2D Path Following for Marine Craft: A Least-Square Approach

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Abstract: We study the problem of straight-line path following for fully actuated marine craft. We propose a controller that adjusts the speed of the marine craft according to the geometric distance and the rate of convergence to the path. The control law is derived using the method of least squares, which is used to find an approximate solution for overdetermined systems. The conditions under which the closed-loop system is globally asymptotically stable are found. Moreover, a method to ensure zero cross-track error in the presence of ocean currents is proposed. The stability proof relies on the theory of cascaded systems. The effectiveness of the method is verified by performing computer simulations.

Keywords: Path following, backstepping, the method of least squares, cascade systems.

1. INTRODUCTION

In marine applications, path following is referred to the task of forcing a marine craft to follow a geometric path without imposing a timing law; i.e. it is not specified when the craft has to be at a given point on the path. Path following of marine craft is required in many operations such as cable laying, towing, and dredging, and control systems must be designed in a way that they act accurately and cost-effectively. In this sort of operations, fully actuated marine craft are typically employed.

A great of number of articles have been published on motion control of marine craft. Much of the work on path following is rooted in the work of Samson (1992) where land robots are considered. The path maneuvering problem addressed in (Hauser and Hindman, 1997) was generalized by Skjetne et al. (2004) where the geometric task of regulating the position and orientation is decoupled from the dynamic task of controlling the speed of the craft along the path.

In maritime applications, a classical method for path following is to define an error space using the concept of Serret-Frenet frame; e.g. see (Encarnacao et al., 2000; Lapierre and Soetanto, 2007). The principles of guidance-based path following were reviewed in (Breivik and Fossen, 2005a). A nonlinear adaptive path-following controller was proposed for fully actuated vessels in (Almeida et al., 2007) to cope with ocean currents. To deal with modeling uncertainties, Kaminer et al. (2005) proposed a robust path-following controller for fully actuated marine vehicles. Fossen (2011) provides a profound insight into marine control systems.

Generally, path-following controllers for marine craft comprise decoupled speed controllers and heading autopilots; e.g. (Fredriksen and Pettersen, 2006; Breivik and Fossen, 2005b). As a result, the controller always tries to maintain the speed as desired even if the marine craft does not move on the path. This is while a captain may change the speed according to the distance to the path and the rate of convergence.

Peymani and Fossen (2012) proposed a controller based on backstepping such that the control system increases the forward velocity when the craft is not on the path. In the present paper, the authors provide an alternative controller which modifies the speed according to the geometric error, which is the error between the craft and a desired point on the path.

The main contribution of this paper is to propose a 2-dimensional path-following controller that is capable of manipulating the speed of the marine craft when the craft is off the path. In fact, the speed of the craft depends on the geometric error and its derivative. It is also shown that the proposed controller enhances robustness with respect to external disturbances. A method is, moreover, introduced to make the craft move on the path in the presence of constant external disturbances by sacrificing the speed assignment task; indeed, we propose a method to resolve the inherent drawback of those path-following controllers that are based on the line-of-sight guidance system.

2. PROBLEM STATEMENT

The paper deals with the path-following problem for 3-DOF marine craft. Particularly, a controller is designed such that a marine vehicle converges to and follows a desired path; it imposes a set of geometric constraints on the position and orientation of the vehicle. In addition, path following requires that the speed of the vehicle tracks a desired nonzero speed profile. As moving on the path is more important than moving with the desired speed, the geometric task takes precedence over the speed-assignment task. According to Skjetne et al. (2004), these two tasks can be executed separately.

2.1 *Model of 3-DOF Marine Craft*

Consider the vehicle pose $q = [p^{\mathrm{T}}, \psi]^{\mathrm{T}}$ where $p = [x, y]^{\mathrm{T}} \in \mathbb{R}^2$ is the earth-fixed position and $\psi \in S$ is the yaw angle. Let $\nu = [u, v, r]^{\mathrm{T}} \in \mathbb{R}^3$ where u and v are the components of the speed expressed in the body-fixed reference frame, denoted $\{b\}$, and r is the angular velocity around the z-axis of $\{b\}$. Let $J(\psi) = \text{diag}\{R(\psi), 1\}$ be the rotation matrix from $\{b\}$ to the inertial reference frame. The matrix $R(\psi)$ is given by

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix} \in SO(2) \tag{1}$$

According to Fossen (2011), the dynamic equations of motion are described by

$$\dot{q} = J(\psi)\nu \tag{2a}$$

$$M_b\dot{\nu} + C_b(\nu)\nu + D_b(\nu)\nu = \tau_b \tag{2b}$$

in which $M_b = M_b^T > 0$, $\dot{M}_b = 0$, $C_b = -C_b^T$, and $D_b > 0$. Homogeneous mass distribution and xz-plane symmetry are presumed, and the surge is assumed to be decoupled from the sway-yaw subsystem; thus, the system matrices take the following structures

$$M_b = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}, \quad D_b = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{23} & d_{33} \end{bmatrix} \tag{2c}$$

$$C_b = \begin{bmatrix} 0 & 0 & -(m_{22}\nu + m_{23}r) \\ 0 & 0 & m_{11}u \\ (m_{22}\nu + m_{23}r) & -m_{11}u & 0 \end{bmatrix} \tag{2d}$$

where damping is assumed to be linear. In (2b), $\tau_b \triangleq [\tau_u, \tau_v, \tau_r]^T$ represents the vector of generalized forces, expressed in $\{b\}$, which captures the forces and moments due to actuators as well as due to external disturbances.

