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Balanceability Analysis of GSS Systems with Unit Heterogeneity*

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Abstract: The generalized switched server (GSS) system model has been proposed to address the task balancing control problems of the multi-units systems with control constraints. Similar to the stability of a control system, the balanceability of a GSS system investigates whether the system could be driven to a task balancing status or not. The heterogeneous GSS systems, in which the dynamical models of the multiple parallel units are not the same, are commonly confronted in practice. This paper mainly concerns with the system balanceabililty analysis, especially, the FRR switching policy for heterogeneous systems. The GSS system model and the related switching control policy are first introduced, then the balanceability problems are elaborated, and finally a sufficient condition for the balanceability of the FRR policy for the heterogeneous systems is presented.

Keywords: GSS systems, Switching control policy, Balanceability, Task balancing, Hybrid dynamic systems, Control constraint.

1. INTRODUCTION

Task balancing problems widely exist in engineering fields, such as in petrochemical and petroleum refinery industries (Cheng et al, 1999, Friedman 1994, Garg 1999, Herzog 2004, Li et al 1994, Wang & Zheng 2005, 2007, Zhang et al 2008), Internet and Web services (Cardellini et al 2002, Kim et al 2007, Liu & Lu 2004, Menasce 2002, Roubos & Bhulai 2009, Son et al 2009, Wu & Starobinski 2008), and water resources scheduling and management (Tricaud & Chen 2007), etc, where the concerned systems consist of multiple, say N, parallel units, each of which has a task input $Task_i$ and a load status output $Load\ Status_i$ respectively, i=1, 2, …, N. As shown in Fig. 1, the N task inputs are distributed from the system task input $Task_{source}$, and there is a control constraint

$$\sum_{i=1}^{N} Task_i = Task_{source} \ . \tag{1}$$

In order to realize the balancing control of the systems with the above constraint, the generalized switched server system model (GSS) and some control policies, such as the FRR, the FMM, the aMM, and the FSS policy, have been proposed (Wang 2008, Wang & Zheng 2006, 2009). Essentially speaking, the GSS system is a switched control system (Cheng et al 2005, Jiang & Voulgaris 2009, Liberzon & Morse 1999, Lin & Antsaklis 2009, Shorten et al 2007, Sun & Ge 2005, Wu et al 2009, Xiang & Wang 2009, Xie & Wang 2009, Xie & Wu 2009), and the basic switching unit is a subsystem that consists of two units and a continuous controller. Based on event generation and driving rules, discrete events are generated through quantized observation of the system output, and the system control is driven by the discrete events.

For balancing control, the balanceability of the control policy deserves efforts, which investigates whether a GSS system could be driven to a balancing status by the control policy. Wang (2008) has obtained some conclusions on the balanceability of the FRR policy for system homogeneity cases where the dynamic models of the multiple parallel units are the same. In practice, it probably appears the situation where not all of the structures of the parallel units are identical, and further, the dynamical models of multiple units may differ from each other even if they have the same structures, due to that the dynamical models may change with the operating point. So, it is necessary to address the problems for the system heterogeneity cases.

This paper mainly studies the balanceabililty of the FRR policy for heterogeneous GSS systems. For the cases where the number of the parallel units is three, a sufficient condition for the balanceability is obtained. The rest of the paper is organized as follows. In Section 2, the GSS model and some control policies are introduced. Section 3 formulates the balanceability problem of the control policy and discusses the heterogeneity issues. Section 4 elaborates the balanceability of the FRR policy for the heterogeneous cases. Finally, some concluding remarks are presented in Section 5.

2. GSS SYSTEM MODEL AND SWITCHING POLICY

A subsystem that consists of two units and a continuous controller is employed as the basic switching unit. The control structure of the subsystem is based on the difference control technique (Li et al 1994, Wang & Zheng 2005). For the convenience of the narration, throughout the following paper, the basic unit subsystem is denoted as the *DCT*,

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especially as the DCT_{ij} when the subsystem is composed of units i and j, i, j =1, 2, …, N.

