

Constant-yield control of the chemostat

Georgios Savoglidis and Costas Kravaris

Abstract—The present work addresses the problem of chemostat stabilization around an optimal steady state, in the sense of enlargement of its stability region. The need for stabilization becomes imperative under conditions where the growth of biomass is subject to substrate inhibition, and the primary concern is to prevent washout of the biomass in the presence of disturbances. Inspired by the empirical concept of constant-yield control, a nonlinear state feedback control law is derived, and the stability basin of resulting closed-loop system is estimated using a Lyapunov function approach. Our analysis extends previous results in the sense that it accounts for biomass decay and endogenous metabolism and, moreover, it covers the case where the product is soluble in the effluent stream.

I. INTRODUCTION

Continuous stirred microbial bioreactors, often called chemostats, cover a wide range of applications; specialized “pure culture” biotechnological processes for the production of specialty chemicals (proteins, antibiotics etc.) as well as large-scale environmental technology processes of mixed cultures such as wastewater treatment. The dynamics of the chemostat [1,2] is often adequately represented by a simple dynamic model with state variables the concentration of the microbial biomass x , the concentration of the limiting organic substrate s and, possibly, the concentration P of the product in the reacting mixture:

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \mu(s)x - K_d x \\ \frac{ds}{dt} &= D(S_0 - s) - \frac{1}{Y_{x/s}} \mu(s)x - mx \\ \frac{dP}{dt} &= -DP + Y_p \mu(s)x \end{aligned} \quad (1)$$

where D is the dilution rate, S_0 is the feed substrate concentration, $Y_{x/s}$ is a biomass yield factor, K_d is the biomass decay rate constant, m is the endogenous metabolism rate constant, Y_p is a yield coefficient of product formation and $\mu(s)$ is the specific growth rate, a given function of s . The most widely used expressions for the specific growth rate are the Haldane equation

$$\mu(s) = \mu_{\max} s / \left(K_S + s + \frac{s^2}{K_I} \right) \quad (2a)$$

and the Monod equation,

$$\mu(s) = \mu_{\max} s / (K_S + s) \quad (2b)$$

where μ_{\max} is the maximum specific growth rate, K_S the saturation constant and K_I the substrate inhibition kinetic constant, with the Monod kinetics being a special case of the Haldane kinetics ($1/K_I = 0$) for biological reactions where substrate inhibitory phenomena can be neglected.

The problem of chemostat stabilization has received considerable attention in the literature [3-9]. In the vast majority of cases, the control input is the dilution rate D . Usually, the objective of control is to regulate the system at specific design conditions, which will net optimal performance.

One important class of applications taking place in a chemostat is related to anaerobic digestion, which is a key process in wastewater treatment, sludge management, production of energy from biomass, etc. During the process of anaerobic digestion, the organic compounds are mineralized to biogas, a useful energy product, consisting primarily of methane and carbon dioxide, through a series of reactions mediated by several groups of microorganisms.

In anaerobic digestion, the product (biogas) is insoluble and the overall dynamics can be described by a second-order model:

$$\begin{aligned} \frac{dx}{dt} &= -Dx + \mu(s)x - K_d x \\ \frac{ds}{dt} &= D(S_0 - s) - \frac{1}{Y_{x/s}} \mu(s)x - mx \\ \dot{P} &= Y_p \mu(s)x \end{aligned} \quad (3)$$

where \dot{P} , the rate of production of biogas, is a nonlinear algebraic function of the states. Control of anaerobic digesters has received significant attention in the literature [4,10-18], including recent work of the authors [19], the aim being stabilization and regulation at proper operating conditions in the presence of disturbances.

Comparing dynamic models (1) and (3), we observe that the dynamic equations of (3) are exactly the first two state equations of (1). In fact, dynamic system (3) can be viewed as a simplification of (1) when the production rate is not directly affected by changes in the dilution rate.

In the present study, we develop a mathematical formulation of constant-yield control for system (1) and we investigate its effectiveness in enhancing the stability properties of the chemostat. The results to be presented in this study can be specialized to system (3) in a direct and immediate way.

