Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Parametric Integrated Perturbation Analysis - Sequential Quadratic Programming Approach for Minimum-time Model Predictive Control

Hyeongjun Park * **Ilya Kolmanovsky** * **Jing Sun** **

* *Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: judepark@umich.edu; ilya@umich.edu)*
** *Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: jingsun@umich.edu)*

Abstract: A minimum-time Model Predictive Control (MPC) problem is considered. By employing a time scaling transformation and cost regularization, it is shown that this problem becomes amenable to the application of parametric Integrated Perturbation Analysis - Sequential Quadratic Programming (IPA-SQP). The IPA-SQP framework exploits neighboring extremal optimal control and sequential quadratic programming based updates to efficiently and rapidly compute approximations to solutions in receding horizon optimal control. An interesting feature of the minimum-time MPC problem is that, after reformulation, the optimization needs to be performed simultaneously with respect to the control sequence and a constant parameter (terminal time) over the prediction horizon. Two examples are considered. The first example is for a double integrator with a control constraint. The second example is based on a two-dimensional model of a hypersonic vehicle.

Keywords: Minimum-time optimal control, model predictive control, neighboring extremal control, parameter optimization, integrated perturbation analysis and sequential quadratic programming

1. INTRODUCTION

In this paper, we consider a minimum-time Model Predictive Control problem. Minimum-time *optimal control* problems are well studied, see e.g., Bryson et al. (1975); Athans et al. (2006); Gao (2004); Bako et al. (2011); Kalman (1957). In simple cases, such as for a double integrator with control constraints, a feedback law can be computed explicitly. In more general cases, the explicit computation of a feedback law is infeasible and a minimum-time open loop (feed-forward) trajectory is generated by applying either direct or indirect computational methods, see e.g., Ben-Asher et al. (2010). To provide robustness to unmeasured uncertainties and disturbances, open-loop control can be augmented with a feedback stabilizer to the computed open-loop trajectory. Keerthi and Gilbert (1987); Mayne and Schroeder (1997) developed another approach to minimum-time control for linear systems in discrete-time based on set theoretic techniques.

Applying Model Predictive Control (MPC) philosophy to minimum-time control involves recomputing the open-loop state and control trajectory subject to pointwise-in-time state and control constraints, terminal state constraint, and the current state as the initial conditions. The computed control trajectory is applied open-loop till the next time instant when it is recomputed. As in traditional MPC, see Camacho et al. (2004), by recomputing the minimum-time control based on updated state information, robustness to unmeasured disturbances and uncertainties is improved.

The minimum-time MPC approach has been used in Starek and Kolmanovsky (2012), Petersen et al. (2013) for low thrust orbital maneuvering and for hypersonic glider guidance, respectively. In these applications, minimum-time MPC has been exploited to improve robustness to unmeasured disturbances and uncertainties. For instance, the ability to perform Earth-to-Mars low thrust orbital transfers despite thrust errors and perturbation forces has been demonstrated in Starek and Kolmanovsky (2012). Additionally, the finite-time convergence of the minimum-time control is advantageous for applications that involve way point following. In hypersonic glider applications, way points have to be reached in minimum-time subject to exclusion zone and control input constraints (see Petersen et al. (2013)).

The observation that the minimum-time feedback control possesses finite-time stability and robustness properties is easily made in continuous-time, as the cost-to-go function, $V(x)$, under appropriate assumptions, satisfies the Bellman equation, $\dot{V}(t) = -1$, leading to finite-time convergence of $V(x(t))$ to 0 and $x(t)$ to x_T, where x_T denotes the target state (e.g. a way point) for the minimum-time control (Starek and Kolmanovsky (2012)).

If the perturbations do not destroy the property, $\dot{V}(t) \leq \varepsilon < 0$, it is shown in Starek and Kolmanovsky (2012) that finite-time convergence is maintained despite these perturbations. The minimum-time MPC solutions represent an approximation to the minimum-time feedback control.

To generate the minimum-time MPC law, a fast nonlinear optimizer is necessary. In this paper, such an optimizer is developed based on an Integrated Perturbation Analysis - Sequential Quadratic Programming (IPA-SQP) framework in Ghaemi (2010); Ghaemi et al. (2007, 2008, 2009). The IPA-SQP uses the method of Neighboring Extremal (NE) optimal control of Bryson et al. (1975) to predict the solution to the optimal control problem at the next time instant based on the solution available at the current time instant; then the prediction is corrected based on an SQP type update. Both the prediction and the correction steps are merged together in an efficient single update, and are exploited the sequential character of the system dynamic model.