For a subsystem DCT, the difference of the load statuses of the two units $Diff$ is controlled by the continuous controller under a zero reference, and the control target is to make the two statuses identical. The output of the controller, Dev, is added to and subtracted from the two task input setvalues simultaneously, which guarantees that the sum of the two inputs does not vary with the Dev. Fig. 2 shows a subsystem DCT_{12}, where the task inputs of the two units are given by

$$\begin{cases} Task_1 = T_{s1} + Dev \\ Task_2 = T_{s2} - Dev \end{cases}. \tag{2}$$

For the GSS systems, only two units are controlled at a instant, and the others are not controlled temporarily. With time evolving, different units are selected to control based on certain control policy, and thus, all the N units can be controlled in a time-sharing manner. A schematic diagram of the GSS system with the basic unit being DCT_{12} is shown in Fig. 3, where $Dev_1 = -Dev_2$, and $Dev_3 = \ldots = Dev_N = 0$.

Now, the following two questions naturally arise: 1) when does the DCT_{ij} switch? and 2) how does it? In fact, different answers to the questions 1) and/or 2) generate the corresponding switching policies. Based on the answers to the two questions, Wang & Zheng (2006) have proposed the FRR, FMM, and aMM policies. Here the FRR policy is briefly reviewed.

For the FRR switching policy, the answer to question 1) is that the DCT switches when it reaches the balancing status, and the answer to question 2) is that the DCT switches in a round-robin manner. That is, at initial time $t = T_0 = 0$, some two adjacent units, say units 1 and 2, are selected to control. At time T_n, n = 1, 2, …, assume that the indices of the two units that are in control are i and i+1 respectively, i.e., the basic unit subsystem is $DCT_{(i)(i+1)}$. From the time T_n, the control run for the units i and i+1 continues to the time T_{n+1} when the $DCT_{(i)(i+1)}$ reaches to the balancing status. At time T_{n+1}, the basic unit is switched to $DCT_{(i+1)(i+2)}$ for i = 1, 2, …, N-2; or $DCT_{(N)(1)}$ for i = N-1; or DCT_{12} for i = N. The regulation and the evolution procedures of the basic unit are repeated like this, and the switching sequence is given by

$$\begin{aligned} & DCT_{12} \to DCT_{23} \to \cdots \quad \cdots \\ & \qquad \to DCT_{(N-1)(N)} \to DCT_{(N)(1)} \to DCT_{12} \end{aligned}. \tag{3}$$

3. BALANCEABILITY PROBLEM FORMULATION

The balanceability of a switching control policy addresses the problem whether the policy could drive the GSS system to reach a balancing status, and here is, as shown in Fig. 4, formulated as that, under a zero input ($Task_{source}$ =0), for the given initial system status $\boldsymbol{L_0}=[l_{01}, l_{02}, \ldots, l_{0N}]$, where the l_{0i} is the initial status of the unit i, i=1, 2, …, N, whether or not the given switching policy could drive the N units to have identical statuses. Denote

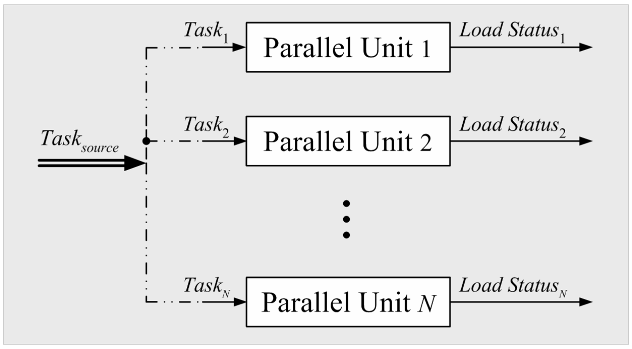

Fig. 1. A multiple units system, where the N inputs are distributed from the system input $Task_{source}$.

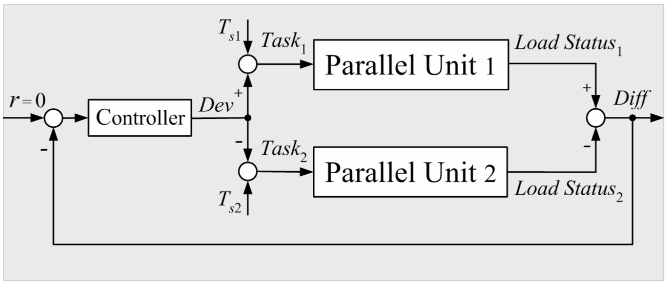

Fig. 2. A basic unit DCT_{12}, where the $Diff$ is controlled under a zero reference, and the Dev is added to the T_{s1} and subtracted from the T_{s2} simultaneously.

