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Dynamic Valuation-Based System for reliability assessment of systems

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Abstract: This paper describes dynamic Valuation-Based System (VBS) for reliability assessment of systems under uncertainty. The reliability data and dependencies between components are represented using variables, sample spaces of variables, a set of valuations represented by probabilities, and basic probability assignments (bpas) that map sample spaces of sets of variables to the set of valuations. The uncertainties considered here are related to the states of components and their dependencies. The imprecise reliability of systems under uncertainty is estimated by an interval composed of upper and lower bounds. The proposed dynamic VBS approach is finally applied on a valve system and compared to the classical Bayesian Network approach.

Keywords: Valuation-Based System (VBS), Reliability assessment, Model uncertainty, Dynamic model, Modeling.

1. INTRODUCTION

Reliability is defined as the ability of a system to perform a required function under given conditions during a given time interval. It is a part of RAMS (Reliability, Availability, Maintainability, Safety) attributes and is used to evaluate the performance of a system (see Birnbaum et al. (1961)). Nowadays, for systems which involve humankind such that railway and aeronautical systems, some safety standards introduce RAMS requirements which have to be satisfied to ensure the safety design of such systems.

In this paper, reliability is used to evaluate the performance of systems and Valuation-Based System (VBS) is used to model systems. VBS was introduced by Shenoy (1989) to provide a framework of knowledge representation and reasoning under uncertainty. Xu et al. (1996) have proposed a decision support system. The system integrates Bayesian decision analysis and reasoning which is based on belief functions theory introduced by Dempster (1967) to suggest the optimal decision or the optimal sequence of decisions. It is applied on a nuclear waste disposal problem to find the location of the leakage of radioactive product in a river. Xu (1997) has also proposed a decision calculus using belief functions and VBS to help the decision maker to select an appropriate decision alternative when there are uncertainties concerning the states of events. This calculus is applied on an oil wildcatter problem to decide either to drill for oil or not. Benavoli et al. (2009) have developed an automatic information fusion system in VBS to support a commander's decision making. The system is applied on a threat assessment problem to assess the probability of a threat which is modeled by a network of entities and relationships between them. Uncertainties in the relationships are represented by belief functions. A sequence of incoming valuations is used to make dynamic inference.

There are already many tools to evaluate the reliability, such as reliability block diagrams, fault trees, etc. Here we choose VBS because of the following reasons:

- VBS provides a compact representation of system components and their dependencies;
- VBS is well adapted to represent and propagate uncertainties in models;
- VBS can model and evaluate performances of multi-state systems.

In our knowledge, VBS has not been used to evaluate the reliability of systems before. Because VBS can model systems and represent the knowledge about systems quantitatively, the reliability of systems is a measurable parameter in VBS modeling. However, VBS is a static modeling tool. It only supports to evaluate the reliability at a given instant. We propose a new dynamic VBS approach which allows evaluating the valuations as a function of time and then computing the reliability of systems over time. We also take into account uncertainties which are related to the states of components and the relationships between them. We compare this approach with a Dynamic Bayesian Network (DBN) approach proposed by Weber and Jouffe (2003).

The reminder of the paper is organized as follows. Section 2 presents VBS briefly. Section 3 presents the method of reliability assessment in dynamic VBS. Section 4 applies the dynamic VBS approach on a valve system, compares the proposed dynamic VBS approach with DBN approach and takes model uncertainties into account. Section 5 gives some conclusions and perspectives.

2. VALUATION-BASED SYSTEM

VBS was introduced by Shenoy (1989) in 1989. A VBS is a framework for representation and reasoning with knowledge under uncertainty. It is made up of two parts: a static part concerned with representation of knowledge, and a dynamic part concerned with reasoning with knowledge. The static part consists of variables and valuations. The dynamic part consists of two operators: combination and marginalization.

2.1 Variables

A real-world problem can be modeled by a finite set of variables. For a variable X, its frame Ω_X is all the possible values of this variable.

Given a finite nonempty set Φ of variables $\{A, B, C, ...\}$, we let Ω_Φ denote the Cartesian product of Ω_X for X in Φ: $\Omega_\Phi = \times\{\Omega_X | X \in \Phi\}$. We call Ω_Φ the frame for Φ. We regard the elements of Ω_Φ as configurations of Φ. Subsets of Φ will be denoted by $r, s, t, \ldots$.

