# Jobs Pre-Allocation on Parallel Unrelated Machines with Sequence Dependent Setup Times: Evidence from a Large Experimentation 

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#### Abstract

In this paper the problem of allocating and scheduling jobs on parallel unrelated machines is studied. Jobs are grouped in families of similar items. A sequence dependent setup is required between batches of jobs belonging to the same and different families, even if in the first case lower time is required. The size of batches is not known a-priori, hence the problem is divided in two different sub problems: a) the allocation of volumes of work on each machine and b) subsequently the scheduling of each item. The focus of the paper is on the first step and consequently on the pre-assignment problem. Three different solving approaches are implemented in several real-life case studies.


Keywords: Manufacturing line, scheduling, parallel unrelated machines, sequence-dependent setup times

## 1. INTRODUCTION

The problems of allocation and scheduling of works on available machines are research areas attracting interests of technicians and academicians, since they influence the capacity of companies to reach their production goals and consequently satisfy customers.

In this paper, a make-to-stock environment is analysed. Items are produced in batches, whose size is not known a-priori, but computed in accordance with demand forecasting data. Specifically, the batch size is defined before each production running, in accordance with the necessity of either reducing items stocked in the warehouse (and consequently reducing the bacth size) or the necessity of minimizing set-up (and consequently augmenting the batch size, in order to limit production stoppages along with the requests of efforts of operators dedicated to production changes). Obviously, the previous optimization occurs while respecting customers demand.

Given the complexity of the analysed operative environment, as similarly as Arnaout et al. (2010), the problem is divided into two different sub-problems: a) the allocation of volumes of work to the available machines and then b) the scheduling that is the sequencing of jobs on each machine. Such a subdivision of the solving algorithm does not assure to reach the optimal solution, but allows the management of numerous and unusual constraints imposed both in the analysed case study and in numerous applications, whose an example is described in Gamberini et al. (2011).

This paper focuses on the first step and, for solving it, a mathematical model and two heuristic algorithms are presented. Moreover, a comparison is carried out, in accordance with data obtained in a large experimentation.

Section 2 presents a literature review, in section 3 the aforementioned solving approaches are briefly presented. Experimental design and computational results are reported in section 4 , while section 5 concludes the paper.

## 2. LITERATURE REVIEW

Jobs scheduling on parallel machines has received considerable attention from researchers. Published results are summarized in several survey papers, i.e. Liaee and Emmons (1997), Yang and Liao (1998), Allahverdi et al. (1999, 2008), Cheng at al. (2000), Potts and Kovalyov (2000), Akyol and Bayhan (2007), Kai and Shanlin (2008), Edis et al. (2013), Tavares Neto and Godinho Filho (2013).

By following the notation proposed in Lawler et al. (1993), scheduling problems are categorized in accordance with shop type, setup information, shop conditions, and performance criteria. The first field describes analysed shop environments: single machine, parallel machines, flow shop, job shop and open shop. In this paper, parallel unrelated machines are considered. The second field discriminates between sequence dependent and sequence independent setup times, along with it highlights cases where setups are not considered. In this paper sequence dependent setup times are studied. Furthermore, the necessity of determining batches dimensions, not given by the customer, arises. There are several performance criteria adopted in different published works, i.e. minimization of total completion time, minimization of makespan and minimization of the number of tardy jobs. In this paper four criteria are selected: the minimization of machine idle capacity, the minimization of the number of batches subdivided between different available machines, the minimization of the unsatisfied demand and the
minimization of the amount of items produced too in advanced.

By summarising, this paper studies optimizing methodologies for a complex manufacturing operative environment, that, to the best of our knowledge, none published research, except for the pioneering work reported in Gamberini et al. (2011), has explored. By starting from the mathematical model and heuristic algorithms reported in Gamberini et al. (2011), a large experimentation is carried out, in order to evaluate their behaviour in a wide set of working conditions.

## 3. PROBLEM FORMULATION AND SOLUTION APPROACHES

In this section, the operative environment analysed is presented in order to underline those features that lead to specific choices during the development of the solving approaches and the experimentation.

First of all, the production environment can be considered to be divided into three different plants, because the products are divided into three main families completely independent from one another (Fam A, Fam B, Fam C). In each department parallel unrelated machines are present. The production cycles of the manufacturing items can be executed only on a subset of the available resources. In particular, mono-line items can be processed only by one specific machine, while switch-line items con be allocated to a subset of the machines containing more than one element. Moreover, each machine requires a pre-specified operators team for assuring its correct run. Pre-defined production equipment is needed as well, so the production of an item is strictly related to its availability. Sequence dependent setup times occur between batches, and each setup requires a prespecified operators team for its correct execution. Furthermore, due to union agreements signed, only a predefined number of setups can be fulfilled each day and each week.