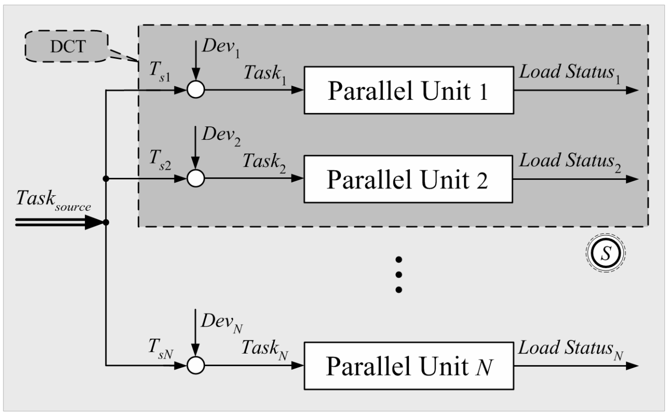

Fig. 3. A schematic diagram of the GSS system with the switching unit being DCT_{12}, where $Dev_1 = -Dev_2$, and $Dev_3 = \ldots = Dev_N = 0$.

$$L(t) = \left[l_1(t), l_2(t), \ldots, l_N(t) \right]^T \tag{4}$$

as the system status at time t, where the $l_i(t)$ is the load status of the unit i, i = 1, 2, …, N.

Definition 1: A switching policy is said to be balanceable, if the limit of (4)

$$\lim_{t\to\infty} L(t) = U \text{ , for } \forall L_0 = \left[l_{01}, l_{02}, \ldots, l_{0N} \right]^T , \tag{5}$$

where the $\boldsymbol{U}$ is a uniform vector whose components are all identical.

Definition 2: A GSS system is said to be *homogeneous*, if all the dynamical models of the N parallel units are identical.

Definition 3: A GSS system is said to be *heterogeneous*, if it is not homogeneous.

Definition 4: For a GSS system, the *heterogeneity factor* of a unit is defined as the ratio of the steady gain to the average of the N units. That is, the heterogeneity factor γ_i is given by

$$\gamma_i = K_i / (\frac{1}{N}\sum_{m=1}^{N} K_m) , \tag{6}$$

where K_i is the steady gain of unit i, $i = 1, 2, \ldots, N$.

4. BALANCEABILITY ANALYSIS

This section investigates the balanceability of the FRR policy for heterogeneity cases. It is first shown that the constraint (1) can be decoupled, and then the balanceability problem is elaborated.

4.1 Constraint Decoupling

Theorem 1: For the FRR policy, the system constraint can be decoupled, if it can at the initial time t_0.

Proof: From (2), it can be found that, for the basic unit DCT_{12}, the sum of the inputs of the two units is given by

$$Task_1 + Task_2 = T_{s1} + T_{s2} , \tag{7}$$

and it does not change with the output of the continuous controller, *Dev*. In addition, for the FRR policy, only some adjacent two units are selected to control during the given time slot, keeping the other N-2 inputs $Task_3$ to $Task_N$ be constant temporarily. Thus, the sum of the N inputs

$$\sum_{i=1}^{N} Task_i \tag{8}$$

does not change with the time. So, if the value of (8) is set to be the system input $Task_{source}$ at initial time t_0, Equation (1) holds, that is, the system input constraint is decoupled. ■

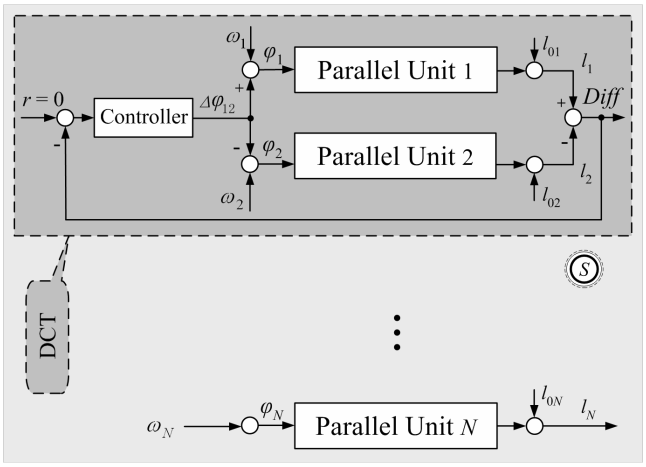

Fig. 4. A schematic diagram showing the balanceability problem formulation, where the φ_i is the input of unit i, and the ω_i is the holding value of the φ_i when the switching takes place, $i = 1, 2, \ldots, N$.