2.2 Guidance System

A guidance system is required to provide the desired heading so that the vessel moves toward the path smoothly. In fact, the guidance system maps the desired position onto the desired heading angle. We employ the line-of-sight (LOS) guidance system (Fossen, 2011, Ch.10). Consider a straight-line path connecting the points p_k and p_{k+1}. The slope of the path is denoted ψ_k. Also consider a path-fixed reference frame, represented by $\{p_k\}$, that originates at p_k. Its x-axis has been rotated by a positive angle ψ_k.

Let $p_{los} = [x_{los}, y_{los}]^T$ be the desired point on the path that the vessel has to reach at each time instant. To find the point p_{los}, the lookahead-based steering method (Fossen, 2011) is utilized, in which p_{los} is a point on the path which is located a lookahead distance $\Delta > 0$ ahead of the direct projection of p onto the path. See Fig. 1. The LOS vector is the vector from p to p_{los}. The LOS angle, denoted ψ_{los}, is the angle that the LOS vector makes with the x-axis of the inertial frame.

Let $e(t)$ denote the cross-track error, and let $s(t)$ be the along-track error. Defining $\varepsilon \triangleq [s, e]^T$, one can find

$$\varepsilon = R(\psi_k)^T (p - p_k) \tag{4}$$

The objective is to align the x-axis of $\{b\}$ with the LOS vector. Equivalently, the heading (yaw) angle has to track the LOS angle, which is computed using:

$$\psi_{los} = \psi_k + \psi_r \tag{5}$$

where the relative angle (approach angle) ψ_r is found using:

$$\psi_r = \arctan\left(-\frac{e}{\Delta}\right) \tag{6}$$

This work focuses on straight-line paths. The result can be extended to waypoint tracking where the path is described by a set of points connected by straight-line segments; see (Fossen, 2011, Ch.10).

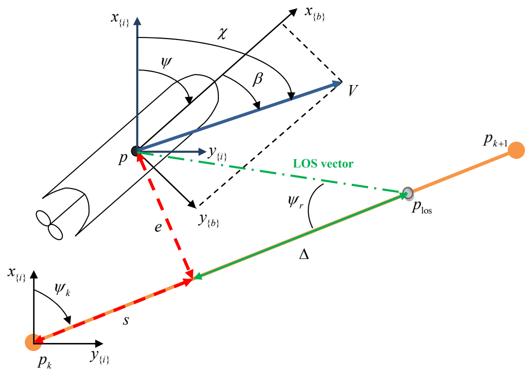

Fig. 1. The geometric representation of the straight-line path-following problem.

2.3 Problem Formulation

The primary objective is to converge to the path and follow it. Convergence to the path, which is referred to as the geometric task, is formulated as

$$\lim_{t\to\infty} e(t) = 0 \tag{7a}$$

The marine craft should converge to the path smoothly; so, the heading angle has to track a desired angle; that is:

$$\lim_{t\to\infty} (\psi - \psi_d) = 0 \tag{7b}$$

We choose the desired heading angle as $\psi_d = \psi_{los}$. The secondary objective is to regulate the speed to a desired value; it is stated as:

$$\lim_{t\to\infty} (u - u_d) = 0, \qquad \lim_{t\to\infty} \nu = 0 \tag{7c}$$

By the secondary objective, we mean that the dynamic task of speed assignment has less importance than the geometric task, and it can be sacrificed so as to have the main objective satisfied.

Path-following Problem. Consider a 3-DOF fully-actuated marine craft described by (2). Given a path and a desired speed u_d, the problem is to find a stabilizing controller such that the objective (7) is achieved. ◀

The standard solution for the path-following problem is to design a speed controller decoupled from a heading autopilot; see e.g. (Fredriksen and Pettersen, 2006; Fossen et al., 2003). However, in this paper, we intend to find a controller such that the speed depends on the cross-track error and its derivative.

Problem 1. Solve the Path-following Problem where u has to track $u_d^*(u_d, e)$ that has to be specified appropriately such that $u_d^* \to u_d$ as time tends to ∞. ◀

3. CONTROL DESIGN METHOD

The control law is designed in two steps. In the first step, the accelerations that are required for an exponential convergence to the path are derived. The second step is devoted to find the accelerations that satisfy (7b) and (7c). Finally, the control laws are derived based on the method of least squares, which is utilized to find the best approximate for the achieved accelerations.

3.1 Accelerations to Make Cross-track Error zero

We aim to make the cross-track error $e(t)$ converge to zero as time tends to infinity. Let $\rho_1(\psi)$, $\rho_2(\psi)$, and ρ_3 be as

$$\begin{array}{ll}\rho_1(\psi)^{\mathrm{T}} &= [\cos(\psi),\ -\sin(\psi),\ 0] \\ \rho_2(\psi)^{\mathrm{T}} &= [\sin(\psi),\quad \cos(\psi),\ 0] \\ \rho_3^{\mathrm{T}} &= [0,\qquad 0,\quad 1]\end{array} \Rightarrow J(\psi) = \begin{bmatrix}\rho_1(\psi)^{\mathrm{T}} \\ \rho_2(\psi)^{\mathrm{T}} \\ \rho_3^{\mathrm{T}}\end{bmatrix}$$