The existing results on IPA-SQP, however, cannot be directly applied to the minimum-time MPC problem since the time horizon does not stay constant over time. We, therefore, employ a time scaling transformation to obtain a fixed end time problem, with the terminal time appearing as a multiplicative parameter in the dynamic equations of the continuous-time model. We then convert the model to discrete-time and formulate an optimization problem where both the control sequence and the time horizon, now appearing as a parameter, have to be optimized. The reformulated variant of the problem fits nicely into a parametric IPA-SQP algorithm developed in Ghaemi et al. (2010). This parametric IPA-SQP algorithm is applicable to simultaneous optimization of the control sequence and a parameter. See Gao et al. (2011) for an application of parametric IPA-SQP to ship path following. Since appropriate regularity properties need to be satisfied to be able to apply the IPA-SQP algorithm, the cost functional is regularized.

The paper is organized as follows. In Section 2, we describe the minimum-time MPC problem, the time scaling transformation and the cost regularization steps necessary to reformulate the problem to make IPA-SQP algorithm in Ghaemi et al. (2010) for simultaneous optimization of control time history and a parameter applicable. The IPA-SQP algorithm is described in Section 3. Two examples are considered. The first example is of a double integrator with a control constraint. This example is treated in Section 4. The results are compared with the minimum-time open-loop and closed-loop control, since in this case the minimum-time control can be easily constructed. The second example, treated in Section 5, is an application of minimum-time MPC to a two dimensional hypersonic vehicle flight model from Jorris (2007) with a control constraint (a similar model arises in generating Dubins' paths for mobile robots and aircraft (Souères and Boissonnat (1998)).

2. PROBLEM FORMULATION AND TRANSFORMATION

The minimum-time MPC is based on solving a minimum-time optimal control problem for steering the system from the current state x_0 at the current time t_0 to the origin at the terminal time t_f:

$$\min \; J = t_f, \tag{1}$$

subject to

$$\begin{aligned} &\dot{x}(t) = f(x(t), u(t)),\\ &x(t_0) = x_0,\\ &x(t_f) = 0,\\ &C(x(t), u(t)) \leq 0, \quad t_0 \leq t \leq t_f, \end{aligned} \tag{2}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are state and control input, respectively. Here, C defines constraints. To be able to apply the IPA-SQP framework, the cost function is regularized by augmenting a control penalty. The cost is modified to

$$\min \; J = t_f + \epsilon \int_{t_0}^{t_f} u(t)^{\mathrm{T}} u(t) dt, \tag{3}$$

where ϵ is a small positive scalar number, i.e., $\epsilon \in \mathbb{R}_+$. This cost must be minimized subject to (2).

A time scaling transformation is now employed to convert the free terminal time problem to a fixed terminal time problem,

$$\tau = \frac{t - t_0}{t_f - t_0}. \tag{4}$$

Since $t_0 \leq t \leq t_f$, it follows that

$$0 \leq \tau \leq 1. \tag{5}$$

The dynamics of the system is then expressed as

$$x' \triangleq \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = (t_f - t_0) f(x, u). \tag{6}$$

The transformed model (6) is converted to discrete-time,

$$x(k+1) = x(k) + \Delta\tau (t_f - t_0) f(x(k), u(k)), \tag{7}$$

where $\Delta\tau = 1/N$, and N is the number of control nodes employed in discretizing the trajectory. The cost functional in (3) is then converted to

$$J_d = t_f + \epsilon \sum_{k=0}^{N-1} \Delta\tau (t_f - t_0) u(k)^{\mathrm{T}} u(k). \tag{8}$$

The adjustable variables are the control time sequence, $\{u(0), u(1), \cdots, u(N-1)\}$ and the parameter $p = t_f - t_0$ that need to be simultaneously optimized.

As a final step of reformulating the problem, we replace the hard terminal constraint, $x(t_f) = 0$, by a penalty added to the cost (8) so that the cost being minimized becomes

$$\bar{J}_d = \rho x(N)^{\mathrm{T}} x(N) + p + \epsilon \sum_{k=0}^{N-1} \Delta\tau p u(k)^{\mathrm{T}} u(k), \tag{9}$$

where $\rho > 0$ is the penalty factor. This change is not essential but simplifies subsequent numerical implementation and mitigates potential terminal constraint infeasibility.

