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An Experimental Study for Delay-Independent State-Feedback Controller Design

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Abstract: In this study, a method for determining delay-independent stability zones of the general LTI dynamics with multiple delays against parametric uncertainties is presented. The method is utilized to design a delay-independent state-feedback controller and verified experimentally for a two-tank liquid level control system. The method is based on extended kronecker summation(EKS) to investigate controller parameter space for delay-independent stability(DIS) of the system. The main aim of the paper is recalling a new sufficient condition for determination of controller parameter space for DIS and presenting the application of methodology for a physical experimental case study.

Keywords: Time-delay systems, multiple delays, delay-independent stability, robust stability, kronecker summation, controller design, two-tank level control

1. INTRODUCTION

In many applications,time delay in feedback is inevitable and effects the performance and the stability of the system drastically. In the literature, there are numerous studies which investigates controller design methods for time delayed systems (Gundes et al. [2007], Suva et al. [2002], Mahmoodi Nia and Sipahi [2013]). Most of the studies focused on the stable operation of the systems when there is a predetermined delay in the feedback or the system itself. In this study, we consider systems with undetermined delays. Our main goal is to determine state feedback controller parameters for stable operation of the system regardless of the delay value. This class of controllers exhibit particular importance with the systems where the delayed feedback may cause critical and dangerous instabilities such as motion control,tank level control, high temperature furnace control. In these applications, controller has to drive the system to a stable operating point regardless of the delay where delay may occur due to a malfunction in the sensory system.

In our study, we consider linear time invariant, retarded multiple time delayed state feedback systems (LTI-MTDS). General state space form is given as,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \sum_{j=1}^{p} \mathbf{B}\mathbf{K}_j(\mathbf{q})\mathbf{x}(t-\tau_j) \qquad (1)$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{n\times n}$,$\mathbf{B} \in \mathbb{R}^{n\times 1}$,$\mathbf{K}_j(\mathbf{q}) \in \mathbb{R}^{1\times n}$, $j = 1 \dots p$, and the vector of time delays $\boldsymbol{\tau}= (\tau_1, \tau_2, \dots, \tau_j, \dots, \tau_p) \in \mathbb{R}^{p+}$ the elements of which are rationally independent from each other. In the text, we use boldface capital notation for vector and matrix quantities, open unit disc, unit circle and outside of unit circle are referred as $\mathbb{D}, \mathbb{T}, \mathbb{S}$, respectively. Naturally, $\mathbb{D} \bigcup \mathbb{T} \bigcup \mathbb{S} = \mathbb{C}$ represents the entire complex plane.

Besides the mainstream of the research such as Hale and Lunel [1993], Niculescu [2001] etc. focusing on the stability analysis of LTI-MTDS, the robust stability against delay and system parameter uncertainties is also investigated as cited in Chapellat and Bhattacharyya [1989], Fu et al. [1989], Kharitonov [1999], Richard [2003], Silva et al. [2001], Gu et al. [2003]. In essence, these studies can be classified in two major topic, delay-independent stability (DIS) and delay-dependent stability (DDS).

In this paper, we focused on DIS systems where researchers approach the problem two distinctive way. First one is Lyapunov based approach depending usually on Krasovskii and Razumikhin functionals (Ivanescu et al. [2000], Xu [2001], Fridman [2001]). Second approach is mainly motivated by the characteristic root crossings on the imaginary axis. Several methods such as frequency (ω) sweeping or matrix pencil approach are used to determine the DIS criteria in many studies like Kamen [1982], Hertz et al. [1984], Chen et al. [1995], Chen and Latchman [1995], Niculescu [1998], Tuzcu and Ahmadian [2002], Michiels and Niculescu [2007], Ergenc [2010], Delice and Sipahi [2012]. As well as analysis of DIS, there are some studies to improve controller design methods guaranteeing DIS both in Lyapunov approach (Baser [2003], He et al. [2011]) and root crossing approach using algebraic tools such as Descartes rule of signs and Sturm Sequences(Delice and Sipahi [2010], Mahmoodi Nia and Sipahi [2013]).