According to (4),

$$\dot{e} = \rho_2(\gamma)^{\mathrm{T}}\nu, \qquad \ddot{e} = \rho_2(\gamma)^{\mathrm{T}}\dot{\nu} + \dot{\rho}_2(\gamma)^{\mathrm{T}}\nu \tag{8}$$

in which $\gamma = \psi - \psi_k$. The objective is recast to make $X = [e, \dot{e}]^{\mathrm{T}}$ globally asymptotically/exponentially stable (GAS/GES) at the origin. The dynamics of X is given by:

$$\dot{X} = AX + Bu_{\mathrm{e}} \quad \text{where } A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, \quad B = \begin{bmatrix}0\\1\end{bmatrix} \tag{9}$$

The virtual control input u_{e} is utilized to stabilize e at the origin. As the pair (A,B) is controllable, there exists a vector $K_{\mathrm{e}} = [k_{\mathrm{e}1},\ k_{\mathrm{e}2}]^{\mathrm{T}}$ such that the state-feedback control law

$$u_e = -K_{\mathrm{e}}^{\mathrm{T}}X \tag{10}$$

renders the equilibrium point ($X = 0$) GES. In fact, there exits $\mathscr{P}_{\mathrm{e}} = \mathscr{P}_{\mathrm{e}}^{\mathrm{T}} > 0$ and $V_{\mathrm{e}} = X^{\mathrm{T}}\mathscr{P}_{\mathrm{e}}X$ such that $\dot{V}_{\mathrm{e}} < 0$ for $X \neq 0$. Closing the loop with (10) and considering (8), it follows that

$$\rho_2(\gamma)^{\mathrm{T}}\dot{\nu} - \sigma_{\mathrm{e}} = 0 \ \text{ where } \ \sigma_{\mathrm{e}} = -\dot{\rho}_2(\gamma)^{\mathrm{T}}\nu - k_{\mathrm{e}2}\dot{e} - k_{\mathrm{e}1}e \tag{11}$$

Eq. (11) yields the *desired* accelerations that make the vehicle converge to the path with an exponential rate. It places no constraints on the rate of rotation (i.e. $\dot{r}$).

3.2 Accelerations to Achieve Heading and Speed Objectives

Define $z_0 \triangleq \psi - \psi_{\mathrm{d}}$. Then, $\dot{z}_0 = \dot{\psi} - \dot{\psi}_{\mathrm{d}} = \rho_3^{\mathrm{T}}\nu - \dot{\psi}_{\mathrm{d}}$. Consider $V_1 = \frac{1}{2}z_0^2$ and differentiate it in time:

$$\dot{V}_1 = z_0\dot{z}_0 = z_0(\rho_3^{\mathrm{T}}\nu - \dot{\psi}_{\mathrm{d}}) \tag{12}$$

To regulate z_0 to zero, the system velocities ν are chosen as virtual control inputs; we define $\nu \triangleq z + \alpha$ where the new state variables z and the vector of stabilizing functions α are as

$$z = [\, z_1,\ z_2,\ z_3 \,]^{\mathrm{T}}, \qquad \alpha = [\, \alpha_1,\ \alpha_2,\ \alpha_3 \,]^{\mathrm{T}}$$

Therefore, (12) can be written as

$$\dot{V}_1 = z_0(\rho_3^{\mathrm{T}}z + \alpha_3 - \dot{\psi}_{\mathrm{d}}) \tag{13}$$

Choosing $\alpha_3 = \dot{\psi}_{\mathrm{d}} - k_0 z_0$ yields

$$\dot{V}_1 = -k_0 z_0^2 + z^{\mathrm{T}}\rho_3 z_0, \qquad k_0 > 0 \tag{14}$$

Now, the goal is to stabilize z at the origin. Choose $\alpha_2 = 0$ and $\alpha_1 = u_{\mathrm{d}}$. It implies that if $z \to 0$, u and v will converge to α_1 and α_2, respectively. The dynamics of z are given by $\dot{z} = \dot{\nu} - \dot{\alpha}$. Consider $V_2 = V_1 + \frac{1}{2}z^{\mathrm{T}}z$ and differentiate V_2 along the trajectory of the system (z_0,z):

$$\dot{V}_2 = -k_0 z_0^2 + z^{\mathrm{T}}(\rho_3 z_0 + \dot{\nu} - \dot{\alpha}) \tag{15}$$

Let $\sigma_{\mathrm{z}} = \dot{\alpha} - \rho_3 z_0 - Kz$ where $K \triangleq \mathrm{diag}\{k_1, k_2, k_3\} > 0$. Therefore, if the constraint

$$\dot{\nu} - \sigma_{\mathrm{z}} = 0 \tag{16}$$

holds, it turns out that

$$\dot{V}_2 = -k_0 z_0^2 - z^{\mathrm{T}}Kz < 0, \qquad \forall z_0 \neq 0, \forall z \neq 0 \tag{17}$$

3.3 Accelerations to Achieve All Objectives

In view of (11) and (16), one may write:

$$H(\gamma)\dot{\nu} = b(\gamma, \phi) \tag{18}$$

in which $\phi \triangleq [e, \dot{e}, z_0, z^{\mathrm{T}}]^{\mathrm{T}}$, and

$$H(\gamma) = \begin{bmatrix}\mathrm{I}_3 \\ \rho_2(\gamma)^{\mathrm{T}}\end{bmatrix}, \qquad b(\gamma, \phi) = \begin{bmatrix}\sigma_{\mathrm{z}} \\ \sigma_{\mathrm{e}}\end{bmatrix} \tag{19}$$

where I_i is the $i \times i$ identity matrix. Define

$$H_{\mathrm{b}}(\gamma) \triangleq H(\gamma)^{\mathrm{T}}H(\gamma) = \mathrm{I}_3 + \rho_2(\gamma)\rho_2(\gamma)^{\mathrm{T}} \tag{20}$$

which is non-singular $\forall\gamma$. Therefore, $H_{\mathrm{b}}^{-1}(\gamma)$ exists and

$$H_{\mathrm{b}}^{-1}(\gamma) = \mathrm{I}_3 - \frac{1}{2}\rho_2(\gamma)\rho_2(\gamma)^{\mathrm{T}} \tag{21}$$