*G. Savoglidis was with University of Patras, 26500 Patras, Greece. He is now with École Polytechnique Fédérale de Lausanne (EPFL), CH 1015 Lausanne, Switzerland (e-mail: georgios.savoglidis@epfl.ch).

C. Kravaris is with University of Patras, 26500 Patras, Greece (phone:+30 2610 969514; fax:+30 2610 993070; e-mail: kravaris@chemeng.upatras.gr).

II. OPEN-LOOP SYSTEM: PROPERTIES AND OPTIMAL OPERATING CONDITIONS

A. System steady states

Consider the dynamic system (1), with $\mu(s)$ given by (2a) or (2b), where the dilution rate D is the input variable of the system and $S_0, K_d, Y_{x/s}, Y_p, \mu_{\max}, K_s, K_I, m$ are constant parameters. The following hypothesis will be made throughout this paper:

$$-mY_{x/s} < K_d < \mu(S_0) \quad (\text{H})$$

Assumption (H) guarantees the existence of positive steady state(s) for the bioreactor ($x_s > 0, s_s > 0, P_s > 0$ corresponding to $D_s > 0$). These are calculated from the set of equations:

$$\begin{aligned} D_s &= \mu(s_s) - K_d \\ x_s &= \frac{(S_0 - s_s)}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} \left(1 - \frac{K_d}{\mu(s_s)} \right) \\ P_s &= Y_p \frac{(S_0 - s_s)}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} \end{aligned} \quad (4)$$

Hence the equilibrium curve of the system is defined by the set of equations

$$\left\{ \begin{aligned} x_s &= \frac{(S_0 - s_s)}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} \left(1 - \frac{K_d}{\mu(s_s)} \right) \\ P_s &= Y_p \frac{(S_0 - s_s)}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} \end{aligned} \right\} \quad (5)$$

Under assumption (H), the equilibrium curve has the shape shown in Fig. 1.

One end of the equilibrium curve is on the s -axis, at the point $(x, s, P) = (0, S_0, 0)$. The other end is on the plane $x=0$,

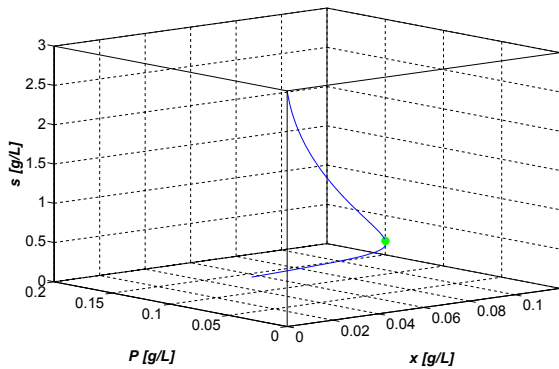


Figure 1. Equilibrium curve of the open loop system.

at the point $(x, s, P) = (0, s_*, Y_p \frac{(S_0 - s_*)}{\frac{1}{Y_{x/s}} + \frac{m}{K_d}})$, where s_* is the

smallest root of the equation $\mu(s) = K_d$ and is given by:

$$s_* = K_s \cdot \frac{2 \frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}} \right) + \left[\left(1 - \frac{K_d}{\mu_{\max}} \right)^2 - 4 \left(\frac{K_s}{K_I} \right) \left(\frac{K_d}{\mu_{\max}} \right)^2 \right]^{1/2}} \quad (6a)$$

In the special case of Monod kinetics ($1/K_I = 0$),

$$s_* = K_s \cdot \left(\frac{K_d}{\mu_{\max}} \right) / \left(1 - \frac{K_d}{\mu_{\max}} \right) \quad (6b)$$

is the unique root of $\mu(s) = K_d$.

Remark: For $1/K_I \neq 0$, the equation $\mu(s) = K_d$ has two roots, which are both real and positive; the smallest root s_* is given by (6a) and the largest root is

$$s^* = K_s \cdot \frac{2 \frac{K_d}{\mu_{\max}}}{\left(1 - \frac{K_d}{\mu_{\max}} \right) - \left[\left(1 - \frac{K_d}{\mu_{\max}} \right)^2 - 4 \left(\frac{K_s}{K_I} \right) \left(\frac{K_d}{\mu_{\max}} \right)^2 \right]^{1/2}} \quad (7)$$

Assumption (H) implies that $S_* < \min\{S_0, \sqrt{K_s \cdot K_I}\}$ and $s^* > \max\{S_0, \sqrt{K_s \cdot K_I}\}$. In the special case of Monod kinetics ($1/K_I = 0$), assumption (H) implies that the unique root s_* of $\mu(s) = K_d$ satisfies $S_* < S_0$.