To summarize, the problem to which IPA-SQP framework will be applied has the following form,

$$\min_{u(\cdot), p} \bar{J}_d, \tag{10}$$

subject to

$$\begin{aligned} &x(k+1) = x(k) + p\Delta\tau f(x(k), u(k)),\\ &x(t_0) = x_0,\\ &C(x(k), u(k), t_f) \le 0. \end{aligned} \tag{11}$$

The minimum-time MPC is a feedback law $u_{MPC}(x_0)$ defined based on the solution of the above optimization problem with t_0 as the current time and x_0 as the current state. The number of control nodes, N, is maintained constant, and, consequently, the control corrections become finer as the state gets closer to the origin. To avoid control time subinterval becoming infinitesimally small after the convergence within a prescribed tolerance of the origin is achieved, the control is no longer recomputed, and an open-loop trajectory is simply followed to completion.

In the following section, we review the IPA-SQP algorithm from Ghaemi et al. (2010) for discrete-time systems with inputs, parameters and constraints.

3. REVIEW OF THE PARAMETRIC IPA-SQP ALGORITHM

The receding horizon optimization problem, at time instant t, treated by IPA-SQP has the following form,

$$\min_{u(\cdot),p} J(x(\cdot), u(\cdot), p),$$

where

$$J(x(\cdot), u(\cdot), p) = \Phi(x(t+N), p) + \sum_{k=t}^{t+N-1} L((x(k), u(k), p),$$

subject to

$$\begin{aligned} &x(k+1) = f(x(k), u(k), p),\ f : \mathbb{R}^{n+m+r} \to \mathbb{R}^n,\\ &x(t) = x_t,\ x_t \in \mathbb{R}^n,\\ &C(x(k), u(k), p) \le 0,\ C : \mathbb{R}^{n+m+r} \to \mathbb{R}^l,\\ &\qquad\qquad k = t, \ldots, t+N-1,\\ &\bar{C}(x(k), p) \le 0,\ \bar{C} : \mathbb{R}^{n+r} \to \mathbb{R}^q,\ k = t, \ldots, t+N, \end{aligned} \tag{12}$$

where C and $\bar{C}$ denote the mixed state-input constraints and state-only constraints, respectively. The parameters are included as optimization variables. We assume that the function L, f, Φ, C, and $\bar{C}$ are twice continuously differentiable with respect to their arguments.

Let $x^0(\cdot)$, $u^0(\cdot)$, and p^0 be the nominal optimal solution of (12). The Hamiltonian function is defined as

$$\begin{aligned} H(k) &= L(x(k), u(k), p) + \lambda(k+1)^{\mathrm{T}} f(x(k), u(k), p)\\ &\quad + \mu(k)^{\mathrm{T}} C^a(x(k), u(k), p) + \bar{\mu}(k)^{\mathrm{T}} \bar{C}^a(x(k), p), \end{aligned} \tag{13}$$

where $\lambda(\cdot)$ is the sequence of co-states associated with the dynamic equations, $C^a(x(k), u(k), p)$ and $\bar{C}^a(x(k), p)$ denote vectors consisting of the active constraints, and $\mu(k)$ and $\bar{\mu}(k)$ are the vectors of the corresponding Lagrange multipliers.

If the nominal solution $x^0(\cdot)$, $u^0(\cdot)$, and p^0 is optimal, the following necessary optimality conditions implied by Karush-Kuhn-Tucker (KKT) conditions hold

$$\lambda(k) = H_x(k),\ k = t, ..., t+N-1, \tag{14}$$

$$H_u(k) = 0,\ k = t, ..., t+N-1, \tag{15}$$

$$\lambda(t+N) = \Phi_x(x(t+N), p) + \bar{\mu}(N)^{\mathrm{T}} \bar{C}^a_x(x(t+N), p), \tag{16}$$

$$\sum_{k=t}^{t+N-1} H_p(k) + \Phi_p(x(t+N), p) + \bar{\mu}(N)^{\mathrm{T}} \bar{C}^a_p(x(t+N), p) = 0, \tag{17}$$

$$x(k+1) = f(x(k), u(k), p),\ f : \mathbb{R}^{n+m+r} \to \mathbb{R}^n, \tag{18}$$

$$x(t) = x_t,\ x_t \in \mathbb{R}^n, \tag{19}$$

$$\mu(k) \ge 0,\ k = t, ..., t+N-1, \tag{20}$$

$$\bar{\mu}(k) \ge 0,\ k = t, ..., t+N, \tag{21}$$

where H_x and H_u denote the partial derivative of H with respect x and u, respectively.