As similarly as in Gamberini et al. (2011), a make-to-stock environment is studied, where production planning has a degree of freedom in the definition of batch size, not strictly related with customer orders. On one hand, the unwillingness to reach the stockout lead to the production of large batches. On the other hand, the increment of holding cost leads to favour batches of small dimensions. Consequently, in the definition of the production batches, a trade-off between opposite necessities is present. Another constraint related to the batch size is its minimum dimension, fixed in order to justify time and resources required for the setup.

For solving the problem, a two-step methodology is proposed. The first step regards the allocation of volumes to machines and the second step is composed by the sequencing of the volumes allocated. Three approaches for managing the first step are presented in detail in Gamberini et al. (2010). Nevertheless, in the following, main characteristics useful for comprehending the experimentation are reported. Specifically, a mathematical model, subsequently solved by a commercial solver and two heuristics ( H 1 and H 2 ) are presented.

In the mathematical model:

- the planning horizon is divided into time buckets of known length;
- the objective function is composed by four different addends that are minimized: the remaining capacity of each machine, the number of volumes split, the unsatisfied demand and the early demand. The last three factors are multiplied by parameters $(a, b, c)$ that have to be set properly in order to give the right priority to objectives, in accordance with what suggested by company managers;
- backorders are managed, along with the possibility of anticipating the products manufacturing, during idle capacity of machines.

In heuristic H1:

- the objectives considered are both the balanced work-load of the machines and the integrity of switch items volumes in order to minimize the total setup number;
- main steps include: 1) the allocation of the mono-line items, 2) the allocation of switch-line items, 3) the reallocation of small volumes, in order to improve the objective functions;
- the possibility of producing in advance some items during periods with idle capacity is disregarded, along with backorders.

In heuristic H2:

- the possibility of producing in advance a portion of the volume is included. Nevertheless, such an advance is allowed only for one period;
- backorders are not managed;
- the algorithm is composed by several steps, whose main actions are cited in the following: 1) allocation of mono-line items, 2) creation of a list of switch-line items in descending order of volume, 3) creation of a list of machines in increasing order of saturation coefficient, 4) allocation of switch-line items without allowing the split of volumes, 5) allocation in advance of the left orders and finally 6) allocation of residual works by allowing switching.


## 4. EXPERIMENTATION

The aforementioned solving approaches are experimentally compared in order to identify the methodology that combines a short solution time with an efficient allocation.

A data set with real-life data, that have been subsequently modified in order to simulate stressful periods and accidents that could also occur in a company, is used. A range of $[2,5]$ switch items out of the total [28,36], allocated on 5 machines, is manufactured in FAM A department. In the FAM B department a set of $[68,98]$ items are produced; a range of $[10,11]$ switch items is included and 6 machines are available. In the FAM C department $[59,69]$ items are manufactured. A set of [10,11] switch items is present again and manufactured on 11 machines.

The analysis has been divided into three different sections The first one where all three approaches are compared, the second where just the two heuristics are taken under consideration and the last one where the parameters fitting for the assignment model is evaluated.

Per each department, different operative scenarios are studied, whose characteristics are described in the following:

- the capacity of machines. Given an initial setting (C0), the available capacity is decreased by a $15 \%$ percent, initially on all machines (C1) and subsequently only on half of them (C2). Alternatively, the available capacity is decreased by a $15 \%$ on all the machines, but only during one time bucket (C3).
- the orders. Given an initial setting (O0), three different modifications have been introduced: O1) increase of the volumes to be produced by a $5 \%$ for all of the items in all three time buckets, O2) increase of the volumes to be manufactured by a $15 \%$ for all of the items but just for one time bucket and O3) increase by a $15 \%$ for the $40 \%$ percent of the items for all three time buckets.
- number of machines suitable for producing switch line items. Given the initial situation (PM0), the number of available machines is increased by three (PM1) and five (PM2) units. However, this modification is implemented only in FAM B and FAM C departments. In FAM A zone all the switch items were already assigned to all the switch lines.

Finally, per each scenario, 5 problems are solved.
In order to compare solutions obtained, by the mathematical model and the two heuristics H 1 and H 2 , four performance indicators are computed:

- Average Number of Split Volumes (ANSV), that evaluates the average number of fractioning per switch item in the scheduling horizon. When each item is allocated to one machine per time bucket, ANSV assumes the value 1. Otherwise an increasing value is recorded.
- Average Early Switch Demand (AESD): average percentage of switch demand produced in advance.
- Average Switch Filling Rate (ASFR): average saturation coefficient of the switch lines in each time period.
- Average Filling Rate (AFR): average saturation coefficient of all machines in each time period.