2.2 Valuations

For a finite set of valuations $\Psi = \{\rho, \sigma, \tau, \ldots\}$ associated to subsets of Φ, each valuation represents knowledge about a subset of variables in Φ. For example, ρ is a valuation for r, where $r \subseteq \Phi$. Valuations are primitives in the description of VBS, so no definition is required. VBS can represent knowledge in different domains including probability theory, belief functions theory (see Dempster (1967), Shafer (1976)), possibility theory (see Dubois and Prade (1999)). According to the operations in the dynamic part, valuations are objects that can be combined, marginalized and solved.

2.3 Combination

A mapping $\oplus : \Psi \times \Psi \rightarrow \Psi$ is called combination which allows aggregating knowledge. The combination operator has three properties:

- Domain: if ρ is a valuation for r, and σ is a valuation for s, then $\rho \oplus \sigma$ is a valuation for $r \cup s$
- Commutativity: $\rho \oplus \sigma = \sigma \oplus \rho$
- Associativity: $\rho \oplus (\sigma \oplus \tau) = (\rho \oplus \sigma) \oplus \tau$

The combination of all valuations, $\oplus\Psi$, is called the joint valuation.

2.4 Marginalization

A mapping $-X : \Psi \rightarrow \Psi$ is called marginalization which allows coarsening knowledge by marginalizing X out of the domain of a valuation. The marginalization operator has three properties:

- Domain: if ρ is a valuation for r , and $X \in r$, then ρ^{-X} is a valuation for $r \backslash \{X\}$.

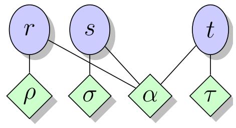


Fig. 1. Illustration of a valuation network

- Order does not matter: if ρ is a valuation for r , $X \in r$, and $Y \in r$, then $(\rho^{-X})^{-Y} = (\rho^{-Y})^{-X} = \rho^{-\{X,Y\}}$
- Local computation: if ρ and σ are valuations for r and s, respectively, $X \in r$, and $X \notin s$, then $(\rho \oplus \sigma)^{-X} = (\rho^{-X}) \oplus \sigma$

Sometimes $\rho^{-\{X,Y\}}$ is denoted by $\rho^{\downarrow r \backslash \{X,Y\}}$.

2.5 Making inference

Making inference with these two operators means finding marginals of the joint valuation for the variables of interest. Thus, if X is a variable of interest, $(\oplus\Psi)^{\downarrow X}$ is computed by marginalizing all the other variables in $\Phi \backslash \{X\}$ out of the joint valuation $\oplus\Psi$.

A graphical representation of a VBS is called a valuation network. Fig. 1 shows a valuation network. In the illustration, r, s, t are variables, $\rho, \sigma, \tau, \alpha$ are valuations.

2.6 Basic probability assignment

VBS can represent uncertainty using different theories. In this paper, probability is chosen as the valuation and is used to represent the uncertainty.

For a variable of interest X, Ω is its frame of discernment. The mapping $m^\Omega : 2^\Omega \rightarrow [0, 1]$ is called bpa on the measurable space $(\Omega, 2^\Omega)$ if $\forall A \in 2^\Omega$, $\sum_{A \subseteq \Omega} m^\Omega(A) = 1$, $m^\Omega(A) \geq 0$, and $m^\Omega(\emptyset) = 0$.

A bpa m^Ω is assigned to each subset of 2^Ω instead of Ω. $m^\Omega(A)$ represents the subjective probability assigned to the information which exactly supports A. The subsets $A \subset \Omega$ such that $m^\Omega(A) > 0$ are called focal sets.