The IPA-SQP algorithm is based on NE optimal control to predict the change in the solution with the change in problem data which is the state of the system in our case. The NE solution (Bryson et al. (1975)) is the first-order correction that approximates the optimal state and control sequences for the perturbed initial state. Given the nominal optimal solution, the NE solution can be shown to minimize the second-order variation of the Hamiltonian function subject to the linearized constraints (i.e., it is a solution of the following problem subject to the linearized constraints):

$$\min_{\delta u(\cdot),\ \delta p} \delta^2 \bar{J},$$

where,

$$\begin{aligned} \delta^2 \bar{J} &= \frac{1}{2} \begin{bmatrix} \delta x(t+N) \\ \delta p \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \delta x(t+N) \\ \delta p \end{bmatrix}\\ &+ \frac{1}{2} \sum_{k=t}^{t+N-1} \begin{bmatrix} \delta x(k) \\ \delta u(k) \\ \delta p \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} H_{xx}(k) & H_{xu}(k) & H_{xp}(k) \\ H_{ux}(k) & H_{uu}(k) & H_{up}(k) \\ H_{px}(k) & H_{pu}(k) & H_{pp}(k) \end{bmatrix} \begin{bmatrix} \delta x(k) \\ \delta u(k) \\ \delta p \end{bmatrix}, \end{aligned} \tag{22}$$

subject to

$$\delta x(k+1) = f_x(k)\delta x(k) + f_u(k)\delta u(k) + f_p(k)\delta p, \tag{23}$$

$$\delta x(t) = \delta x_t, \tag{24}$$

$$\begin{aligned} &C^a_x(x(k), u(k), p)\delta x(k) + C^a_u(x(k), u(k), p)\delta u(k)\\ &\qquad + C^a_p(x(k), u(k), p)\delta p = 0, \end{aligned} \tag{25}$$

$$\bar{C}^a_x(x(k), p)\delta x(k) + \bar{C}^a_p(x(k), p)\delta p = 0, \tag{26}$$

where,

$$\Phi_{11} = \Phi_{xx}(x(t{+}N), p) + \frac{\partial}{\partial x}(\bar{\mu}(N)^{\mathrm{T}} \bar{C}_x(x(t{+}N), p)), \tag{27}$$

$$\Phi_{12} = \Phi_{xp}(x(t{+}N), p) + \frac{\partial}{\partial p}(\bar{\mu}(N)^{\mathrm{T}} \bar{C}_x(x(t{+}N), p)), \tag{28}$$

$$\Phi_{21} = \Phi_{px}(x(t{+}N), p) + \frac{\partial}{\partial x}(\bar{\mu}(N)^{\mathrm{T}} \bar{C}_p(x(t{+}N), p)), \tag{29}$$

$$\Phi_{22} = \Phi_{pp}(x(t{+}N), p) + \frac{\partial}{\partial p}(\bar{\mu}(N)^{\mathrm{T}} \bar{C}_p(x(t{+}N), p)). \tag{30}$$

The subscript letters in (22)-(30) denote the variables with respect to which the partial derivatives are taken.

We then obtain the closed-form solution of (22)-(26) for the initial state perturbation δx_t,

$$\begin{aligned} \delta u(k) &= K(k)\delta x(k) + K_p(k)\delta p,\\ \delta p &= -W_{22}^{-1}(t) W_{21}(t)\delta x_t. \end{aligned} \tag{31}$$

Detailed calculation and the expressions for $K(k)$, $K_p(k)$, $W_{22}(t)$, and $W_{21}(t)$ are found in Ghaemi et al. (2010).