### 4.1 Experimentation - Part I: comparison of the three solving approaches

Initially, the three solving approaches are compared. In the mathematical model, the penalty parameters have been set as described in the following: $a$ and $c$ assume value 1 , otherwise $b$ assumes value 98 , since the primary goal of the company is not to have unsatisfied demand and completely follow customers' requests.

Performance parameters are depicted in figures 1, 2, 3, 4 and 5. Concerning the solution time, for the mathematical model the average needed time is 30 minutes. Alternatively, in regards to the two heuristics, up to 3 seconds are required.

As depicted in figure 1, ANSV register the lowest value with H 2 . The mathematical model devotes its efforts to the minimisation of unsatisfied demand and consequently does not reaches the best solution anymore.

As depicted in figure 2, the augmented capacity of splitting volumes demonstrated by the mathematical model in figure 1, is associated with the best performance as concerns AESD.

The heuristic H 1 does not allow the anticipation of volumes, hence AESD is always equal to zero. However, as depicted in figures 3 and 4, ASFR and AFR do not register consistent modifications, by changing the solving approach.

As a consequence, the mathematical model and the heuristic H2 appear as the best performing approaches. However, the heuristic H 2 is characterised by shorter solving time, hence particularly addressed in changeable environments where rearrangements in scheduling are required.

Figure 1. Trend of ANSV in Part I of the experimentation


Figure 2. Trend of AESD in Part I of the experimentation


Figure 3. Trend of ASFR in Part I of the experimentation

|  | ASFR |
| :---: | :---: |
| 94,00\% |  |
| 92,00\% |  |
|  |  |
| 88,00\% |  |
|  |  |
| 86,00\% | 84,00\% |
| 80,00\% |  |
|  |  |  |
| 78,00\% |  |
|  |  |  |
| $74,00 \%$ |  |
| 72,00\% |  |
|  |  |

Figure 4. Trend of AFR in Part I of the experimentation


### 4.2 Experimentation - Part II: comparison of heuristics H1 and H 2

Similar considerations are traced by comparing H1 and H2.
Results obtained are depicted in figures 5, 6, 7, 8 .

### 4.3 Experimentation - Part II: Effects of alternative penalty parameters in the mathematical model

The penalty parameters have a consistent effect on the behaviour of the mathematical model. In this paragraph, such an impact is explored.

Figure 5. Trend of ANSV in Part II of the experimentation


Figure 6. Trend of AESD in Part II of the experimentation


Figure 7. Trend of ASFR in Part II of the experimentation

|  | ASFR |
| :---: | :---: |
| 94,00\% |  |
| 92,00\% $\square$ |  |
|  |  |
| 88,00\% |  |
| $84,00 \%$ |  |
|  |  |
| 82,00\% |  |
| 80,00\% |  |
|  |  |

Figure 8. Trend of AFR in Part II of the experimentation


Four different combinations of the penalty parameters have been chosen and reported in table 1.

Results obtained are depicted in figures 9, 10, 11, 12. Furthermore, the solving time is monitored, too and registered in table 2.

FAM A requires a solving time lower than families FAM B and FAM C, given its simpler structure. Moreover, the solving time augments when penalty parameters are similar to one another (i.e. when M1 parameters set is chosen).

Table 1: The penalty parameters set in the mathematical model in Part III of the experimentation

|  | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: |
| M0 | 1 | 98 | 1 |
| M1 | 30 | 40 | 30 |
| M2 | 1 | 69 | 30 |
| M3 | 30 | 69 | 1 |

Table 2: The solving time of the mathematical model in Part III of the experimentation

|  | average total <br> [minutes] <br> solution time | average solution <br> time for M0 | average solution <br> time for M1 | average solution <br> time for M2 | average solution <br> time for M3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| FAM A | 2.725 | 2.800 | 2.300 | 1.800 | 4.000 |
| FAM B | 44.925 | 18.600 | 56.100 | 26.750 | 78.250 |
| FAM C | 51.1875 | 13.250 | 95.050 | 8.550 | 87.900 |

Figure 9. Trend of ANSV in Part III of the experimentation


Figure 10. Trend of AESD in Part III of the experimentation

|  | AESD |
| :---: | :---: |
| 8,00\% |  |
| 7,00\% |  |
|  |  |
| $\begin{array}{l\|l\|l} 6,00 \% \\ 5,00 \% \end{array} \perp$ |  |
| 4,00\% | $\wedge$ |
| 3,00\% |  |
| 2,00\% |  |
| 1,00\% | 1 - |
| 0,00\% | - |
|  |  <br>  <br>  |

Figure 11. Trend of ASFR in Part III of the experimentation


Figure 12. Trend of AFR in Part III of the experimentation

|  | AFR |
| :---: | :---: |
| 95,0\% |  |
| 90,0\%\% |  |
| 80,0\% |  |
| 75,0\% |  |
| 70,00\% |  |
|  |  <br>  |

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