In this study, the problem of dictating delay-independent stability criteria is transformed into assigning a certain number of the zeros in $\mathbb{D}$ of a polynomial derived from the system equations. The key novelty introduced by this method is that there are no restrictions on the number of the delays (p) and the number of the parametric uncertainties (r). It is based on unique properties of a self-inversive polynomial which represents the infinite dimensional delayed system in terms of a finite dimensional polynomial with interspersed zeros on the unit-circle.

The paper is organized as follows: In section 2 preliminary definitions and statements of the study are given. Section 3 presents the controller design methodology for delay-independent stability for LTI system with multiple delays. Section 4 contains experimental example case studies. In the last section, conclusive remarks about methodology is given.

2. PRELIMINARIES

The characteristic equation of the system in (1) is derived as follows

$$CE(s,\mathbf{k},\tau_1,\ldots,\tau_p) = \det\left[s\mathbf{I}-\mathbf{A}-\sum_{j=1}^{p}\mathbf{BK}_j(\mathbf{q})e^{-\tau_j s}\right] = \sum_{l=0}^{n} a_l(\mathbf{q})s^l + \sum_{j=1}^{p}\left(\sum_{l=0}^{n-1} a_l(\mathbf{q})\ s^l\right)e^{-\tau_j s} = 0 \tag{2}$$

The characteristic equation is an n^{th} degree polynomial in s, and retarded system has n_j as the highest order of commensuracy of delay τ_j in the dynamics ($n_j \le n$).

Definition: The number of characteristic roots of (2) in the complex open right half plane ($\mathbb{C}_+$) states the stability of the system in (1). This number is a function of the delays and the controller gains, which are the parameters of (1). Non-existing roots on $\mathbb{C}_+$ assure that the system is "stable". The stability of the system fails when a characteristic root crosses $\mathbb{C}_0$ at a point.

The system is delay-independent stable when all the characteristic roots lie on the complex open left half plane ($\mathbb{C}_-$) regardless of the delay values. To achieve DIS operation of the system we need to determine the full set of state feedback controller gains which locates the characteristic roots in $\mathbb{C}_-$. Examination of infinitely many roots of the delayed system is very cumbersome and time demanding process even with the best numerical methods available(Breda et al. [2004], Vyhlidal and Zitek [2009], Engelborghs et al. [2002]). Thus we transform the problem of examining imaginary axis crossing of infinitely many characteristic roots into determination of the root locations of the auxiliary equation of which represents the system at the stability switching points. The Extended Kronecker Summation method is used to convert the infinite imaginary axis crossing problem into finite unit circle ($\mathbb{T}$) crossing (Ergenc et al. [2007], Ergenc [2010]).

Definition: Auxiliary Characteristic Equation (ACE) of the system in (1), with $z_j = e^{-\tau_j s}$ is defined as a determinant of a matrix derived from system equations using EKS in (Ergenc et al. [2007]) as follows:

$$ACE(\mathbf{z},\mathbf{q}) = \left|\begin{matrix}\mathbf{A}\otimes\mathbf{I}+\mathbf{I}\otimes\mathbf{A^T}+ \\ \sum_{j=1}^{p}\left(\mathbf{BK}_j(\mathbf{q})\right)\otimes\mathbf{I}z_j+\mathbf{I}\otimes\left(\mathbf{BK}_j(\mathbf{q})\right)^T z_j^{-1}\right)\end{matrix}\right| = 0 \tag{3}$$

Theorem 1. (Ergenc et al. [2007]) For the system given in (1) if there exists at least one pair of imaginary characteristic roots, $\pm\omega i$, of (2) with corresponding delay vector $\boldsymbol{\tau}=\{\tau_j\} \in \mathbb{R}^{p+}$ and a controller gain vector $\mathbf{q} \in \mathbb{R}^r$ then a vector of p-dimensional unitary complex numbers $\mathbf{z} = \{z_j\} \in \mathbb{T}^P$, $|z_j| = 1, \forall j = 1\ldots p$ satisfies ACE.