B. Optimal steady state corresponding to maximal production rate

From a practical point of view, it makes sense to try to operate the bioreactor at steady state conditions corresponding to maximal production rate (which corresponds to the product $D \cdot P$). A simple calculation gives:

$$D_s P_s = Y_p \mu(s_s) x_s = Y_p \frac{(S_0 - s_s)}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s)}} (\mu(s_s) - K_d)$$

In this case the production rate is maximized when:

$$\begin{aligned} \frac{d}{ds_s} (D_s P_s) &= 0 \Leftrightarrow \\ \frac{d\mu(s_s)}{ds_s} (S_0 - s_s) &= \frac{\mu(s_s)}{\frac{1}{1 - K_d/\mu(s_s)} + \frac{mY_{x/s}/\mu(s_s)}{1 + mY_{x/s}/\mu(s_s)}} \end{aligned} \quad (8)$$

For $\mu(s)$ given by (2a) or (2b), condition (8) leads to a polynomial equation with respect to s_s , of up to 6th-degree. Only one of its roots corresponds to a positive steady state ($x_s > 0, s_s > 0, P_s > 0, D_s > 0$) and this is the optimal steady state value s_s^{opt} .

For the following values of the parameters

$$\begin{aligned}\mu_{\max} &= 0.1 d^{-1}, K_S = 1 g/L, Y_{X/S} = 0.05 g/g, Y_P = 1 g/g, \\ S_0 &= 3 g/L, K_I = 1 g/L, K_d = 0.01 d^{-1}, m = 0.03 d^{-1}\end{aligned}$$

The optimal nontrivial, positive steady state calculated through (8) and (5) is

$$(x_s^{opt} = 0.112 g/L, s_s^{opt} = 0.259 g/L, P_s^{opt} = 0.134 g/L)$$

The optimal steady state is achieved for the value of the dilution rate of $D_s^{opt} = 0.051 d^{-1}$. The optimal steady state for a process characterized by the above set of parameters is indicated by the green dot on the equilibrium line of Fig. 1.

C. Local asymptotic stability

The dynamics of the open-loop system (1) has serial structure, consisting of the dynamics of x and s , followed by the dynamics of P . The eigenvalues of the linearization of (1) associated with the dynamics of x and s are the roots of the quadratic polynomial

$$\begin{aligned}\lambda^2 + \left(\mu(s_s) - K_d + \frac{\mu(s_s) - K_d}{\mu(s_s) + mY_{X/S}} (S_0 - s_s) \frac{d\mu}{ds}(s_s) \right) \lambda + \\ (\mu(s_s) - K_d)(S_0 - s_s) \frac{d\mu}{ds}(s_s)\end{aligned}$$

whereas the eigenvalue associated with the P dynamics is $\lambda = -(\mu(s_s) - K_d)$.

Because hypothesis (H) guarantees that $\left\{ \begin{array}{l} \mu(s_s) > K_d \\ \mu(s_s) > -mY_{X/S} \\ s_s < S_0 \end{array} \right\}$,

we conclude that a positive equilibrium is locally asymptotically stable if $\frac{d\mu}{ds}(s_s) > 0$, unstable if $\frac{d\mu}{ds}(s_s) < 0$.

Note that from (8) the optimal steady state satisfies $\frac{d\mu}{ds}(s_s^{opt}) > 0$, therefore it is locally asymptotically stable.