4.2 Balanceability of FRR Policy

Following the descriptions and the notations presented in Section 2, denote

$$DCT_{ij}^{\{k,k+1\}} \tag{9}$$

as the basic unit subsystem which is composed of units i and j, $i \neq j$, $i, j = 1, 2, \ldots, N$, during the time interval $(kh, (k+1)h)$, k=0, 1, 2, …, where the h is the control time slot. For the FRR control policy with the N being three, the evolution of basic unit (9), without loss of the generality, can be given by

$$\begin{cases} DCT_{12}^{\{6n,\ 6n+1\}}, \ t \in (6nh, (6n+1)h) \\ DCT_{23}^{\{6n+1,\ 6n+2\}}, \ t \in ((6n+1)h, (6n+2)h) \\ DCT_{31}^{\{6n+2,\ 6n+3\}}, \ t \in ((6n+2)h, (6n+3)h) \\ DCT_{12}^{\{6n+3,\ 6n+4\}}, \ t \in ((6n+3)h, (6n+4)h) \\ DCT_{23}^{\{6n+4,\ 6n+5\}}, \ t \in ((6n+4)h, (6n+5)h) \\ DCT_{31}^{\{6n+5,\ 6n+6\}}, \ t \in ((6n+5)h, (6n+6)h) \end{cases} , \tag{10}$$

where n=0, 1, 2, ….

Lemma 1: For a basic unit subsystem with zero input, if the subsystem can be controlled by its continuous controller, then, the identical statuses of the two units would be given by

$$\overline{l_1} = \overline{l_2} = \frac{\gamma_2 \cdot l_{01} + \gamma_1 \cdot l_{02}}{\gamma_1 + \gamma_2} , \tag{11}$$

where the l_{01} and l_{02} are the two initial statuses, and the γ_1 and γ_2 are the two heterogeneity factors accordingly.

Proof: Consider a subsystem with zero input (as shown in Fig. 5). From the condition, it is known that the two statuses $l_1(t)$ and $l_2(t)$ would be identical at and after the time t_s when the subsystem reaches to a balancing status, that is, $l_1(t) = l_2(t)$ for time $t > t_s$.

It can be found from Fig. 5 that $r_2(t) = -r_1(t)$ and

$$\begin{cases} l_1(t) = y_1(t) + l_{01} \\ l_2(t) = y_2(t) + l_{02} \end{cases} . \tag{12}$$

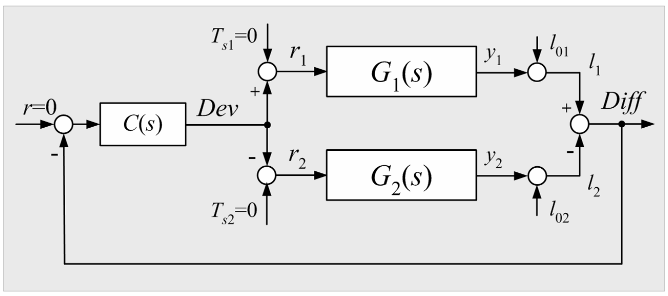

Fig. 5. A schematic diagram for the performance analysis of the basic switching unit subsystem.

In addition, for time $t > t_s$, the following holds.

$$y_i(t) = K_i \cdot r_i(t), \tag{13}$$

where the K_i is the steady gain of parallel unit i, $i = 1, 2$.