To find $\dot{\nu}$, both sides of (18) are pre-multiplied by $\bar{H}(\gamma) = H_{\mathrm{b}}^{-1}(\gamma)H(\gamma)^{\mathrm{T}}$. It gives rise to:

$$\dot{\nu} = \bar{H}(\gamma)\, b(\gamma, \phi) \tag{22}$$

One may perceive $\bar{H}$ as the Moore-Penrose psuedoinverse of H. Substituting (22) in the equations of motion (2b) gives the control forces that are required to make the marine craft have the acquired acceleration (22). The control laws are given by

$$\tau_{\mathrm{b}}^{\mathrm{p}} = M_{\mathrm{b}}\bar{H}(\gamma)b(\gamma, \phi) + C_{\mathrm{b}}(\nu)\nu + D_{\mathrm{b}}(\nu)\nu \tag{23}$$

The solution for $\dot{\nu}$ exists if and only if $b \in \mathrm{im}\, H$, which is not valid in general. Equation (22) yields the best approximate for $\dot{\nu}$ such that the function $\|H(\gamma)\dot{\nu} - b(\gamma, \phi)\|^2$ is minimized. One should notice that taking the Lyapunov function $V = V_{\mathrm{e}} + V_2$ is therefore meaningless, and it cannot be used to establish the stability of the closed-loop system. Hence, it is crucial to investigate the stability of the closed-loop system under the derived control law.

4. MAIN RESULT

In this section, we study the stability of the closed-loop system. To facilitate analysis, we make a change in the control law (23). Clearly, $\dot{\rho}_2(\gamma)^{\mathrm{T}} = \dot{\gamma}\rho_1(\gamma)^{\mathrm{T}}$. On the other hand, according to (4), $\rho_1(\gamma)^{\mathrm{T}}\nu = \dot{s}(t)$ which is the speed of the craft along the path. We replace $\rho_1(\gamma)^{\mathrm{T}}\nu$ with u_{d}, which is reasonable since the vehicle is supposed to move along the path with the desired speed. Accordingly, (11) is altered to (is replaced with)

$$\sigma_{\mathrm{e}}^* = -\dot{\gamma}u_{\mathrm{d}} - k_{\mathrm{e}2}\dot{e} - k_{\mathrm{e}1}e \tag{24}$$

Then, we define $b^* = [\sigma_{\mathrm{z}}^{\mathrm{T}}, \sigma_{\mathrm{e}}^*]^{\mathrm{T}}$ and use b^* instead of b in (18). It gives rise to the following control law

$$\tau_{\mathrm{b}}^* = M_{\mathrm{b}}\bar{H}(\gamma)b^*(\gamma, \phi) + C_{\mathrm{b}}(\nu)\nu + D_{\mathrm{b}}(\nu)\nu \tag{25}$$

Theorem 1 provides a solution for Problem 1.

Theorem 1. Let u_{d} and Δ be positive constants. Apply the control law (25) to the system (2). The origin $(e, z_0, z) = 0$ is globally asymptotically stable if

T1.1 $k_0, k_3, k_{\mathrm{e}1} > 0$ and $k_{\mathrm{e}2} > u_{\mathrm{d}}/\Delta$;
T1.2 $k_1 = k_2 = k$ such that $k > 3u_{\mathrm{d}}k_{\mathrm{e}2}^2/(4k_{\mathrm{e}1}\Delta)$.

Proof. The proof of the theorem relies on the theory of nonlinear composite systems (Jankovic et al., 1996). We find the closed-loop equations. In view of (21), one can write

$$\bar{H} = (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}} = \left[\, \mathrm{I}_3 - \tfrac{1}{2}\rho_2\rho_2^{\mathrm{T}}, \quad \tfrac{1}{2}\rho_2 \,\right] \tag{26}$$

where we have dropped the argument γ. From $\dot{\nu} = \bar{H}(\gamma)\, b^*(\gamma, \phi)$, it follows that

$$\dot{\nu} = (\mathrm{I}_3 - \tfrac{1}{2}\rho_2\rho_2^{\mathrm{T}})(\dot{\alpha} - \rho_3 z_0 - Kz) - \tfrac{1}{2}\rho_2(\dot{\gamma}u_{\mathrm{d}} + k_{\mathrm{e}2}\dot{e} + k_{\mathrm{e}1}e)$$

One may find $\dot{\gamma} = z_3 + \dot{\psi}_{\mathrm{d}} - k_0 z_0$. Recalling $\psi_{\mathrm{d}} = \psi_{\mathrm{los}}$, it is straightforward to show that $\dot{\psi}_{\mathrm{d}} = \dot{\psi}_{\mathrm{r}}$; thus, we obtain

$$\dot{\psi}_{\mathrm{d}} = -\frac{\Delta}{e^2 + \Delta^2}\dot{e} \tag{27}$$

Also, notice that

$$\rho_2\rho_2^{\mathrm{T}} = \begin{bmatrix}\sin^2(\gamma) & \sin(\gamma)\cos(\gamma) & 0 \\ \sin(\gamma)\cos(\gamma) & \cos^2(\gamma) & 0 \\ 0 & 0 & 0\end{bmatrix}$$