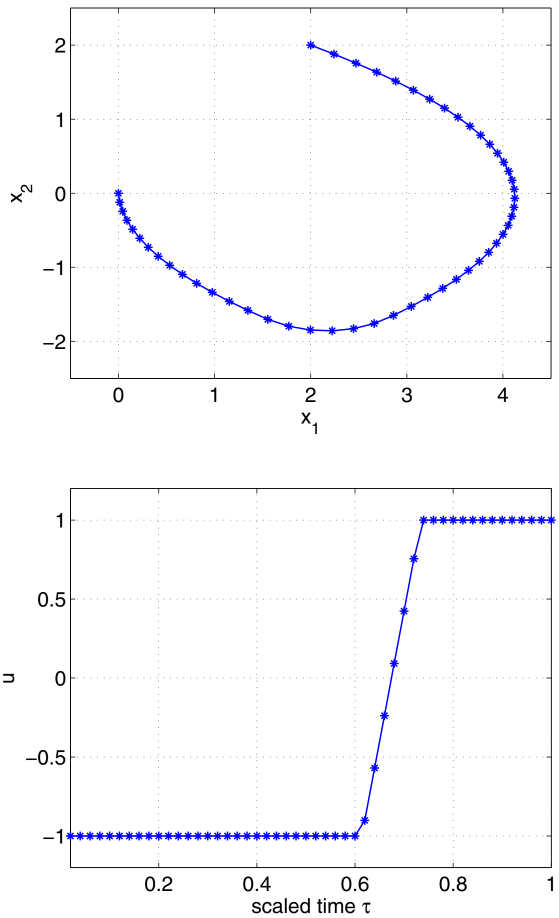

Fig. 1. Open-loop control by fmincon. Top: Phase plot of the state. Bottom: Control input history.

When δx_t is large and causes activity status changes in constraints, we handle such changes using the procedure in Ghaemi et al. (2007, 2009).

Finally, the IPA-SQP algorithm represents the modification of the predictor update (31) with a corrector update.

4. SIMULATIONS OF DOUBLE INTEGRATOR SYSTEM

To evaluate the performance of the IPA-SQP algorithm, we consider a double integrator system with control input constraints. The minimum-time MPC problem for transferring a nonzero initial state to the origin for the double integrator system has the form,

$$\min \bar{J}_d = \rho x(t+N)^{\mathrm{T}} x(t+N) + p + p\Delta\tau\epsilon \sum_{k=t}^{t+N-1} u(k)^{\mathrm{T}} u(k), \quad (32)$$

subject to

$$\begin{aligned} x(k+1) &= x(k) + p\Delta\tau(Ax(k) + Bu(k)), \\ x(t) &= x_t, \\ |u(k)| &\leq 1, \end{aligned} \quad (33)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \; A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \text{and } B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We choose the number of control nodes as $N = 50$, $\rho = 10^6$, and $\epsilon = 0.1$. The initial state is $x_0 = (2,\ 2)^{\mathrm{T}}$.

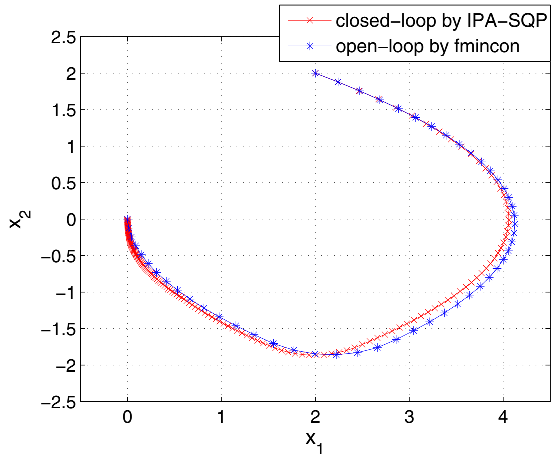

Fig. 2. Phase plots of the states.

4.1 Open-loop Control

Before implementing the IPA-SQP approach, we obtain the nominal optimal solution of (32)-(33) at the initial time using MATLAB nonlinear programming solver fmincon. Fig. 1 shows the phase plot of the open-loop state and control input sequences. The minimum-time of the maneuver is $p^0 = 6.083$ sec.

4.2 Closed-loop Control

Let (x^0, u^0, p^0) denote the solution from Section 4.1 approximated as a piecewise constant function in time with nodes at time instants t_0^0 , $t_1^0, \cdots,$ t_N^0. Then, the time interval between t_0^0 and t_1^0 is $p^0/N = 6.083/50 = 0.122$ sec.