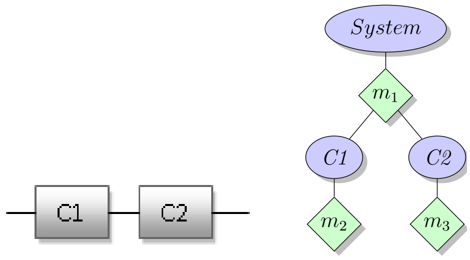
Given a set Ω and a bpa m^Ω on $(\Omega, 2^\Omega)$, the lower bound of the probability of a set A on Ω represents the sum of all probabilities of subsets that support A as follows (see Shafer (1976))

$$\underline{P}(A) = \sum_{B | B \subseteq A} m^\Omega(B), A \subseteq \Omega \tag{1}$$

The upper bound of the probability of A on Ω is defined as the total amount of probabilities of subsets that are consistent with A as follows

$$\overline{P}(A) = \sum_{B | B \cap A \neq \emptyset} m^\Omega(B), A \subseteq \Omega \tag{2}$$

The meaning of $\underline{P}(A)$ and $\overline{P}(A)$ can be explained by the following example. Suppose that a component has three states: s1, s2 and s3. s1 is supposed to be a working state. s2 is supposed to be a failed state. s3 is an unknown state. It is either a working state or a failed state. An expert gives probabilities at time t to the three states of the component as follows: $m^\Omega(s1) = 0.8, m^\Omega(s2) = 0.1, m^\Omega(s3) = 0.1$. In this case, a probability can be given to an event A:



(a) (b)

Fig. 2. System S. (a) Reliability block diagram. (b) Valuation network.

"*the component is in the working state at time t*", in the form $[\underline{P}(working), \overline{P}(working)] = [m^{\Omega}(s1), m^{\Omega}(s1) + m^{\Omega}(s3)] = [0.8, 0.9]$. The value 0.8 represents all the information that totally supports the event A, whereas the value 0.9 represents all the information that totally or partially supports the event A according to the expert. The length of the interval $\overline{P}(A) - \underline{P}(A)$ represents the imprecision about A. The probability of the event A is included in the closed interval composed of the lower and upper bounds.

3. RELIABILITY ASSESSMENT IN VBS

In this section, first, the method of reliability assessment at a given instant in VBS is presented. Then, the method of reliability assessment over time in dynamic VBS is proposed. Later, uncertainty is taken into account in the dynamic VBS model.

'Dynamic' means that a process or system is characterized by constant change, activity, or progress. In this paper, we aim to compute reliability over time. Thus, the proposed dynamic model of the system should include a temporal dimension.

3.1 Reliability assessment at a given instant

VBS is a static modeling tool. It models systems and represents the knowledge about systems quantitatively, so the reliability of systems can be measured.

Here we take a system S depicted in Fig. 2(a) as an example. The system is made up of two components connected in series. Each component has two states: "0" denotes the failed state and "1" denotes the working state.

Fig. 2(b) shows the valuation network of the system. There are three variables represented by circular nodes: the decision variable *System*, *C1* and *C2*. There are 3 bpas which represent the valuations by diamond-shaped nodes. m_1 represents the knowledge about the relationship between all the variables. It can be expressed by the structure function in Table 1 which can be represented by the following bpa

$$m_1^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(0,0,0),(0,1,0),(1,0,0),(1,1,1)\}) = 1$$

where the first parameter represents the state of *C1*, the second one represents the state of *C2*, and the last one represents the state of *System*. $\Omega_{C1} = \Omega_{C2} = \Omega_S = \{0, 1\}$.

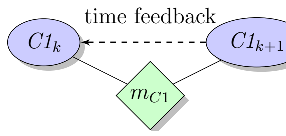


Fig. 3. Dynamic valuation network of the variable *C1*

m_2 and m_3 represent the knowledge about the variables *C1* and *C2*. Suppose that an expert gives the following bpas at time t:

- $m_2^{\Omega_{C1}}(\{0\}) = 0.05$ There is 0.05 chance that C1 is failed.
- $m_2^{\Omega_{C1}}(\{1\}) = 0.95$ There is 0.95 chance that C1 is working.
- $m_3^{\Omega_{C2}}(\{0\}) = 0.02$ There is 0.02 chance that C2 is failed.
- $m_3^{\Omega_{C2}}(\{1\}) = 0.98$ There is 0.98 chance that C2 is working.