Since $\dot{\alpha}_1 = \dot{\alpha}_2 = 0$, we obtain $\rho_2\rho_2^{\mathrm{T}}\dot{\alpha} = 0$. Moreover, one may find that $\rho_2\rho_2^{\mathrm{T}}\sigma_{\mathrm{z}}$ and $\rho_2\sigma_{\mathrm{e}}$ do not influence $\dot{z}_3$. Find $\bar{\rho}_i(\psi)$ for $i = 1,2$ such that

$$\rho_i(\psi) = [\bar{\rho}_i(\psi)^{\mathrm{T}}, 0]^{\mathrm{T}} \quad \text{for} \quad i = 1,2 \tag{28}$$

Define $\bar{z} \triangleq [z_1, z_2]^{\mathrm{T}}$. The closed-loop equations are:

$$\Sigma_2 : \begin{cases} \dot{z}_0 = -k_0 z_0 + z_3 & (29a) \\ \dot{z}_3 = -z_0 - k_3 z_3 & (29b) \end{cases}$$

$$\dot{\bar{z}} = -(\mathrm{I}_2 - \frac{1}{2}\bar{\rho}_2(\gamma)\bar{\rho}_2(\gamma)^{\mathrm{T}})\bar{K}\bar{z} - \frac{1}{2}\bar{\rho}_2(k_{\mathrm{e}2}\dot{e} + k_{\mathrm{e}1}e) + \frac{1}{2}\bar{\rho}_2 u_{\mathrm{d}}\frac{\Delta\dot{e}}{e^2+\Delta^2} - \frac{1}{2}\bar{\rho}_2(z_3 - k_0 z_0)u_{\mathrm{d}} \tag{30}$$

It is required to include $\dot{e}$ since (30) depends on it. According to (8), $\dot{e} = u\sin(\gamma) + v\cos(\gamma)$. Note that $\gamma = \psi - \psi_k = z_0 + \psi_{\mathrm{r}}$. Thus, one can write

$$\sin(z_0 + \psi_{\mathrm{r}}) = \sin(\psi_{\mathrm{r}}) + g_{\sin}(e, z_0)z_0 \tag{31a}$$

$$\cos(z_0 + \psi_{\mathrm{r}}) = \cos(\psi_{\mathrm{r}}) + g_{\cos}(e, z_0)z_0 \tag{31b}$$

Functions $g_{\sin}(e,z_0)$ and $g_{\cos}(e,z_0)$, given in Appendix A, are globally bounded. According to the guidance system, we have

$$\sin(\psi_{\mathrm{r}}) = \frac{-e}{\sqrt{e^2+\Delta^2}}, \qquad \cos(\psi_{\mathrm{r}}) = \frac{\Delta}{\sqrt{e^2+\Delta^2}} \tag{32}$$

Define $\zeta \triangleq [z_0, z_3]^{\mathrm{T}}$ and $\xi \triangleq [e, z_1, z_2]^{\mathrm{T}}$. Now, we are ready to express the dynamics of the cross-track error and recast (30):

$$\dot{e} = -\frac{u_{\mathrm{d}} + z_1}{\sqrt{e^2+\Delta^2}}e + \frac{z_2\Delta}{\sqrt{e^2+\Delta^2}} + g_e(\xi,\zeta)\zeta \tag{33a}$$

$$\dot{\bar{z}} = -\bar{K}_{\bar{z}}(\psi_{\mathrm{r}})\bar{z} - \tfrac{1}{2}\bar{\rho}_2(\psi_{\mathrm{r}})k_{\mathrm{e}1}e + \tfrac{1}{2}\bar{\rho}_2(\psi_{\mathrm{r}})\Omega_1\left(\frac{u_{\mathrm{d}} + z_1}{\sqrt{e^2+\Delta^2}}e - \frac{z_2\Delta}{\sqrt{e^2+\Delta^2}}\right) + g_{\bar{z}}(\xi,\zeta)\zeta \tag{33b}$$

where $g_{\bar{z}}(\xi,\zeta)$ and $g_e(\xi,\zeta)$ are given in Appendix A, and

$$\bar{K}_{\bar{z}}(\psi_{\mathrm{r}}) = (\mathrm{I}_2 - \frac{1}{2}\bar{\rho}_2(\psi_{\mathrm{r}})\bar{\rho}_2^{\mathrm{T}}(\psi_{\mathrm{r}}))\bar{K} \tag{34}$$

$$\Omega_1 = k_{\mathrm{e}2} - \frac{\Delta}{e^2+\Delta^2}u_{\mathrm{d}} \tag{35}$$

in which $\bar{K} = \mathrm{diag}\{k_1, k_2\}$. Hence, the closed-loop system, comprising (29) and (33), is a nonlinear composite system, which can be written as

$$\dot{\xi} = f(\xi) + g(\xi,\zeta) \tag{36a}$$

$$\dot{\zeta} = A_2\zeta \tag{36b}$$

where $f(\xi)$, $g(\xi,\zeta)$ and A_2 are found from (33) and (29). In other words, the system described by (33) is regarded as a nonlinear system cascaded with the linear system described by (29) through the interconnection term

$$g(\xi,\zeta) = \begin{bmatrix} g_e(\xi,\zeta) \\ g_{\bar{z}}(\xi,\zeta) \end{bmatrix}\zeta \tag{37}$$

To prove the global asymptotic stability of $(\xi,\zeta) = 0$, we invoke (Seibert and Suarez, 1990, Corollary 4.3) and (Jankovic et al., 1996, Lemma 1).

