliability, i.e., the swing phase and gait transition. Similarly, ω_{high} was set by optimization during stance phase only (i.e., initial contact, mid stance, and terminal stance). The off-line optimization is conducted by MATLAB *fmincon* function. ω_{low} and ω_{high} were 0.03 rad/s and 5.36 rad/s, respectively.

3.3 Experimental Results

For the verification of the TVCF method, the subject was asked to walk at a normal speed, a slow speed, and a fast speed so that various disturbances and conditions are imposed for effective optimization and validation.

Figure 5 shows the result of gait phase/transition detection and time-varying cut-off frequency for each walking speed. In the figure, S, SIc, Ic, Ms, Ts, TSS, and S are denoted as $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $x_5(t)$, and $x_6(t)$, respectively.

Table 3. Fuzzy rule basis for time-varying cut-off frequency determination

Swing	Swing to Initial contact	Initial contact	Mid stance	Terminal stance	Terminal stance to Swing	$\xi(t)$
high	*N/A*	*low*	*low*	*low*	*N/A*	0
low	*high*	*low*	*low*	*low*	*low*	0
low	*low*	*low*	*low*	*low*	*high*	0
low	*low*	*high*	*low*	*low*	*low*	1
low	*low*	*low*	*high*	*low*	*low*	1
low	*low*	*low*	*low*	*high*	*low*	1

$\omega_c(t)$ was determined by ω_L, ω_H, and ξ as in (11). With the obtained $\omega_c(t)$, the TVCF was applied.

At the same time, the CF method was also applied to compare its performance to that of the TVCF. To determine the fixed cut-off frequency for each walking experiment, optimization was performed to minimize the 2-norm of the error between the orientation angle estimate by vision system and that by CF method. The optimized cut-off frequencies for each experiment are presented in Table 4. It should be noted that the optimized cut-off frequencies are low, making the algorithm independent on the estimate by the accelerometer.

Figure 6 shows the orientation angle estimate of the gyroscope, the accelerometer, the vision system, the CF method, and the TVCF method for each walking speed, respectively. Since the angle estimate by the gyroscope is obtained by integrating the angular velocity, drift is observed in a way that the estimate becomes decreased as the time goes by. Also the initial value is required for integration, which is set as the initial value of the angle estimate by the accelerometer. Meanwhile, the angle estimate by the accelerometer is very noisy, and it is perturbed largely during the gait transition and in the swing phase.

As shown in Fig. 6, the estimation results of both the CF and TVCF are close to the orientation angle estimated by the vision system. For more quantitative comparison, the root-mean-square (RMS) values of errors between the angles estimated by the vision system and those by CF/TVCF method are presented in Table 4.

For the normal and slow speed walking, the TVCF method brought a better estimation result from the viewpoint of the RMS values of errors. For the fast walking, however, the CF resulted in better estimation performance. These results arise due to the worse angle estimate by the accelerometer during fast speed walking as in Fig. 6c. Since the CF method rarely relied on the accelerometer because of the low cut-off frequency, the angle estimated by the CF was more accurate than that by the TVCF in this case.

Table 4. Comparison of orientation angle estimation performance

	Normal speed		Slow speed		Fast speed	
Error	CF	TVCF	CF	TVCF	CF	TVCF
RMS (deg)	2.908	1.172	1.636	1.593	2.951	6.218
Optimal	CF	TVCF	CF	TVCF	CF	TVCF
ω_c (rad/s)	0.001	N/A	0.688	N/A	0.959	N/A

However, the low fixed cut-off frequency leds to the drift problem. The drift on the estimate by the CF and the gyroscope are observed in Fig. 7, and it is apparent that the effect of the drift would be further increased due to integration. On the other hand, the TVCF was able to get rid of the drift as in the figure. Therefore, although the angle estimation result of the TVCF method during the fast speed walking experiment was worse than that of the CF method, it is expected that the TVCF will give a better result for prolonged operation time. In addition, the walking speed of the amputees is not as fast as the normal, and thus the proposed TVCF method is effective for the development of an AAFP.

4. CONCLUSION

In this paper, a new approach for an orientation angle estimation was proposed using a TVCF method. By using the gait phase analysis function of an AAFP, to change the cut-off frequency of the filter, the proposed method showed improved the estimation performance while maintaining ease of implementation. The effectiveness was verified with experimental results, and the accurately estimated orientation angle will be used to estimate the slope of ground as a future work.