The reliability of the system can be calculated by $(m_2^{\Omega_{C1}} \oplus m_3^{\Omega_{C2}} \oplus m_1^{\Omega_{C1}\Omega_{C2}\Omega_S})^{\downarrow\Omega_S}$.

There are many rules of combination in the literature. Here we use the Dempster's rule of combination (see Smets and Kennes (1994)) which is given by

$$m_{i\oplus j}^{\Omega}(H) = \frac{\sum_{A\cap B=H, \forall A,B\subseteq\Omega} m_i^{\Omega}(A) m_j^{\Omega}(B)}{1 - \sum_{A\cap B=\emptyset, \forall A,B\subseteq\Omega} m_i^{\Omega}(A) m_j^{\Omega}(B)} \tag{3}$$

$\forall\, A, B, H \subseteq \Omega$.

In this rule, it is assumed that all bpas stem from fully reliable and independent sources. Note that other rules are defined when the sources are not independent or reliable (see Denoeux (2006)).

First, we combine the bpas together to get the following joint valuation

$$m_{1,2,3}^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(1,1,1)\}) = 0.931$$
$$m_{1,2,3}^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(0,1,0)\}) = 0.049$$
$$m_{1,2,3}^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(1,0,0)\}) = 0.019$$
$$m_{1,2,3}^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(0,0,0)\}) = 0.001$$

Then, we marginalize the joint valuation to the variable *System* as follows

$$m^{\Omega_S}(\{0\}) = 0.069$$
$$m^{\Omega_S}(\{1\}) = 0.931$$

Thus, the reliability of system S at time t is 0.931.

3.2 Reliability assessment over time

In this subsection, we propose a dynamic VBS approach to evaluate the reliability of systems over time. A dynamic VBS is a VBS including a temporal dimension.

Table 1. Structure function of system S

C1	*C2*	*System*
0	0	0
0	1	0
1	0	0
1	1	1

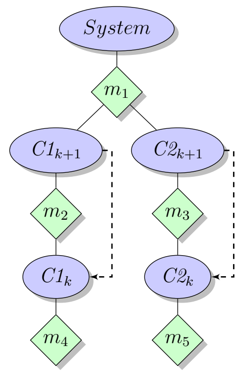


Fig. 4. Dynamic valuation network of system S

We suppose that the state of *C1* depends totally on its previous state. For example, the state probability of *C1(k+1)* depends on the state probability of *C1(k)*. It means that the state probability of *C1* can be calculated by iterative inferences.

The dynamic valuation network of *C1* with two time slices is shown in Fig. 3. Every time the state probability at time step *k+1* is computed, it will be sent to the state probability at time step *k* to realize the iterative inferences. Thus, to obtain the state probability at time step *k+1*, only the state probability at time step *0* and the conditional probability between *C1(k+1)* and *C1(k)* are required. m_{C1} in the valuation network represents the relationship between *C1(k+1)* and *C1(k)*. The dashed line with an arrowhead means the time feedback.

The dynamic valuation network of system S is shown in Fig. 4. To obtain the state probability of *System* at time step *k*, only the state probabilities of *C1* and *C2* at time step *0* and the conditional probabilities of *C1* and *C2* are required.

Table 2. CPT of *C1* and *C2*

Ci_k \ Ci_{k+1}	Failed	Working
Failed	1	0
Working	0.001	0.999

Table 2 gives the Conditional Probability Table (CPT) of *C1* and *C2*. We suppose that both of the two components are working at time step *0*. The solid line in Fig. 5 shows the reliability of system S during 1000 time steps.

3.3 Uncertainty model

In this subsection, uncertainty is taken into account in the dynamic VBS model of system S. The uncertainty is supposed to exist in the structure function. It means that we have a doubt about the model. For example, the probability that we are sure of the structure function is supposed to be 0.9. Thus, in the valuation network, m_1 is changed as follows

$$m_1^{\Omega_{C1}\Omega_{C2}\Omega_S}(\{(0,0,0),(0,1,0),(1,0,0),(1,1,1)\}) = 0.9$$
$$m_1^{\Omega_{C1}\Omega_{C2}\Omega_S}(\Omega) = 0.1$$

where $m_1^{\Omega_{C1}\Omega_{C2}\Omega_S}(\Omega)$ represents our part of ignorance about the model (i.e. all the combinations of the states of components & system are possible).