Considering the initial state perturbation $\delta x_0^0 = x^0(t_1^0) - x^0(t_0^0)$ where x^0 denotes the measured state, the optimal control sequence and minimum-time are obtained for the next control cycle using the IPA-SQP approach,

$$\begin{aligned} u^1 &= u^0 + \delta u^0, \\ p^1 &= p^0 + \delta p^0. \end{aligned}$$

The nodes of u^1 at the next control cycle are t_0^1, $t_1^1, \cdots,$ t_N^1. The first element of the computed optimal control sequence u^1 is applied to the system between the time instant t_0^1 and t_1^1.

By repeating this computing procedure with the fixed number of control nodes, the optimal trajectory is obtained as shown in Fig. 2. In this simulation, the threshold time to terminate the algorithm for recomputing is 0.5 sec; this avoids infinitesimally spaced control nodes. Thus, if the minimum-time computed at a certain control cycle reaches the threshold time, the optimal control sequence obtained in the control cycle is applied in the open-loop without further recalculation.

In Fig. 2, the phase plot of the state of the open-loop control by fmincon and that of the closed-loop control by the IPA-SQP are compared. There are slight differences between two trajectories.

The time of the IPA-SQP trajectory is computed as

$$t_f^* = (t_N^s - t_0^s) + \sum_{j=0}^{s-1}(t_1^j - t_0^j), \tag{34}$$

where s is the first control cycle whose minimum-time is less than the threshold time. The maneuver time is 5.814 sec. It is less than the time for the open-loop minimum-time trajectory due to the control trajectory refinements effects. Specifically, IPA-SQP exploits the same number of nodes in each control cycle, thus the effective time interval between control changes decreases with time.

In Fig. 3, the trajectories by the minimum-time MPC based on IPA-SQP is presented as the solid line and the results of the minimum-time MPC based on MATLAB fmincon solver are illustrated by the dot line. Fig. 3 shows that the solutions by the IPA-SQP and fmincon in closed-loop control are quite similar in performance.

The total computation time of the IPA-SQP, however, is 26.4 sec while the computation time of fmincon is 472.8 sec. Thus, the IPA-SQP improved computational efficiency. The total computation time is measured by CPU time usage. The simulations are performed by the controller codes implemented in MATLAB on a computer with Intel® CPU @ 2.10 GHz.

5. MINIMUM-TIME MPC FOR HYPERSONIC VEHICLE FLIGHT

In this section, the parametric IPA-SQP is applied to a two-dimensional flight model (Jorris (2007)) for a hypersonic glider. The equations of motion of the two-dimensional hypersonic glider model are given in Jorris (2007); Baldwin and Kolmanovsky (2013),

$$\dot{x} = V\cos(\beta),\ \dot{y} = V\sin(\beta),\ \dot{\beta} = \frac{\tan(\alpha_{max})}{V}u,\ \dot{V} = a, \tag{35}$$

where x and y denote the glider position in x and y direction, β is the glider heading angle, V is the velocity of the vehicle, a is the acceleration, α_{max} is the maximum bank angle, and u, which is constrained as $-1 \le u \le 1$, is the normalized bank angle control signal.

Note that the variables in this flight model are scaled:

$$\begin{aligned} &x = \frac{x_u}{r_0},\ y = \frac{y_u}{r_0},\ \beta = \beta_u, \\ &V = \frac{V_u}{\sqrt{g_0 r_0}},\ a = \frac{a_u}{g_0},\ t = \frac{t_u}{\sqrt{r_0/g_0}}, \end{aligned} \tag{36}$$

where r_0 is the radius of the Earth, g_0 is the gravitational acceleration, x_u, y_u ,β_u, V_u, a_u, and t_u are the x-coordinate (km), the y-coordinate (km), heading angle (rad), velocity (km/s), acceleration (km/s^2) and time (s) in physical units.

We consider a minimum-time problem to maneuver the glider to the origin subject to the control constraint. The values $a = 0$ and $\alpha_{max} = 10\ deg$ were used. Fig. 4