A sufficient condition to determine the pairs of vectors $\langle\boldsymbol{\tau},\mathbf{q}\rangle$ that generates $\omega i \in \mathbb{C}_0$ roots of (2) can be derived using equation (3). Since this equation is completely free of delay terms, and only a function of $\mathbf{z}$ and $\mathbf{q}$, the method is now considerably simplified to find $\mathbf{z} \in \mathbb{T}^P$ solutions of (3) with respect to certain $\mathbf{q}$. Following the procedure, imaginary characteristic roots $s = \pm\omega i$ of (2) can be computed by evaluating $\mathbf{z}$ and $\mathbf{q}$ in (2). These crossing frequencies we are interested in, form the set, i.e.,

$$\Omega = \{\omega | CE(s=\omega i,\mathbf{z},\mathbf{q}) = 0,\ \mathbf{z}\in\mathbf{Z},\ \mathbf{q}\in\mathbf{Q}\} \tag{4}$$

Time delay values corresponding each crossing frequency are determined as follows:

$$\tau_{jk} = \frac{\arg(z_j) \mp 2k\pi}{\omega}\ j = 1\ldots p,\ k = 0,1,2,\ldots \tag{5}$$

where τ_{jk} implies the j^{th} delay value for various k values.

In the next section, we propose a new approach for the determination of controller gains ($\mathbf{q}$) in the controller gain space ($\mathbf{Q}$) where the system (1) is delay-independent stable.

3. DELAY-INDEPENDENT CONTROLLER DESIGN

A LTI-MTDS is delay-independent stable (in other words, robust to delays) when all the characteristic roots of (2) lie on the $\mathbb{C}^-, \forall\tau \in \mathbb{R}^{p+}$. It is well known that exhaustively calculating characteristic roots for *all* τ is not computationally possible for determination controller gains for DIS operation. In this work, we present a theorem which states necessary and sufficient conditions for delay-independent stability of (1). Similar theorems exist in an earlier studies (Delice and Sipahi [2010], Mahmoodi Nia and Sipahi [2013]) which lead computationally heavier algorithms.

Theorem 2. (Delay-Independent Stability) A LTI system given in (1) is delay-independent stable if following conditions are satisfied simultaneously.

(1) $Re\left(eig\left[\mathbf{A}+\sum_{j=1}^{p}\mathbf{BK}_j(\mathbf{q})\right]\right) < 0$
(2) The roots of ACE, $\mathbf{z} = \{z_j\} \notin \mathbb{T}^P$ for $\mathbf{q}\in\mathbf{Q}$.

Proof. In the first condition, the stability of the non-delayed system is guaranteed for the certain set of controller gains $\mathbf{q}$. In the second one, stating ACE has no roots on $\mathbb{T}^p$ assures that characteristic equation (2) has no $i\omega$ roots on the imaginary axis. This condition is the result of root continuity property.

These two conditions are the framework of the DIS. In following part of the paper, we would like to mention about distinctive properties of ACE which are utilized very conveniently in the robustness analysis of the time delayed systems against time delay variations. The outcome of the analysis presented controller gain space which provides control scheme for delay-independent stable operation. ACE is a special multinomial of z_j ($j = 1\ldots p$). We form

ACE in another arrangement, where we embed $z_j \quad j = 1 \ldots k-1, k+1 \ldots p$ in $b_j(\mathbf{q}, z_1, z_2, \ldots, z_{k-1}, z_{k+1}, \ldots, z_p)$. Here, b_j's indicate the coefficients of a complex polynomial in terms of z_k. In a formal display:

$$ACE(\mathbf{z}, \mathbf{q}) = \sum_{j=0}^{m} b_j\left(\mathbf{q}, z_1, z_2, \ldots, z_{k-1}, z_{k+1}, \ldots, z_p\right) z_k^j \tag{6}$$

where $m < 2n^2$. At this point, it is beneficial to mention about the properties of ACE which is a self-inversive polynomial (Ergenc [2010]). Equation (6) which is generated by determinant of Kronecker summation of two conjugate matrices is inherently a self-inversive polynomial in terms of z_k^j's. The zeros of this type of polynomials lie either on the unit circle $\mathbb{T}$ or occur in pairs conjugate to $\mathbb{T}$ (symmetric pair of roots wrt unit circle). This is an instrumental property of ACE which we utilize to determine crossing points of (2). Meanwhile our ultimate aim is to design a delay-independent stable state feedback controller for the system, we desire that none of the roots of (6) even cross the unit circle $\mathbb{T}$. At this point, we should indicate that investigation of $\mathbf{q}$ space which generates non-unitary zeros for (6) is a very cumbersome problem. Here, we employ the remarkable relationship of the critical points of the self-inversive polynomial (zeros of the derivative wrt z) and the zeros of the polynomial itself. It is stated as in the theorem below (Sheil-Small [2002]).