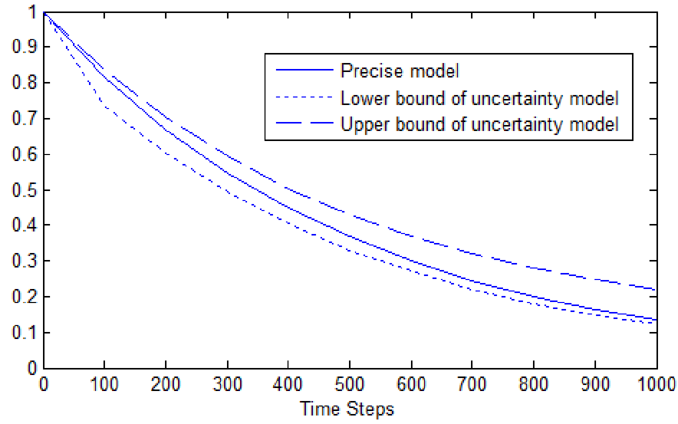


Fig. 5. Reliability of system S

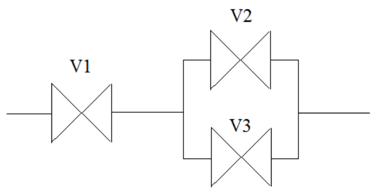


Fig. 6. Valve system

The simulation result is drawn in Fig. 5. The reliability of the system under uncertainty over time is estimated by an interval composed of the lower and upper bounds. As we can see, the precise reliability is always included in the interval.

4. APPLICATION

4.1 Description of the case study

Weber and Jouffe (2003) have proposed a DBN approach to evaluate the reliability of a valve system over time. Reliability in this application represents the probability that the system is operational at time t. In this section, our proposed dynamic VBS approach is also applied on the valve system. Results of these two approaches will be compared to validate the proposed dynamic VBS approach.

Fig. 6 depicts the structure of the valve system. Three valves are used to distribute or not a fluid. Every valve has a working state and two failed states: remains closed (RC) and remains opened (RO). Their corresponding failure rates are listed as follows:

$$\lambda_{1RC} = 1 * 10^{-3} \quad \lambda_{2RC} = 2 * 10^{-3} \quad \lambda_{3RC} = 3 * 10^{-3}$$
$$\lambda_{1RO} = 2 * 10^{-3} \quad \lambda_{2RO} = 3 * 10^{-3} \quad \lambda_{3RO} = 4 * 10^{-3}$$

The time step $\Delta t = 1$. Thus, the corresponding probability of failure is $\lambda * \Delta t$.

If V2 and V3 remain closed, the fluid cannot pass. If V2 and V3 remain open, then V1 can control the passage of the fluid (if V1 is working). Remains Open and Remains Closed can be used to classify the system's failure. The object is to determine whether or not the system is controllable. The structure function of the valve system is given in Table 3.

Table 3. Structure function of the valve system

V1	*V2*	*V3*	*System*
OK	OK	OK	Working
OK	OK	RC	Working
OK	OK	RO	Working
OK	RC	OK	Working
OK	RC	RC	Failed
OK	RC	RO	Working
OK	RO	OK	Working
OK	RO	RC	Working
OK	RO	RO	Working
RC	OK	OK	Failed
RC	OK	RC	Failed
RC	OK	RO	Failed
RC	RC	OK	Failed
RC	RC	RC	Failed
RC	RC	RO	Failed
RC	RO	OK	Failed
RC	RO	RC	Failed
RC	RO	RO	Failed
RO	OK	OK	Working
RO	OK	RC	Working
RO	OK	RO	Failed
RO	RC	OK	Working
RO	RC	RC	Failed
RO	RC	RO	Failed
RO	RO	OK	Failed
RO	RO	RC	Failed
RO	RO	RO	Failed