D. The need for control

Fig. 2 depicts the three-dimensional trajectories of the system dynamics under constant dilution rate and in particular for the optimal value of the dilution rate D_s^{opt} calculated in the previous subsection. Fig. 3 is a projection of the three-dimensional phase portrait on the $x-s$ plane. For this value of the dilution rate, the system has two nontrivial steady states; one is the optimal steady state (marked with green dot), which is stable, and the other one is an unstable steady state (marked with red dot):

$$(x_s^{unstable} = 0.107 g/L, s_s^{unstable} = 0.386 g/L, P_s^{unstable} = 0.128 g/L)$$

There is also a trivial steady state corresponding to washout of the biomass ($x_s^{w/o} = 0, s_s^{w/o} = S_0 = 3 g/L, P_s^{w/o} = 0$), which represents a completely undesirable situation. One can immediately observe that the washout steady state is stable and has a large region of attraction that extends up to the vicinity of the stable optimal steady state. This makes the

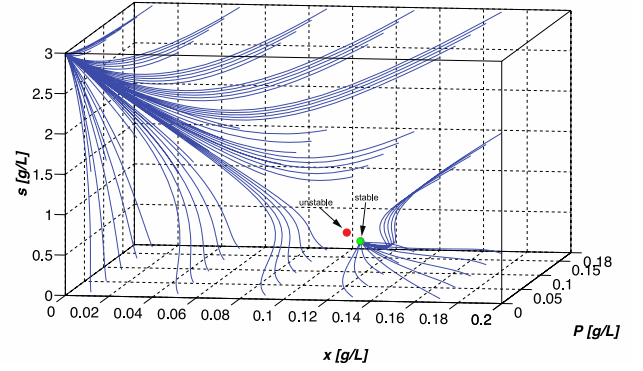


Figure 2. Three-dimensional phase portrait of the open loop system under constant dilution rate. With green dot is the stable design steady state. With red dot is the unstable steady state.

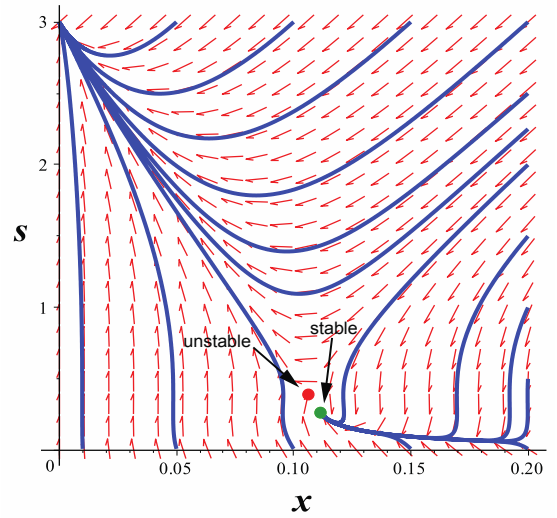


Figure 3. Phase portrait of the open loop system in $x-s$ plane. With green dot is the stable design steady state. With red dot is the unstable steady state.

operation of the chemostat at the optimal steady state conditions very sensitive to disturbances.

The goal of the proposed nonlinear controller is the stabilization of the system in the sense of enlargement of the stability region.

III. CONSTANT-YIELD CONTROL POLICY

The intuitive concept of constant yield control [14] has been motivated by anaerobic digestion applications, based on the intuitive idea that the controller must try to keep

$$\frac{\text{biogas production rate}}{\text{organic substrate feed rate}} = \text{constant}$$

When this intuitive idea is applied to a bioreactor in the absence of biomass decay or endogenous metabolism ($K_d = 0, m = 0$), it leads to a proportional control law in the biogas production rate that has very attractive theoretical properties. In particular, it was shown that it leads to global asymptotic stability of the resulting closed loop system

[4,5], with robustness with respect to bounded errors in S_0 [8]. In what follows, we will generalize the concept of constant yield control in the presence of death rate of the biomass $K_d \neq 0$ and/or consumption of the substrate for endogenous metabolism $m \neq 0$, which will lead to a similar control law, proportional to the reaction rate.