Considering (12), (13), that $l_1(t) = l_2(t)$, and that $r_2(t) = -r_1(t)$, it can be found that

$$l_1(t) = l_2(t) = \frac{\gamma_2 \cdot l_{01} + \gamma_1 \cdot l_{02}}{\gamma_1 + \gamma_2} \tag{14}$$

for time $t > t_s$. This completes the proof of the lemma. ■

Theorem 2: For a heterogeneous GSS system, if the basic unit subsystems can be controlled by the continuous controller during the time slot h, then, for the evolution procedures presented by (10), the system status (4) at time $t = kh$

$$L(t)|_{t=kh} \triangleq \left[l_1^{(k)}, l_2^{(k)}, l_3^{(k)} \right]^T \tag{15}$$

can be given respectively by

1) for $n = 0$,

$$\begin{cases} l_1^{(1)} = \alpha l_{01} + (1-\alpha) l_{02} \\ l_2^{(1)} = l_1^{(1)} \\ l_3^{(1)} = l_{03} \end{cases}, \tag{16-1}$$

where the α is given by $\alpha = \gamma_2 / (\gamma_1 + \gamma_2)$;

$$\begin{cases} l_1^{(2)} = \alpha l_{01} + (1-\alpha) l_{02} \\ l_2^{(2)} = \beta \cdot \alpha l_{01} + \beta \cdot (1-\alpha) l_{02} + (1-\beta) l_{03}, \\ l_3^{(2)} = l_2^{(2)} \end{cases} \tag{16-2}$$

where α is as above, and β is given by $\beta = \gamma_3 / (\gamma_2 + \gamma_3)$;

$$\begin{cases} l_1^{(3)} = [\gamma \cdot \alpha + (1-\gamma)\beta \cdot \alpha] l_{01} + [\gamma(1-\alpha) + \\ \qquad (1-\gamma)\beta \cdot (1-\alpha)] l_{02} + (1-\gamma)(1-\beta) l_{03} \\ l_2^{(3)} = \beta \cdot \alpha l_{01} + \beta \cdot (1-\alpha) l_{02} + (1-\beta) l_{03} \\ l_3^{(3)} = l_1^{(3)} \end{cases}, \tag{16-3}$$

where the α and β are as in (16-1) and (16-2), and the γ is given by $\gamma = \gamma_3 / (\gamma_1 + \gamma_3)$;

$$\begin{cases} l_1^{(4)} = [\alpha^2 \cdot \gamma + \alpha^2 (1-\gamma)\beta + (1-\alpha)\beta \cdot \alpha] l_{01} + \\ \qquad [\alpha \cdot \gamma(1-\alpha) + \alpha(1-\gamma)\beta(1-\alpha) + (1-\alpha)^2 \beta] l_{02} + \\ \qquad [\alpha(1-\gamma)(1-\beta) + (1-\alpha)(1-\beta)] l_{03} \\ l_2^{(4)} = l_1^{(4)} \\ l_3^{(4)} = [\gamma \cdot \alpha + (1-\gamma)\beta \cdot \alpha] l_{01} + [\gamma(1-\alpha) + \\ \qquad (1-\gamma)\beta(1-\alpha)] l_{02} + (1-\gamma)(1-\beta) l_{03} \end{cases} \tag{16-4}$$

$$\begin{cases} l_1^{(5)} = [\alpha^2 \cdot \gamma + \alpha^2 (1-\gamma)\beta + (1-\alpha)\beta \cdot \alpha] l_{01} + \\ \qquad [\alpha \cdot \gamma(1-\alpha) + \alpha(1-\gamma)\beta(1-\alpha) + (1-\alpha)^2 \beta] l_{02} + \\ \qquad [\alpha(1-\gamma)(1-\beta) + (1-\alpha)(1-\beta)] l_{03} \\ l_2^{(5)} = [\beta \cdot \alpha^2 \cdot \gamma + \beta^2 \cdot \alpha^2 (1-\gamma) + \beta^2 (1-\alpha)\alpha + \\ \qquad (1-\beta)\gamma \cdot \alpha + (1-\beta)(1-\gamma)\beta \cdot \alpha] l_{01} + \\ \qquad [\beta \cdot \alpha \cdot \gamma(1-\alpha) + \beta^2 \cdot \alpha(1-\gamma)(1-\alpha) + \\ \qquad \beta^2 (1-\alpha)^2 + (1-\beta)\gamma(1-\alpha) + \\ \qquad (1-\beta)(1-\gamma)\beta(1-\alpha)] l_{02} + \\ \qquad [\beta \cdot \alpha(1-\gamma)(1-\beta) + \beta(1-\alpha)(1-\beta) + \\ \qquad (1-\beta)^2 (1-\gamma)] l_{03} \\ l_3^{(5)} = l_2^{(5)} \end{cases} \tag{16-5}$$