Theorem 3. Let P is a self-inversive polynomial of degree p. Suppose that P has exactly β zeros on the unit circle $\mathbb{T}$ (multiplicity included) and exactly μ critical points on the closed unit disc $\mathbb{D} \bigcup \mathbb{T}$ (counted according to multiplicity). Then

$$\beta = 2(\mu + 1) - p. \tag{7}$$

This theorem is the novel point for establishing the criterion for delay-independent stability. It is stated before that a system as described in (1) is delay-independent stable if its ACE has no zeros on $\mathbb{T}$ and it is cumbersome to check if zeros are unitary. Notice that the number of the unitary roots of ACE is related to the number of critical points in $\mathbb{U}$. In the literature, there are several methods to establish relations between the number of the roots in a certain region and the coefficients of the polynomials (Marden [1949]), which are extensions to the Pellet's Theorem. In essence, theorem 3 is combined with a theorem presented in (Rajan and Reddy [1985]) and (Mori [1984]), then converted into the main instrument for controller synthesis. The second theorem is as follows;

Theorem 4. Let P(z) a polynomial equation,

$$P(z) = \sum_{j=0}^{p} b_j z^j \tag{8}$$

where $b_j \in \mathbb{C}$ and $b_p \neq 0$
If

$$|b_k| > \sum_{j \neq k}^{p} |b_j| \tag{9}$$

then P(z) has exactly k zeros in the unit circle and noting that P(z) , under the above condition has no unitary zeros (i.e. $z \in \mathbb{T}$).

Proof. Proof of this theorem is easily achieved by substituting $r = R = 1$ in Pellet's Theorem in Marden [1949].

After stating the theorems, we present delay-independent stability conditions for the controller design. The system given in (1) is delay-independent stable if its ACE has certain number of critical roots(i.e. roots of the derivative of ACE on the $\mathbb{D}$). According to theorem(4) the condition of delay-independent stability is the following corollary.

Corollary 5. A linear time invariant system with multiple time delays described in (1) is delay-independent stable if

(1) $Re\left(eig\left[\mathbf{A}(\mathbf{q}) + \sum_{j=1}^{p}(\mathbf{B}_j(\mathbf{q}))\right]\right) < 0$
(2) Critical equation of ACE satisfies

$$\begin{aligned} &|\, b_\mu\left(\mathbf{q}, z_1, z_2, \ldots, z_{k-1}, z_{k+1}, \ldots, z_p\right) | > \\ &\sum_{j \neq \mu}^{p} \left|b_j\left(\mathbf{q}, z_1, z_2, \ldots, z_{k-1}, z_{k+1}, \ldots, z_p\right)\right| \end{aligned} \tag{10}$$

where $\mu \leq (p/2) - 1$ and μ is an integer number.

In the corollary, a sufficient condition is given for a self inversive polynomial has no unitary roots. Since ACE is inherently self-inversive polynomial, the controller gain set $\mathbf{q} \in \mathbf{Q}$ that satisfies the inequality above offers delay-independent stability of the system (1). In other words, if the inequality is satisfied, ACE has roots $\mathbf{z} \notin \mathbb{T}^P$ and $\boldsymbol{\tau}$ is an empty set. Thus, the system (1) is stable for all $\tau \in \mathbb{R}^{p+}$.

For the practicality of the controller design we would like to express the method as a procedure:

Procedure:

(1) Compute ACE of the system using Extended Kronecker Summation Method as described in (3)
(2) Find an initial point for $\mathbf{q}$ that satisfies $\forall \mathbf{z} \in \mathbb{D}$ roots of (2)
(3) Find the number of the roots of the critical equation of ACE that should lie in the $\mathbb{D}$ using theorem 3
(4) Evaluate the inequality (10) and investigate the parameter space that the inequality is satisfied.