Consider the dynamic system (1). Requesting that the control law must keep

$$\frac{\text{Production Rate}}{\text{Organic Feed Rate}} = \text{constant}$$

and equal to its value at design conditions, leads to

$$\begin{aligned} \frac{Y_p \mu(s) x}{DS_0} = \text{constant} &= \frac{Y_p \mu(s_s^{des}) x_s^{des}}{D_s^{des} S_0} = \\ &= \frac{Y_p \frac{(S_0 - s_s^{des})}{1 + \frac{m}{Y_{x/s}}} (\mu(s_s^{des}) - K_d)}{(\mu(s_s^{des}) - K_d) S_0} = \frac{Y_p (S_0 - s_s^{des})}{\left(\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})} \right) S_0} \end{aligned}$$

from which, solving for the dilution rate, leads to the following control law:

$$D = \frac{1}{S_0 - s_s^{des}} + \frac{m}{Y_{x/s} \mu(s_s^{des})} \mu(s) x \quad (9)$$

The superscript “des”, wherever it is used, denotes design steady state conditions; these can be arbitrary, even though the most meaningful is the optimal steady state calculated in section II-B.

It is important to observe at this point that control law (9):

- i. is a nonlinear state feedback law, proportional to the production rate $Y_p \mu(s) x$;
- ii. depends only on the states x and s but not on P .

IV. PROPERTIES OF THE CONSTANT-YIELD CONTROLLER

A. The closed-loop system under constant-yield control

Consider the dynamic system (1) under the control law (9). The resulting closed-loop system is

$$\begin{aligned} \frac{dx}{dt} &= \left(1 - \frac{1 + \frac{m}{Y_{x/s}}}{S_0 - s_s^{des}} x \right) \mu(s) x - K_d x \\ \frac{ds}{dt} &= \left(\frac{1 + \frac{m}{Y_{x/s}}}{S_0 - s_s^{des}} (S_0 - s) - \frac{1}{Y_{x/s}} \right) \mu(s) x - mx \\ \frac{dP}{dt} &= \left(Y_p - \frac{1 + \frac{m}{Y_{x/s}}}{S_0 - s_s^{des}} P \right) \mu(s) x \end{aligned} \quad (10)$$

for which

$$\left\{ \begin{array}{l} s_s = s_s^{des} \\ x_s = \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} \left(1 - \frac{K_d}{\mu(s_s^{des})} \right) = x_s^{des} \\ P_s = Y_p \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} = P_s^{des} \end{array} \right.$$

is a steady state (the design steady state). Note, however, that (10) may have an additional positive steady state. In particular, let s_u be the solution of the equation

$$\frac{1}{Y_{x/s} \mu(s_s^{des})} + m \frac{1}{S_0 - s_s^{des}} - \frac{1}{s - s_s^{des}} = 0 \quad \text{with respect to } s.$$

(For $\mu(s)$ given by (2a) or (2b), it is easy to see that existence, uniqueness and positivity of s_u is guaranteed.)

If it so happens that $\frac{K_d}{\mu(s_u)} < 1$, then system (10) possesses an additional positive steady state:

$$\left\{ \begin{array}{l} s_s = s_u \\ x_s = \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} \left(1 - \frac{K_d}{\mu(s_u)} \right) = x_u \\ P_s = Y_p \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} = P_u \end{array} \right.$$

It is not difficult to see that this additional steady state is unstable when the design steady state is chosen to be the optimal steady state, and moreover, that $s_u < s_s^{des}$.

B. Estimation of the size of the stability basin

Consider the closed loop system (10) and notice that it possesses the same serial structure as the open-loop system: x and s dynamics, followed by P dynamics. Moreover, integrating the differential equation for the P dynamics, gives

$$\begin{aligned} P(t) &= Y_p \frac{S_0 - s_s^{des}}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} \\ &+ \left(P(0) - Y_p \frac{S_0 - s_s^{des}}{\frac{1}{Y_{x/s}} + \frac{m}{\mu(s_s^{des})}} \right) \cdot \exp \left(- \frac{1 + \frac{m}{Y_{x/s}}}{S_0 - s_s^{des}} \int_0^t \mu(s(\tau)) x(\tau) d\tau \right) \end{aligned}$$