The perturbing system Σ_2 described by (29) is globally exponentially stable if $k_0, k_3 > 0$. This is established by choosing a positive definite, radially unbounded Lyapunov function $W_2 = z_0^2 + z_3^2$.

Lemma 1 formally expresses the circumstances under which the origin of (33) when $\zeta = 0$ (i.e. the origin of the system $\dot{\xi} = f(\xi)$) is established to be globally asymptotically stable.

Lemma 1. Under conditions **T1.1** and **T1.2**, the origin of the unperturbed system (33) (i.e. when $\zeta = 0$) is globally asymptotically stable. It is established using a quadratic Lyapunov function.

Proof. See Appendix B.

Thus, from (Seibert and Suarez, 1990, Corollary 4.3), it is observed that the origin of the closed-loop system (36) is GAS if all the solutions are bounded. To prove boundedness of all the solutions, we show that the conditions of (Jankovic et al., 1996, Lemma 1) hold.

The interconnection term $g(\xi,\zeta)$, given by (37), vanishes at $\zeta = 0$ and is globally Lipschitz in ξ for any fixed ζ. It follows from Property 1 in Appendix A that $g(\xi,\zeta)$ has linear growth in ξ. According to Lemma 1, a radially unbounded polynomial Lyapunov function is used to prove GAS of the unperturbed system (33). Hence, all the solutions are globally bounded according to (Jankovic et al., 1996, Lemma 1) and $(\xi,\zeta) = 0$ is GAS according to (Seibert and Suarez, 1990, Corollary 4.3). The proof is now complete. ■

According to Assumptions **T1.1** and **T1.2**, the controller gains $k_{\mathrm{e}1}$, $k_{\mathrm{e}2}$, k_0, k_1, k_2 and k_3 can be found for *any* choice of the desired speed $u_{\mathrm{d}} > 0$ and the lookahead distance $\Delta > 0$.

4.1 Properties of Proposed Controller

Now that we have established that the proposed control law accomplishes the objectives, we elucidate the properties of the controller.

a. Manipulation of Speed The proposed controller (25) modifies the speed of the marine craft when the cross-track error, e, is nonzero. To see how it happens, using (26), one may show that the control law can be decomposed into two distinct parts:

$$\tau_{\mathrm{b}}^* = \tau_{\mathrm{n}} + \tau_{\mathrm{e}} \tag{38}$$

where

$$\tau_{\mathrm{n}} = M_{\mathrm{b}}\sigma_{\mathrm{z}} + (C_{\mathrm{b}}(\nu) + D_{\mathrm{b}}(\nu))\,\nu$$

$$\tau_{\mathrm{e}} = \frac{1}{2}M_{\mathrm{b}}\rho_2(\gamma)\left(-\rho_2(\gamma)^{\mathrm{T}}\sigma_{\mathrm{z}} + \sigma_{\mathrm{e}}^*\right)$$

The control law τ_{n} is the control law that one obtains if (11) is not considered. Much of work on path following of marine craft introduces such controllers; for example, (Fredriksen and Pettersen, 2006; Fossen et al., 2003) which can be adapted easily for fully actuated vehicles. The control force τ_{n} intends to regulate z_0 and z.

However, in the proposed path-following controller, τ_{e} makes a difference. The term σ_{e}^* is nonzero when e and $\dot{e}$ are nonzero. On the other hand, due to the structure of $\rho_2(\gamma)$, τ_{e} only affects the dynamics of the linear velocities, and does not influence the heading dynamics. Hence, the proposed controller modifies the speed of the craft in case the geometric error is nonzero, and the speed assignment objective is sacrificed so as to fulfil the path-following (geometric) task.

b. Robustness to Ocean Currents The proposed method makes the geometric task robust with respect to external disturbances to some extent because the controller changes the vehicle's speed when $e \neq 0$. More important, it is possible to obtain zero cross-track error in the presence of constant disturbances by means of augmentation of integral action to the control system. Augment

$$\dot{e}_{\mathrm{I}} = u_{\mathrm{d}} \frac{e}{\sqrt{e^2 + \Delta^2}} \tag{39}$$

to the system described by (8). In (11), replace σ_{e} with $\sigma_{\mathrm{e,I}}$ which is given by

$$\sigma_{\mathrm{e,I}} = \dot{\gamma} u_{\mathrm{d}} + k_{\mathrm{e2}} \dot{e} + k_{\mathrm{e1}} e + k_{\mathrm{e0}} e_{\mathrm{I}} \tag{40}$$

in which, with an argument similar to the previous section, $\dot{\rho}_2(\gamma)^{\mathrm{T}} \nu$ has been replaced with $\dot{\gamma} u_{\mathrm{d}}$. Then, form $b_{\mathrm{I}}^* = [\sigma_z^{\mathrm{T}}, \sigma_{\mathrm{e,I}}]^{\mathrm{T}}$ and use it to derive the control law

$$\tau_{\mathrm{b,I}}^* = M_{\mathrm{b}} \bar{H}(\gamma) b_{\mathrm{I}}^*(\gamma, \phi) + C_{\mathrm{b}}(\nu)\nu + D_{\mathrm{b}}(\nu)\nu \tag{41}$$

Theorem 2 states the result formally.