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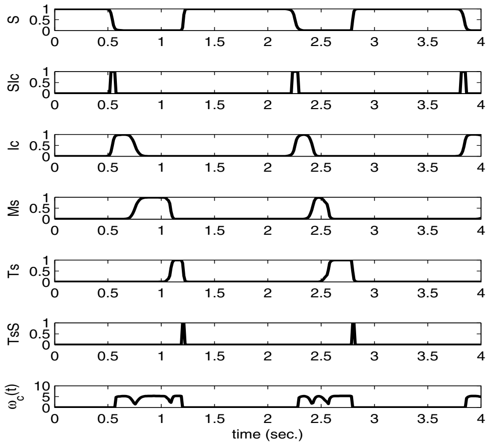

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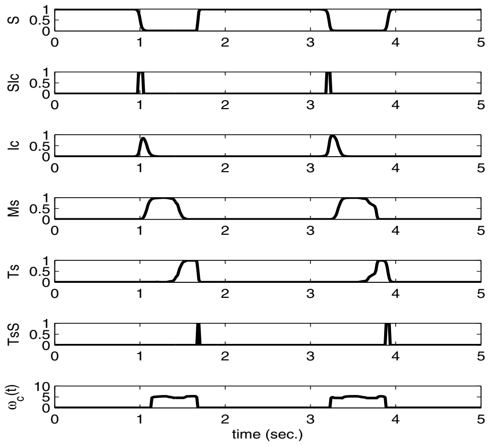

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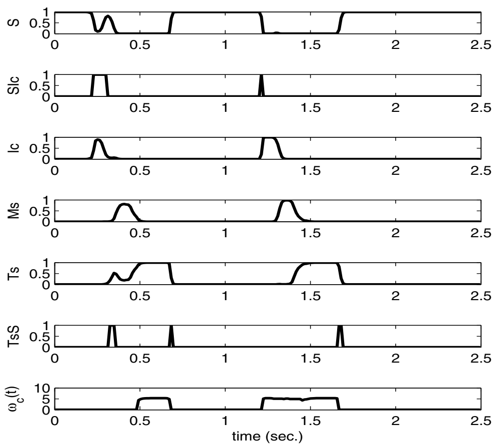

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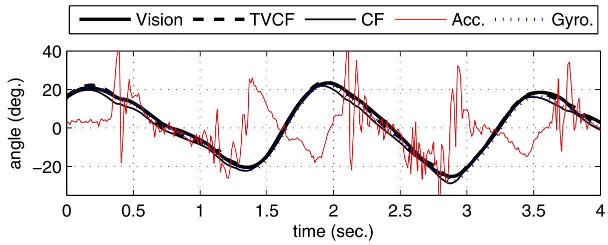
Fig. 5. Experimental results of gait phase detection; (a) normal speed walking, (b) slow speed walking, and (c) fast speed walking.



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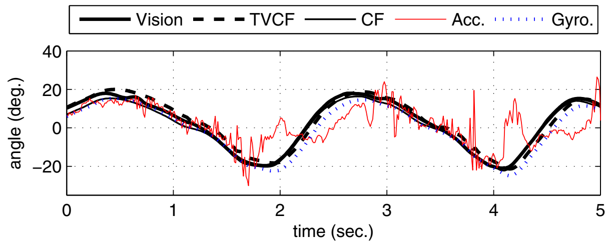

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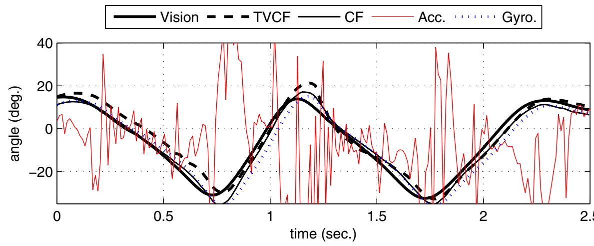

(c)

Fig. 6. Estimation of orientation angle; (a) normal speed walking, (b) slow speed walking, and (c) fast speed walking.

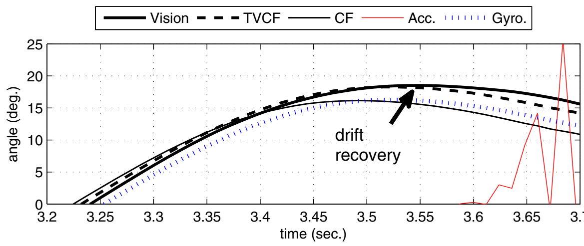

Fig. 7. Zoomed plot of Fig. 6a.

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