If dynamic subsystem of the first two differential equations (dynamics of x and s)

$$\begin{aligned} \frac{dx}{dt} &= \left(1 - \frac{1 + \frac{m}{Y_{x/s}} \mu(s_s^{des})}{S_0 - s_s^{des}} \right) x \mu(s)x - K_d x \\ \frac{ds}{dt} &= \left(\frac{1 + \frac{m}{Y_{x/s}} \mu(s_s^{des})}{S_0 - s_s^{des}} (S_0 - s) - \frac{1}{Y_{x/s}} \right) \mu(s)x - mx \end{aligned} \quad (11)$$

is asymptotically stable, with $x(t) \rightarrow x_s^{des}$, $s(t) \rightarrow s_s^{des}$, then

$$\begin{aligned} \mu(s(t))x(t) &\rightarrow \mu(s_s^{des})x_s^{des} = \left(\mu(s_s^{des}) - K_d \right) \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \mu(s_s^{des})} \\ &= D_s^{des} \frac{(S_0 - s_s^{des})}{\frac{1}{Y_{x/s}} + \mu(s_s^{des})} \end{aligned}$$

and therefore

$$\frac{1}{Y_{x/s}} + \frac{m}{S_0 - s_s^{des}} \int_0^t \mu(s(\tau))x(\tau) d\tau \approx D_s^{des} t, \text{ for large } t$$

which implies that $P(t) \rightarrow Y_p \frac{S_0 - s_s^{des}}{\frac{1}{Y_{x/s}} + \mu(s_s^{des})} = P_s^{des}$, hence

the entire system is stable. Thus, the development of sufficient stability conditions for the closed loop system (10) reduces to finding appropriate conditions for the subsystem (11).

For this purpose, it is possible to use a quadratic Lyapunov function, accounting for the distance from the design steady state and prove the following proposition (see [19]):

Proposition: Assume that $\mu(s)$ is given by (2a) or (2b) and $m \geq 0$. Also, assume that s_s^{des} is chosen to be the optimal steady state that corresponds to maximal steady state production rate. Then, the set

$$I = \left\{ (x, s) \in \mathbb{R}^2 \mid x > 0, \mu(s) > K_d, \frac{1 + \frac{m}{Y_{x/s}} \mu(s_s^{des})}{S_0 - s_s^{des}} + m \frac{1 - \mu(s)}{s - s_s^{des}} > 0 \right\}$$

is contained in the stability basin of (11).

Remark: When both $K_d > 0$ and $m > 0$, the set I can be equivalently represented as

$$I = \left\{ (x, s) \in \mathbb{R}^2 \mid x > 0, s^* > s > \max(s_*, s_u) \right\}$$

This representation is useful since it defines explicit upper and lower bounds on the initial substrate concentration that guarantee that the system trajectory will asymptotically approach the design steady state.

V. SIMULATION STUDY

Numerical simulations of the closed loop system (10) were performed with the parameter values given in section II-B. Fig. 4 depicts a three-dimensional phase portrait of the system. From Fig. 4, it is also clear that there is no unstable steady state in the vicinity of the stable steady state. As a result, a large region of attraction is obtained.

Specifically, for the values of the parameters used, the unstable steady state is now pushed further towards the $x-P$ plane creating a lower limit of the stability region at $s_u = 0.015 \text{ g/L}$. This lower limit (green horizontal line) along with the unstable steady state (red dot) can be seen in Fig. 5, which is a projection of the three-dimensional phase portrait of Fig. 4 on the $x-s$ plane.

A similar graph in the region of very high initial concentrations of the substrate reveals that there exists an upper limit of the stability region, which corresponds to unrealistically high substrate concentrations in practice. As

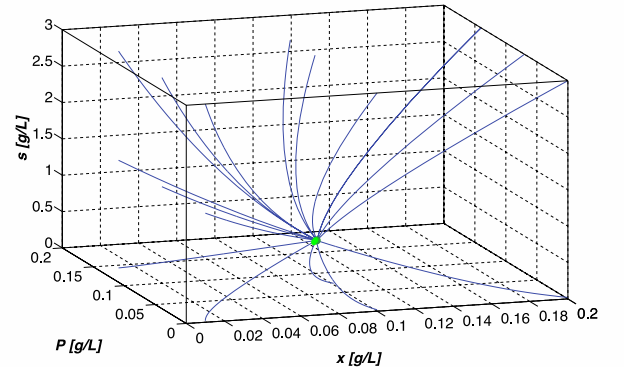


Figure 4. Three-dimensional phase portrait under the nonlinear state feedback controller. The green dot marks the design steady state.