Theorem 2. Let u_{d} and Δ be positive constants. Apply the control law (41) to system (2). Global asymptotic stability (GAS) of $(e_{\mathrm{I}}, e, z_0, z) = 0$ is guaranteed if

T2.1 $k_0, k_3 > 0$, $k_{\mathrm{e1}} > k_{\mathrm{e0}} > 0$ and $k_{\mathrm{e2}} > u_{\mathrm{d}}/\Delta$;
T2.2 $k_1 = k_2 = k$ such that $k > 3u_{\mathrm{d}} k_{\mathrm{e2}}^2 / (4\Delta(k_{\mathrm{e1}} - k_{\mathrm{e0}}))$.

Proof. See Peymani (2013).

5. SIMULATION RESULTS

A ship's model is chosen according to (2) where

$$M_{\mathrm{b}} = \begin{bmatrix} 2376.4 & 0 & 0 \\ 0 & 3949.9 & 2891.8 \\ 0 & 2891.8 & 3349.8 \end{bmatrix}, D_{\mathrm{b}} = \begin{bmatrix} 354 & 0 & 0 \\ 0 & 346.8 & -435.8 \\ 0 & 686.1 & 1427.2 \end{bmatrix}$$

The initial conditions are chosen as $q(0) = [10,\ -250,\ \pi/4]^{\mathrm{T}}$ and $\nu(0) = [1,\ 0,\ 0]^{\mathrm{T}}$. The objective is to converge to and follow a straight-line path which is parallel to the y-axis of $\{i\}$, 40 meters to the north. It is assumed that there exists a current flow whose speed expressed in $\{i\}$ is $U_c = [+.75,\ 0,\ 0]^{\mathrm{T}}$(m/s). The relative velocity $\nu_r = \nu - J(\psi)^{\mathrm{T}} U_{\mathrm{c}}$ is considered in the simulation model, which is different from the control model. We choose $u_{\mathrm{d}} = 2$ (m/s) and $\Delta = 20$ (m). Thus, $k_{\mathrm{e2}} > 0.1$; we choose $k_{\mathrm{e1}} = 2$ and $k_{\mathrm{e2}} = 1$. If integrator is considered, $k_{\mathrm{e0}} = 0.5$. Then, **T2.2** implies that $k > 3$; we choose $k = 10$. Also, $k_0 = 3$ and $k_3 = 1$. It is observed that the conditions of the theorems are not restrictive for practical situations.

We make a comparison between a controller with integral action (labeled with 'LS with integral action') and a controller without considering integral action (labeled with 'LS without integral action') to discern the disturbance rejection properties of the control systems. We also run simulations with the method presented in (Fossen et al., 2003) (labeled with 'Standard method') but we adapted the method for fully actuated vessels. The result is shown in Figs. 2 and 3.

As expected, the least-squares approach with augmentation of integral action results in zero steady-state cross-track error while the other errors are nonzero. It is also realized that the least-square approach leads to faster convergence with respect to the standard method. As explained before, the explicit incorporation of the geometric error in the design of the speed loop will lead to more robust response to external disturbances as the steady-state cross-track error is smaller than that of the standard method.

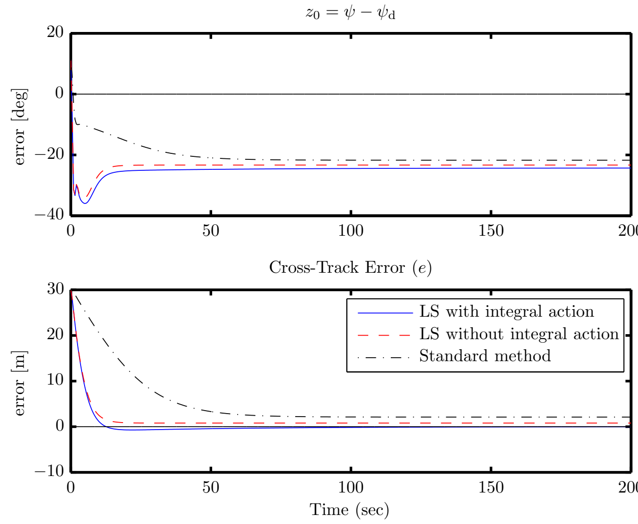

Fig. 2. The heading error and the cross-track error.

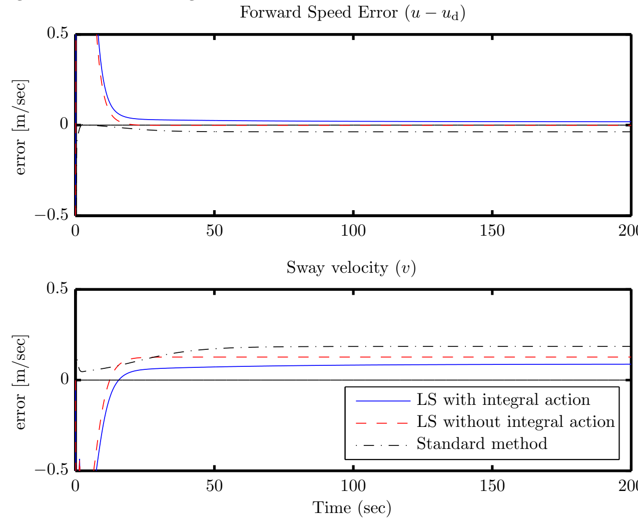

Fig. 3. The speed assignment task.