The procedure above is a tool to analyze the controller gain space $\mathbf{q} \in \mathbf{Q}$ that provides DIS of the state-feedback system. In the corollary (10), it is stated that if the condition is satisfied, ACE of the system has no unimodular roots. It is the crucial condition for our aim considering that we desire to find delay-robust controller gains for the system.

In the following section we give some example cases with the experimental results to emphasize our claims.

4. EXPERIMENTAL CASE STUDY

In industry, tanks are intensively used either for keeping the liquids or housing the reaction for the processes. In many processes, coupled tanks are used and the levels of the tanks are important to maintain the health of the process. In our experimental study, we installed a two tank liquid level control process with an industrial controller made by Allen-Bradley ®. The experimental- setup shown

Fig. 1. The picture of the experimental-setup

in Figure 1 consists of two identical tanks coupled with a transition pipe enabling flow between the tanks. Figure 2 is depicted to explain the principle of the operation of the system. A water pump feeds Tank 1 and the flow rate of the water is controlled using an electro-pneumatic pressure regulator driven proportional valve. Water is drained from the bottom of Tank 1. The feedback mechanism of the liquid levels of Tank 1 and Tank 2 utilizes pressure/current and weight/current transducers, respectively.

The main objective in this process is to control the level of Tank 2. The level readings are transmitted to the industrial controller as the feedback and state-feedback controller generates the control signal which is applied to the regulator to drive the proportional valve. In the mathematical model, h_1, h_2 represents the liquid levels of the Tank 1 and the Tank 2 respectively. q_i is the flow rate of the liquid input to Tank 1 and q_1, q_2 are the flow rates of the liquid in the drain pipe of Tank 1 and transition pipe between the tanks. A_t is the area of each cylindrical tank. Note that R_1, R_2 are the flow resistances of the drain pipe and the transition pipe, respectively. The linearized relationships are given below:

$$q_1 = \frac{1}{R_1} h_1, \quad q_2 = \frac{1}{R_2}(h_2 - h_1) \tag{11}$$

for small variations of q_i and h_j. Considering the conservation of the liquid volume and the basic relations given in (11), the state space representation of the linearized system is as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\, q_i(t) \tag{12}$$

where

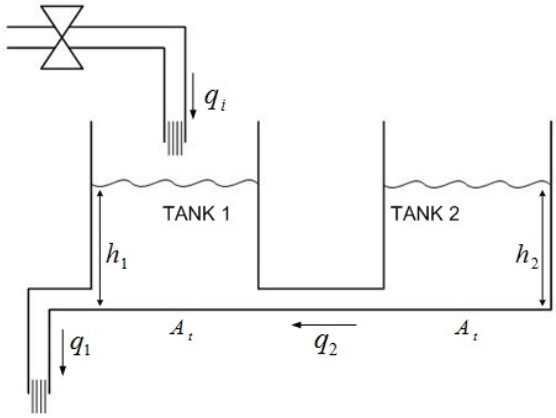

Fig. 2. An illustration of two tank liquid level process

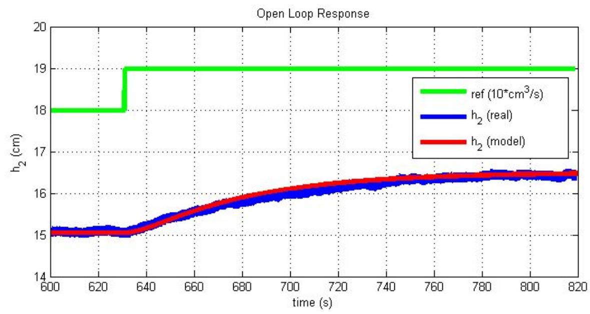

Fig. 3. Open loop response of the system

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{A_t}(\frac{1}{R_1} + \frac{1}{R_2}) & \frac{1}{A_t R_2} \\ \frac{1}{A_t R_2} & -\frac{1}{A_t R_2} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{1}{A_t} \\ 0 \end{bmatrix} \tag{13}$$

and the state vector is $\mathbf{x}(t) = [\, h_1(t) \;\; h_2(t)\,]^T$. The parameters of the system are as follows:

- $A_t = \pi r^2 = 25\pi$ cm^2
- R_1 and R_2 are experimentally derived as 0.14 s/cm^2 and 0.4 s/cm^2, while the operating levels of the tanks are $h_1 = h_2 = 15$ cm .