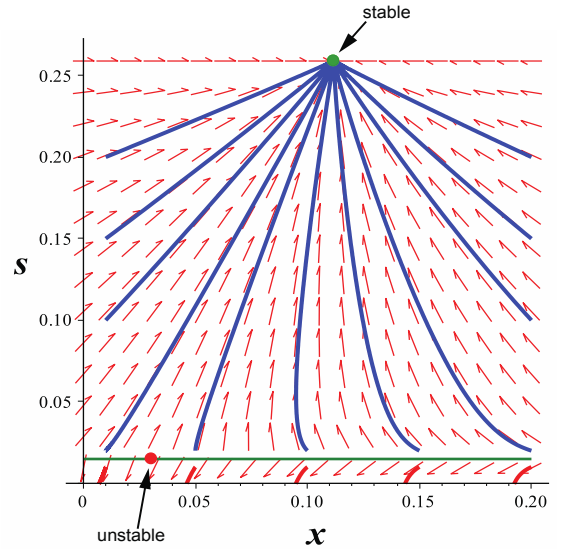


Figure 5. Phase portrait of the closed loop system in the $x-s$ plane close to the lower threshold of stability. With green line is the stability threshold. With red dot is the unstable steady state. With green dot is the stable steady state.

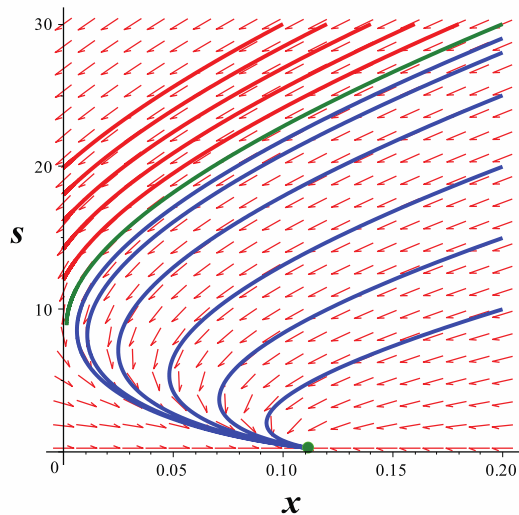


Figure 6. Phase portrait of the closed loop system in the $x-s$ plane close to the upper threshold of stability. With green line is the stability threshold. With green dot is the stable steady state.

one can see in Fig. 6, there exists a trajectory (green line) starting from very high values of the substrate and terminating on the s -axis at the point $s^* = 8.989 \text{ g/L}$.

Note that in the case considered, the closed loop system possesses an unstable steady state in addition to the stable steady state. If we had a lower value of endogenous metabolism rate constant, e.g. $m = 0.015 \text{ d}^{-1}$, and the rest of the parameters were the same, the closed loop system would have only one positive steady state (the design steady state). Fig. 7 shows the corresponding phase portrait in the $x-s$ plane, and for low values of s . We observe that there is still a lower limit in the region of stability, despite the non-existence of an additional positive steady state (in fact, the steady state (s_u, x_u) is now pushed into the second quadrant). The stability region is now limited by the trajectory that terminates at the point $s_* = 0.011 \text{ g/L}$ of the s -axis.

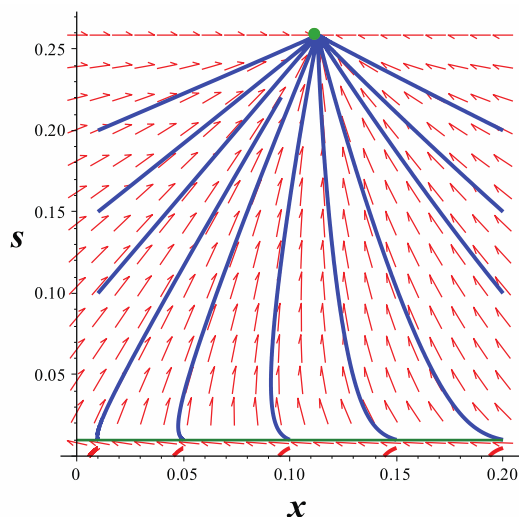


Figure 7. Phase portrait of the closed loop system in the $x-s$ plane close to the lower threshold of stability for the case of low endogenous metabolism.