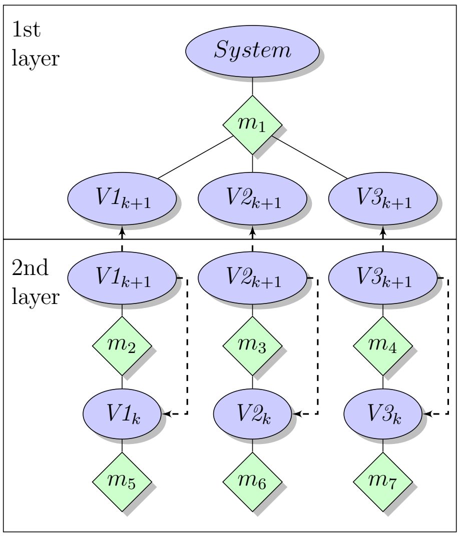


Fig. 7. Dynamic valuation network of the valve system

4.2 Dynamic VBS approach

In this subsection, the proposed dynamic VBS approach is applied on the valve system. Fig. 7 shows the dynamic valuation network of the valve system. In the valuation network, there are 7 variables represented by circular nodes: the decision variable $System$, $V1_{k+1}$, $V2_{k+1}$, $V3_{k+1}$, $V1_k$, $V2_k$, $V3_k$. The frame of discernment of the variable $System$ is given by

$$\Omega_S = \{Failed, Working\}$$

The frame of discernment of the variables Vi is given by

$$\Omega_{Vi} = \{OK, RC, RO\}$$

There are 7 bpas which represent the valuations by diamond-shaped nodes: $m_1, m_2, m_3, m_4, m_5, m_6, m_7$.

The bpas m_1, m_2, m_3, m_4 represent the knowledge about the relations between all the variables. m_1 can be represented by the structure function in Table 3. Conditional probabilities in Table 4 can be represented by bpas m_2, m_3, m_4. It means BN can be transformed into VBS.

Table 4. CPT of $m_2^{\Omega_{V1}}, m_3^{\Omega_{V2}}, m_4^{\Omega_{V3}}$

Vi_k \ Vi_{k+1}	OK	RC	RO
OK	$1-(\lambda_{iRC}-\lambda_{iRO})*\Delta t$	$\lambda_{iRC}*\Delta t$	$\lambda_{iRO}*\Delta t$
RC	0	1	0
RO	0	0	1

The bpas m_5, m_6, m_7 represent the knowledge about the variables. They will be directly affected to the elements of the frames of discernment of the variables $V1_k$, $V2_k$, $V3_k$.

For example, the bpa $m_5^{\Omega_{V1}}$ will be affected directly from observations concerning the valve V1 or experts to the elements of Ω_{V1_k} such that

$$m_5^{\Omega_{V1}}(OK) + m_5^{\Omega_{V1}}(RC) + m_5^{\Omega_{V1}}(RO) = 1$$

The dynamic valuation network of the valve system is composed of two layers. In the 2nd layer, the state probabilities of the valve Vi_{k+1} can be deduced recursively by the CPT and its initial value Vi_0. All the valves are supposed to be working at the time step *0*. Then, the state probabilities of Vi_{k+1} obtained in the 2nd layer are sent to the corresponding variables in the 1st layer. The state probability of the system can be deduced by the state probabilities of all the three valves.

The solid line in Fig. 8 shows the reliability obtained by the dynamic VBS approach.

4.3 Uncertainty model

In this subsection, uncertainty is supposed to exist in the structure function. It means that we have a doubt about this structure function. The probability that we are sure of this structure function is supposed to be 0.8. Thus, in the dynamic valuation network, m_1 is changed as follows

$$m_1^{\Omega_{V1}\Omega_{V2}\Omega_{V3}\Omega_S}(\{(OK, OK, OK, Working), (OK, OK, RC, Working), \ldots, (RO, RO, RO, Failed)\}) = 0.8$$

$$m_1^{\Omega_{V1}\Omega_{V2}\Omega_{V3}\Omega_S}(\Omega) = 0.2$$

The simulation result is drawn in Fig. 8. The reliability of the system under uncertainty over time is estimated by an interval composed of the lower and upper bounds. As we can see, the precise reliability is always included in the interval. The fact that the precise reliability is included in the interval follows by the fact that the set of probabilities associated to the bpas includes the one that gives the precise reliability.