The step response of the open loop real system and the obtained model is given in Figure. 3 to indicate validation.

The objective of the study is to control the input flow rate of the system using proportional valve under the drainage disturbance effects and attain a stable operating point for $h_2(t)$. Here, full state feedback controller is used to achieve the stability regardless of the time delays caused by unavoidable actuator delay and the transducer delay. First, delay-independent stabilizing full state feedback controller gains are investigated analytically and then they are validated experimentally.

The state space model of the process controlled with full state feedback having multiple delays is given as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K_1}\mathbf{x}(t - \tau_1) + \mathbf{B}\mathbf{K_2}\mathbf{x}(t - \tau_2) \tag{14}$$

where

$$\mathbf{A} = \begin{bmatrix} -0.122 & 0.0318 \\ 0.0318 & -0.0318 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.0127 \\ 0 \end{bmatrix} \tag{15}$$

and $\mathbf{K_1} = [\,k_1 \quad 0\,], \mathbf{K_2} = [\,0 \quad k_2\,]$. In order to design delay-independent stable controller we utilize our procedure. The corresponding auxiliary characteristic equation:

$$
\begin{aligned}
&ACE(z_1, z_2, k_1, k_2) = \\
&(0.0000014233{k_1}^2{z_2}^2 - 0.000000065696{k_1}^2 z_2 k_2){z_1}^4 + \\
&(-0.000041667 k_1 {z_2}^2 + 0.0000020528 z_2 k_2 k_1 + \\
&0.0000011223{z_2}^3 k_2 k_1 - 0.00000013139{k_1}^3{z_2}^2){z_1}^3 + \\
&(0.00000016424{z_2}^4{k_2}^2 + 0.00000016424{k_2}^2 + \\
&0.0000053647{k_1}^2{z_2}^2 - 0.00001918 z_2 k_2 - 0.00001918{z_2}^3 k_2 - \\
&0.000000065695{k_1}^2{z_2}^3 k_2 - 0.00000032848{z_2}^2{k_2}^2 + \\
&0.00027166{z_2}^2 - 0.000000065696{k_1}^2 z_2 k_2){z_1}^2 + \\
&(0.0000011222 z_2 k_2 k_1 - 0.00000013139{k_1}^3{z_2}^2 - \\
&0.000041667 k_1 {z_2}^2 + 0.0000020528{z_2}^3 k_2 k_1) z_1 - \\
&0.000000065695{k_1}^2{z_2}^3 k_2 + 0.0000014233{k_1}^2{z_2}^2
\end{aligned} \tag{16}
$$

Using the procedure inequality which provides the DIS region is derived as:

$$
\begin{aligned}
&Ineq(k_1, k_2, z_2) = |-0.00054332\, {z_2}^2 + 0.000038362\, {z_2}^3 k_2 - \\
&0.00000032848\, {k_2}^2 - 0.000010729\, {k_1}^2{z_2}^2 - \\
&0.00000032848\, {z_2}^4{k_2}^2 + 0.00000065696\, {z_2}^2{k_2}^2 + \\
&0.00000013139\, {k_1}^2{z_2}^3 k_2 + 0.000038362\, z_2 k_2 + \\
&0.00000013139\, {k_1}^2 z_2 k_2| - |-0.0000011222\, z_2 k_2 k_1 + \\
&0.00000013139\, {k_1}^3{z_2}^2 - 0.0000020528\, {z_2}^3 k_2 k_1 + \\
&0.000041667\, k_1 {z_2}^2| - |0.00012500\, k_1 {z_2}^2 - \\
&0.0000033669\, {z_2}^3 k_2 k_1 - 0.0000061584\, z_2 k_2 k_1 + \\
&0.00000039417\, {k_1}^3{z_2}^2| - |-0.0000056932\, {k_1}^2{z_2}^2 + \\
&0.00000026278\, {k_1}^2 z_2 k_2| > 0
\end{aligned} \tag{17}
$$