The graphs near the upper limit of the stability region are similar and are skipped due to space limitations.

VI. CONCLUSION

In the present study, a nonlinear state feedback control law has been developed for the control of chemostats. The control law has been derived on the basis of intuitive considerations, in particular on the idea of trying to maintain constant yield between the feeding rate of the nutrients and the rate of production of the product. Even though the controller cannot guarantee global stability in closed loop, it manages to significantly increase the stability basin of the system and thus prevent the washout of the biomass.

REFERENCES

- [1] H.L. Smith, P. Waltman, The theory of the chemostat: dynamics of microbial competition, Cambridge University Press, 1995.
- [2] J.E. Bailey, D.F. Ollis, Biochemical engineering fundamentals, McGraw-Hill, 1986.
- [3] P. De Leenheer, H.L. Smith, Virus dynamics: A global analysis, SIAM J Appl Math, 63 (2003) 1313-1327.
- [4] L. Mailleret, O. Bernard, A simple robust controller to stabilize an anaerobic digestion process, in: 8th Conference on Computer Applications in Biotechnology, 2001, pp. 213-218.
- [5] L. Syrou, I. Karafyllis, K. Stamatelatos, G. Lyberatos, C. Kravaris, Robust global stabilization of continuous bioreactors, in: 7th International Conference on Dynamics and Control of Process Systems (DYCOPS 7), Boston, MA, 2004.
- [6] J.L. Gouze, G. Robledo, Feedback control for competition models with mortality in the chemostat, IEEE Decis Contr, (2006) 2098-2103.
- [7] J. Harmand, A. Rapaport, F. Mazenc, Output tracking of continuous bioreactors through recirculation and by-pass, Automatica, 42 (2006) 1025-1032.
- [8] I. Karafyllis, C. Kravaris, L. Syrou, G. Lyberatos, A vector Lyapunov function characterization of input-to-state stability with application to robust global stabilization of the chemostat, Eur J Control, 14 (2008) 47-61.
- [9] I. Karafyllis, C. Kravaris, N. Kalogerakis, Relaxed Lyapunov criteria for robust global stabilisation of non-linear systems, Int J Control, 82 (2009) 2077-2094.
- [10] D. Dochain, G. Bastin, Stable Adaptive Controllers for Waste Treatment by Anaerobic-Digestion, Environ Technol Lett, 6 (1985) 584-593.
- [11] P. Renard, D. Douchain, G. Bastin, H. Naveau, E.J. Nyns, Adaptive-Control of Anaerobic-Digestion Processes - a Pilot-Scale Application, Biotechnol Bioeng, 31 (1988) 287-294.
- [12] D. Dochain, M. Perrier, Control Design for Nonlinear Waste-Water Treatment Processes, Water Science and Technology, 28 (1993) 283-293.
- [13] M. Perrier, D. Dochain, Evaluation of Control Strategies for Anaerobic-Digestion Processes, Int J Adapt Control, 7 (1993) 309-321.
- [14] P.C. Pullammanappallil, S.A. Svoronos, D.P. Chynoweth, G. Lyberatos, Expert system for control of anaerobic digesters, Biotechnol Bioeng, 58 (1998) 13-22.
- [15] R. Antonelli, J. Harmand, J.P. Steyer, A. Astolfi, Set-point regulation of an anaerobic digestion process with bounded output feedback, IEEE T Contr Syst T, 11 (2003) 495-504.
- [16] P.F. Pind, I. Angelidaki, B.K. Ahring, K. Stamatelatos, G. Lyberatos, Monitoring and control of anaerobic reactors, Adv Biochem Eng Biotechnol, 82 (2003) 135-182.
- [17] L. Mailleret, O. Bernard, J.P. Steyer, Robust regulation of anaerobic digestion processes, Water Sci Technol, 48 (2003) 87-94.
- [18] N. Dimitrova, M. Krastanov, Nonlinear adaptive stabilizing control of an anaerobic digestion model with unknown kinetics, Int J Robust Nonlin Control, 22 (2012) 1743-1752.
- [19] G. Savoglidis, C. Kravaris, Constant-yield control of continuous bioreactors, Chem Eng J, in press.