4.4 Discussion and comparison

Fig. 9 shows the DBN model proposed by Weber and Jouffe (2003). The solid line in Fig. 10 represents the reliability of the valve system during 2000 time steps computed in Weber and Jouffe (2003). The other two curves represent the evolution of the probabilities of the two types of failure. We find that the reliability is exactly the same with the reliability obtained by the proposed dynamic VBS approach.

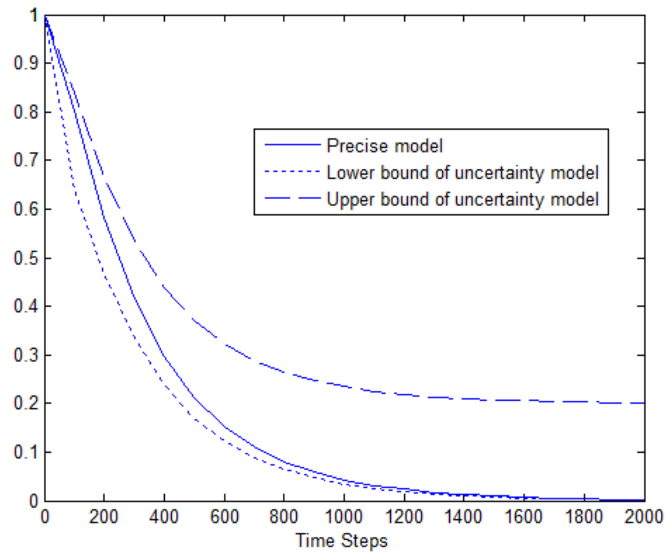


Fig. 8. Reliability obtained by dynamic VBS approach

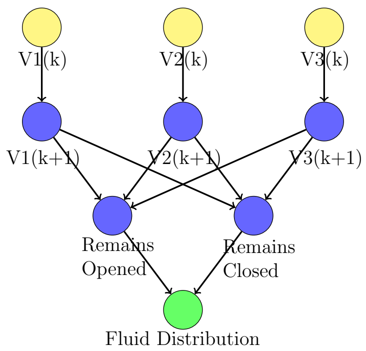


Fig. 9. DBN model of the valve system

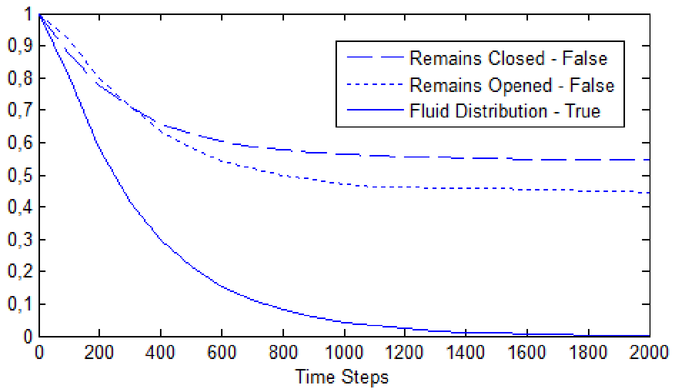


Fig. 10. Reliability obtained by DBN approach

Both of Bayesian Network (BN) and VBS can be used to represent variables and their conditional dependencies via graphical models. However, there are some obvious differences between these two approaches. Above all, BN is a probabilistic directed graphical model, while VBS can be probabilistic or non-probabilistic, and it is non-directed. Besides, the operations in BN are based on the conditional probability formula, while the operations in VBS are based on combination and marginalization operators.

VBS can represent and propagate easily model uncertainties. It allocates bpas over subsets of Ω instead of singletons as done in BN and thus it can represent the uncertainty of components (model uncertainty).

5. CONCLUSION

In this paper, we propose a dynamic VBS approach to evaluate the reliability of systems over time. BN can be transformed into VBS. Thus VBS can be considered as a generalization of BN. The dynamic VBS approach is proved to be as effective as DBN. However, compared to DBN, the dynamic VBS has an advantage: it allows modeling and quantifying many kinds of uncertainties in systems. In the future, we will apply the proposed dynamic VBS approach on more complex systems, such as railway signalling systems, to study the reliability over time and analyze different kinds of model uncertainties in systems.

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