Keeping in mind that ACE is fourth order polynomial, the number of roots of $D(z, \alpha)$ that should lie in unit circle is equal to 1. Using the constructed inequality (17) the delay-independent stability map is generated w.r.t. the controller gains k_1 and k_2 and depicted as in Figure 4. The rhombus region which is generated by direct calculation of roots of characteristic equation of the system equations (14) using the numerical method presented in Breda et al. [2004] which is a rough illustration of DIS region. The green shaded region inside the rhombus is the guaranteed delay-independent stabilizing pairs of controller gains where inequality (17) is satisfied. To validate the results experimentally, two pairs of gain parameters are selected and the closed loop responses of the full state feedback controlled system are given in Figures 5 and 6 for various time delays. In Figure 5, it is seen that the closed loop system is stable for large multiple delays when $k_1 = 4, k_2 = 2$ which is a delay-independent stabilizing gain pair. On the other hand, the closed loop response of the system is unstable when $k_1 = k_2 = 4$ for the same delay values as shown in Figure 6.

5. CONCLUSION

This paper is on the delay-independent stable state feedback controller design for LTI systems with multiple delays. In the center of the design, distinctive properties of ACE, which is an inherently self-inversive polynomial, are utilized. The problem of delay-independent stability of LTI systems with time delays is converted to root distribution of a self-inversive polynomial relative to the unit-circle. The stability analysis technique then employed as controller synthesis. It is also presented that critical equation of the self-inversive polynomial is a very handy tool to assign the roots characteristic equation of the system. An inequality condition is presented, which is the main result of the study to design the controller for delay-independent stability of LTI Systems with multiple delays. The concept

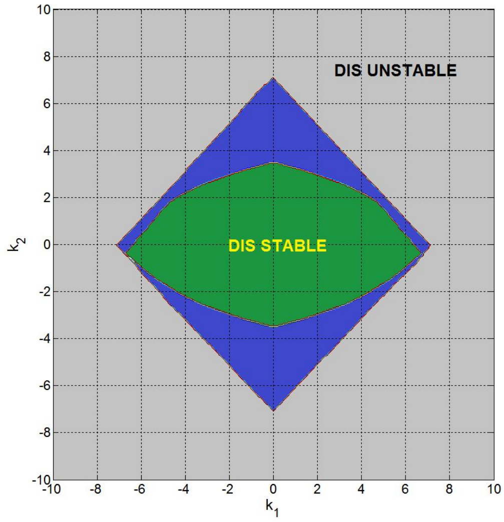

Fig. 4. The region of delay-independent stabilizing controller parameters k_1 and k_2

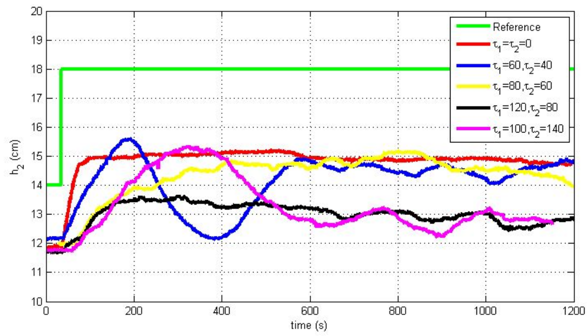

Fig. 5. Closed loop response for $k_1 = 4, k_2 = 2$

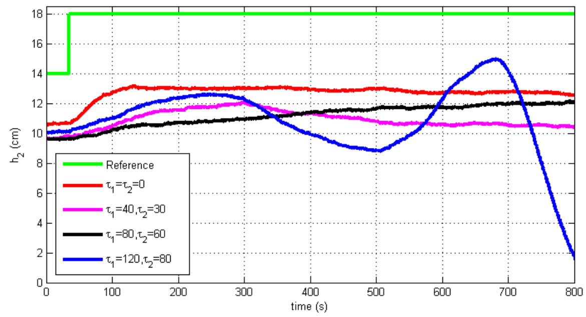

Fig. 6. Closed loop response for $k_1 = k_2 = 4$

is explained and tested by using an experimental example case study.

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