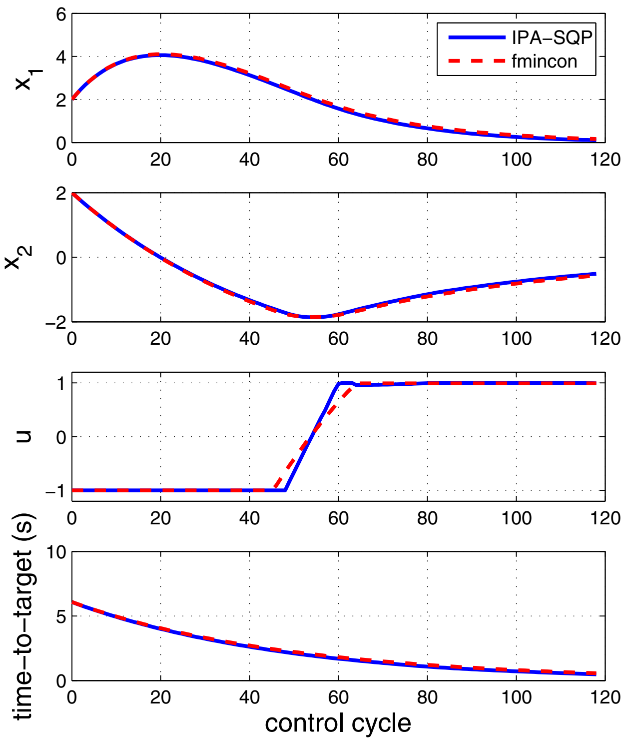

Fig. 3. Closed-loop control of the minimum-time MPC. From top to bottom: Trajectories of x_1, x_2, control input, and minimum-time.

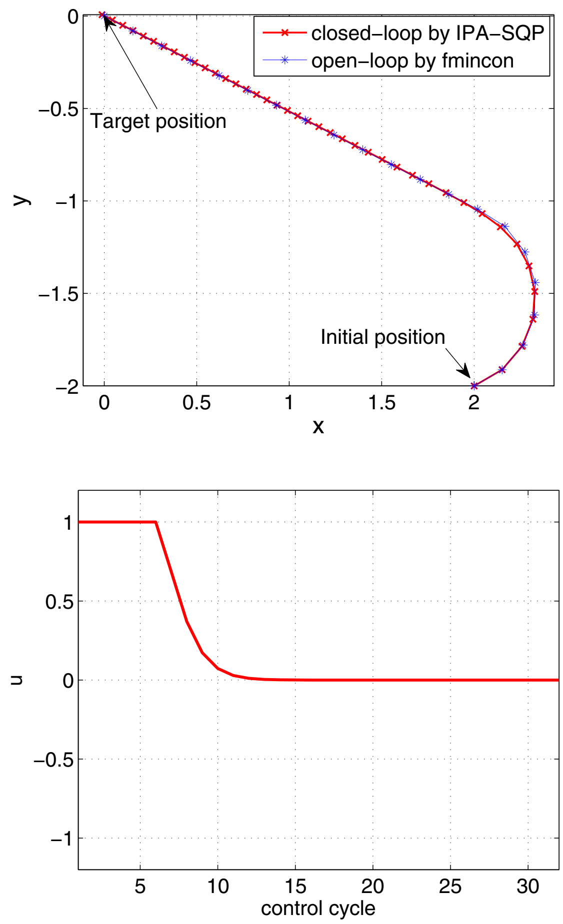

Fig. 4. Trajectories of the hypersonic vehicle model. Top: Phase plot of the state, Bottom: Control input history by IPA-SQP.

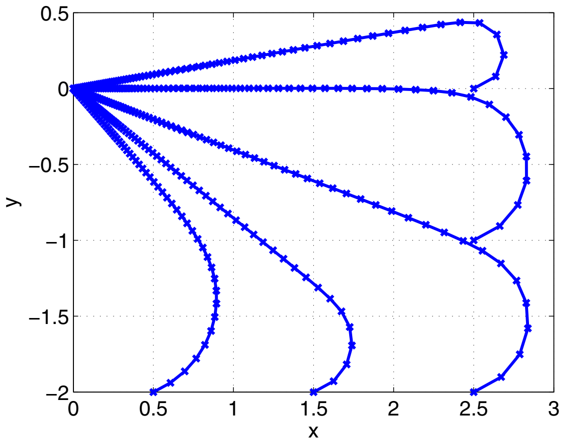

Fig. 5. Trajectories from various initial positions with the same heading angle.

illustrates the simulation results for the initial position of $(2, -2)$, the velocity of 0.3 and the initial heading angle $\beta = 30$ *deg*. The number of the control nodes is chosen as $N = 20$. The minimum-time t^* is 11.656, translating to 9398 sec in unscaled time.