$$\begin{cases} l_1^{(6)} = [\alpha^2 \cdot \gamma^2 + \gamma\alpha^2 (1-\gamma)\beta + \gamma(1-\alpha)\beta \cdot \alpha + \\ \qquad (1-\gamma)\beta \cdot \alpha^2 \cdot \gamma + \beta^2 \cdot \alpha^2 (1-\gamma)^2 + \\ \qquad (1-\gamma)\beta^2 (1-\alpha)\alpha + (1-\gamma)(1-\beta)\gamma \cdot \alpha + \\ \qquad (1-\beta)(1-\gamma)^2 \beta \cdot \alpha] l_{01} \\ \qquad + \\ \qquad [\gamma \cdot \alpha \cdot \gamma(1-\alpha) + \gamma \cdot \alpha(1-\gamma)\beta(1-\alpha) + \\ \qquad \gamma(1-\alpha)^2 \beta + (1-\gamma)\beta \cdot \alpha \cdot \gamma(1-\alpha) + \\ \qquad \beta^2 \cdot \alpha(1-\gamma)^2 (1-\alpha) + (1-\gamma)\beta^2 (1-\alpha)^2 + \\ \qquad (1-\gamma)(1-\beta)\gamma(1-\alpha) + \\ \qquad (1-\beta)(1-\gamma)^2 \beta(1-\alpha)] l_{02} \\ \qquad + \\ \qquad [\gamma \cdot \alpha(1-\gamma)(1-\beta) + \gamma \cdot (1-\alpha)(1-\beta) + \\ \qquad \beta \cdot \alpha(1-\gamma)^2 (1-\beta) + (1-\gamma)\beta(1-\alpha)(1-\beta) + \\ \qquad (1-\beta)^2 (1-\gamma)^2] l_{03} \\ l_2^{(6)} = [\beta \cdot \alpha^2 \cdot \gamma + \beta^2 \cdot \alpha^2 (1-\gamma) + \beta^2 (1-\alpha)\alpha + \\ \qquad (1-\beta)\gamma \cdot \alpha + (1-\beta)(1-\gamma)\beta \cdot \alpha] l_{01} \\ \qquad + \\ \qquad [\beta \cdot \alpha \cdot \gamma(1-\alpha) + \beta^2 \cdot \alpha(1-\gamma)(1-\alpha) + \\ \qquad \beta^2 (1-\alpha)^2 + (1-\beta)\gamma(1-\alpha) + \\ \qquad (1-\beta)(1-\gamma)\beta(1-\alpha)] l_{02} \\ \qquad + \\ \qquad [\beta \cdot \alpha(1-\gamma)(1-\beta) + \beta(1-\alpha)(1-\beta) + \\ \qquad (1-\beta)^2 (1-\gamma)] l_{03} \\ l_3^{(6)} = l_1^{(6)} \end{cases} \tag{16-6}$$

or 2) for $n \geq 1$,

$$\begin{cases} l_1^{(6n+1)} = \alpha[1 - \frac{c_1(1-c_2{}^n)}{1-c_2}] l_{01} + \\ \qquad (1-\alpha)[1 - \frac{c_1(1-c_2{}^n)}{1-c_2}] l_{02} + \frac{c_1(1-c_2{}^n)}{1-c_2} l_{03} \\ l_2^{(6n+1)} = l_1^{(6n+1)} \\ l_3^{(6n+1)} = \alpha[1 - \frac{c_1(1-c_2{}^n)}{1-c_2} - c_2{}^n] l_{01} + \\ \qquad (1-\alpha)[1 - \frac{c_1(1-c_2{}^n)}{1-c_2} - c_2{}^n] l_{02} + \\ \qquad [\frac{c_1(1-c_2{}^n)}{1-c_2} + c_2{}^n] l_{03} \end{cases} \tag{17-1}$$

where the c_1 and c_2 are as follows

$$\begin{cases} c_1 = (1-\beta)(1-\alpha\cdot\gamma)[1-\gamma(1-\beta)(1-\alpha)] \\ c_2 = \gamma^2(1-\beta)^2(1-\alpha)^2 \end{cases};$$

$$\begin{cases} l_1^{(6n+2)} = \alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]l_{01} + (1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]l_{02} + \frac{c_1(1-c_2{}^n)}{1-c_2}l_{03} \\ l_2^{(6n+2)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-(1-\alpha)(1-\beta)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)c_2{}^n\}l_{03} \\ l_3^{(6n+2)} = l_2^{(6n+2)} \end{cases} \quad (17\text{-}2)$$

$$\begin{cases} l_1^{(6n+3)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)(1-\gamma)c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-(1-\alpha)(1-\beta)(1-\gamma)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)(1-\gamma)c_2{}^n\}l_{03} \\ l_2^{(6n+3)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-(1-\alpha)(1-\beta)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)c_2{}^n\}l_{03} \\ l_3^{(6n+3)} = l_1^{(6n+3)} \end{cases}$$

(17-3)

$$\begin{cases} l_1^{(6n+4)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)(1-\alpha\cdot\gamma)c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}] -(1-\alpha)(1-\beta)(1-\alpha\cdot\gamma)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)(1-\alpha\cdot\gamma)c_2{}^n\}l_{03} \\ l_2^{(6n+4)} = l_1^{(6n+4)} \\ l_3^{(6n+4)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)(1-\gamma)c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-(1-\alpha)(1-\beta)(1-\gamma)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)(1-\gamma)c_2{}^n\}l_{03} \end{cases}$$

(17-4)

$$\begin{cases} l_1^{(6n+5)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)(1-\alpha\cdot\gamma) c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}] -(1-\alpha)(1-\beta)(1-\alpha\cdot\gamma)c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)(1-\alpha\cdot\gamma)c_2{}^n\}l_{03} \\ l_2^{(6n+5)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]- \alpha(1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma) c_2{}^n\}l_{01} + \{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]- (1-\alpha)(1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma) c_2{}^n\}l_{02} + \{\frac{c_1(1-c_2{}^n)}{1-c_2}+ (1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)c_2{}^n\}l_{03} \\ l_3^{(6n+5)} = l_2^{(6n+5)} \end{cases}; \quad (17\text{-}5)$$

$$\begin{cases} l_1^{(6n+6)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]-\alpha(1-\beta)[\gamma^2(1-\alpha) +(1-\gamma)(1-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)]c_2{}^n\}l_{01} +\{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]- (1-\alpha)(1-\beta)[\gamma^2(1-\alpha)+ (1-\gamma)(1-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)]c_2{}^n\}l_{02} +\{\frac{c_1(1-c_2{}^n)}{1-c_2}+(1-\beta)[\gamma^2(1-\alpha)+ (1-\gamma)(1-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)]c_2{}^n\}l_{03} \\ l_2^{(6n+6)} = \{\alpha[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]- \alpha(1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)c_2{}^n\}l_{01} +\{(1-\alpha)[1-\frac{c_1(1-c_2{}^n)}{1-c_2}]- (1-\alpha)(1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)c_2{}^n\}l_{02} +\{\frac{c_1(1-c_2{}^n)}{1-c_2}+ (1-\beta)(1-\gamma-\alpha\cdot\beta\cdot\gamma+\beta\cdot\gamma)c_2{}^n\}l_{03} \\ l_3^{(6n+6)} = l_1^{(6n+6)} \end{cases}; \quad (17\text{-}6)$$

Proof: The proof can be finished by using the mathematical induction approach, however, due to the page limit, the proof is omitted. ■

Theorem 3: For a heterogeneous system with the number of the units being three, the FRR policy is balanceable, if the basic unit subsystems can be balancing controlled by the continuous controller during the switching time slot h.

Proof: Consider the series of the system status (4) at the switching time $t = kh$, k=0, 1, 2, …, which can be completely disassembled into the following six sub-series.

$$L_{SS(p)}^{(k)} \triangleq L^{(6n+p)} = \left[l_1^{(6n+p)},\ l_2^{(6n+p)},\ l_3^{(6n+p)} \right]^T, \quad (18)$$

where p=1, 2, …, 6, and n = 0, 1, 2, …

The proof can be finished by analysing the above sub-series and Theorem 2, here due to page limit, the remaining proof is omitted. ■

5. CONCLUSIONS

This paper has investigated the GSS system related issues, with a focus on the balanceabililty of the FRR policy for the system heterogeneity cases. In the paper, it has been shown that, for the cases where the number of the units is three, the FRR policy is balanceable, and accordingly, a sufficient condition for the balanceability of the FRR policy for the heterogeneous GSS systems has been obtained.

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