Fig. 5 illustrates the trajectories on the $x-y$ plane starting from various initial positions with the same heading angle of 30 *deg*, and approaching the same terminal position $(0, 0)$ in minimum time.

6. CONCLUSION

In the paper, we considered an application of an IPA-SQP algorithm to a minimum-time nonlinear MPC problem. Minimum-time MPC is of interest due to its ability to perform way point following, improve robustness to model uncertainties and disturbances, satisfy constraints, and provide automatic control refinements near the target. On the other hand, the IPA-SQP algorithm provides a mechanism for fast control updates. In the paper, we have shown that the minimum-time MPC problem can be appropriately transformed to make the IPA-SQP algorithm applicable. The double integrator example was considered where we showed computational advantages of performing control updates using IPA-SQP over MATLAB nonlinear programming solver fmincon. Another example of a nonlinear system corresponding to a two dimensional model of a hypersonic glider has also been treated.

REFERENCES

M. Athans and P. Falb. *Optimal control*, Dover, 2006.

J. Ben-Asher. *Optimal control theory with aerospace applications*, AIAA Education Series, 2010.

L. Bako, D. Chen, and S. Lecoeuche. A numerical solution to the minimum-time control problem for linear discrete-time systems. *arXiv preprint arXiv*, 1109.3772, 2011.

M. Baldwin and I. Kolmanovsky. Hypersonic glider guidance using model predictive control. *American Control Conference*, Washington DC, USA, 2013.

A. Bryson and Y. Ho. *Applied optimal control: optimization, estimation, and control*, Taylor & Francis, 1975.

E. Camacho and C. Bordons. *Model Predictive Control*, Springer, 2004.

R. Ghaemi. *Robust model based control of constrained systems*, Ph.D. dissertation, The University of Michigan, 2010.

R. Ghaemi, J. Sun, and I. Kolmanovsky. Model predictive control for constrained discrete time systems: An optimal perturbation analysis approach. *American Control Conference*, New York City, USA, 2007.

R. Ghaemi, J. Sun, and I. Kolmanovsky. Overcoming singularity and degeneracy in neighboring extremal solutions of discrete-time optimal control problem with mixed input-state constraints. *Proc. IFAC World Congress*, 2009.

R. Ghaemi, J. Sun, and I. Kolmanovsky. An integrated perturbation analysis and sequential quadratic programming approach for model predictive control. *Automatica*, vol. 45, no. 10, pp. 2412–2418, 2009.

R. Ghaemi, J. Sun, and I. Kolmanovsky. A neighboring extremal approach to nonlinear model predictive control. *8th IFAC Symposium on Nonlinear Control Systems*, pp. 747–752, Italy, 2010.

L. Gao, R. Ghaemi, and J. Sun. Nonlinear adaptive model predictive control for ship path-following using parametric neighboring extremal approach. *11th International Conference on Fast Sea Transportation*, Honolulu, USA, 2011.

Z. Gao. On discrete-time optimal control: A closed-form solution. *American Control Conference*, Boston, USA, 2004.

T. Jorris. *Common aero vehicle autonomous reentry trajectory optimization satisfying waypoint and no-fly zone constraints*, Ph.D. dissertation, Air Force Institute of Technology, 2007.

R. Kalman. Optimal nonlinear control of saturating systems by intermittent action. *IRE Wescon Convention Record*, vol. 4, pp. 130–135, 1957.

S. Keerthi and E. Gilbert. Computation of minimum-time feedback control laws for discrete-time systems with state-control constraints. *IEEE Transactions on Automatic Control*, vol. 32, no. 5, pp. 432–435, 1987.

D. Mayne and W.R. Schroeder. Robust time-optimal control of constrained linear systems. *Automatica*, vol. 33, no. 12, pp. 2103–2118, 1997.

C. Petersen, M. Baldwin and I. Kolmanovsky. Model predictive control guidance with extended command governor inner-loop flight control for hypersonic vehicles. *AIAA Guidance, Navigation and Control*, Boston, USA, 2013.

P. Souerès and J. Boissonnat. Optimal trajectories for nonholonomic mobile robots. *Robot motion planning and control*, pp 93–170, Springer, 1998.

J. Starek and I. Kolmanovsky. Nonlinear model predictive control strategy for low thrust spacecraft missions. *Optimal Control Applications and Methods*, DOI: 10.1002/oca.2049, 2012.