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Appendix A. REQUIRED RELATIONS

Denote $q_1 = \frac{\sin(z_0)}{z_0}$ and $q_2 = \frac{\cos(z_0)-1}{z_0}$, which are well-defined functions and globally bounded. Considering (31), we have

$$g_{\sin}(\psi_r, z_0) = q_1\cos(\psi_r) + q_2\sin(\psi_r)$$
$$g_{\cos}(\psi_r, z_0) = -q_1\sin(\psi_r) + q_2\cos(\psi_r)$$

In this regard, one may find

$$\bar{\rho}_2(\gamma) = \bar{\rho}_2(\psi_r) + \mathscr{R}_g(\psi_r, z_0)\zeta$$

in which $\mathscr{R}_g = [\bar{\rho}_{2g}(\psi_r, z_0),\ 0]$ where

$$\bar{\rho}_{2g}(\psi_r, z_0) = \begin{bmatrix} g_{\sin}(\psi_r, z_0) \\ g_{\cos}(\psi_r, z_0) \end{bmatrix}$$

Accordingly, in (33a), one may find

$$g_e(\xi,\zeta) = [(u_d + z_1)g_{\sin}(\xi,\zeta) + z_2 g_{\cos}(\xi,\zeta), \quad 0]$$

The function $g_{\bar{z}}(\xi,\zeta)$ in (33b) is equal to

$$g_{\bar{z}}(\xi,\zeta) = \tfrac{1}{2}\bar{\rho}_2(\gamma)([k_0 u_d, -u_d] - \Omega_1 g_e(\xi,\zeta)) - \tfrac{1}{2}G^*_{\bar{z}}(\xi,\zeta)$$
$$- \tfrac{1}{2}\mathscr{R}_g(k_{e1}e + \Omega_1(u_d + z_1)\sin(\psi_r) + \Omega_1 z_2\cos(\psi_r)))$$

where $G^*_{\bar{z}}(\xi,\zeta) = [G_{z_0}(\xi,\zeta)\bar{K}\bar{z},\ 0]$ in which

$$G_{z_0}(\xi,\zeta)z_0 = \bar{\rho}_2(\gamma)\bar{\rho}_2^{\mathrm{T}}(\gamma) - \bar{\rho}_2(\psi_r)\bar{\rho}_2^{\mathrm{T}}(\psi_r)$$

is a 2×2 matrix. Let $g_{\bar{z},ij}$ be element (i,j) of $G_{z_0}(\xi,\zeta)$. Then, one may find

$$g_{\bar{z},11} = z_0 g^2_{\sin}(\psi_r, z_0) + 2g_{\sin}(\psi_r, z_0)\sin(\psi_r)$$
$$g_{\bar{z},22} = z_0 g^2_{\cos}(\psi_r, z_0) + 2g_{\cos}(\psi_r, z_0)\cos(\psi_r)$$
$$g_{\bar{z},12} = g_{\bar{z},21} = g_{\sin}(2\psi_r, 2z_0)$$

The following property is easily established.

Property 1. $g_e(\xi,\zeta)$ and $g_{\bar{z}}(\xi,\zeta)$ grow linearly in ξ; i.e.

$$\|g_x(\xi,\zeta)\| \le \sigma_{x1}(\|\zeta\|) + \sigma_{x2}(\|\zeta\|)\|\xi\|, \quad x = e, \bar{z}$$

where $\sigma_{x1}, \sigma_{x2} : [0,\infty) \to [0,\infty)$ are continuous. □

Appendix B. PROOF OF LEMMA 1

According to **T1.1**, Ω_1, given by (35), is always a positive value; i.e. $k_{e2} > \Omega_1 \ge \Omega_1^* > 0, \forall e$.

Property 2. Considering a nonzero vector $x = [x,y]^{\mathrm{T}}$, the next inequality holds

$$x^{\mathrm{T}}(\mathrm{I}_2 - \frac{1}{2}\bar{\rho}_2(\psi_r)\bar{\rho}_2^{\mathrm{T}}(\psi_r))x \ge \frac{1}{2}x^{\mathrm{T}}x \qquad \square$$

Rewrite $\bar{\rho}_2(\psi_r)$ in view of (32). For the sake of clarity, define $\Gamma \triangleq \frac{1}{\sqrt{e^2+\Delta^2}}$ and $\Pi \triangleq \Gamma^2$. Then, (33b) is recast as:

$$\dot{\bar{z}} = -\bar{K}_{\bar{z}}(\psi_r)\bar{z} - \begin{bmatrix} -e \\ \Delta \end{bmatrix}\left(\frac{k_{e1}\Gamma e}{2} - \frac{\Omega_1\Pi}{2}((u_d + z_1)e - \Delta z_2)\right)$$

Let $k_1 = k_2 = k$. Choose $V_1 = \frac{1}{2}\bar{z}^{\mathrm{T}}\bar{K}\bar{z}$ where $\bar{K} = k\mathrm{I}_2 > 0$. Differentiation yields

$$\dot{V}_1 = -\bar{K}\bar{K}_{\bar{z}}(\psi_r)\bar{z} + \frac{k}{2}(k_{e1}\Gamma - \Omega_1\Pi u_d)(e^2 z_1 - e z_2\Delta)$$
$$- \frac{k\Omega_1}{2}\left(\frac{e^2 z_1^2}{e^2+\Delta^2} - \frac{2e z_1 z_2\Delta}{e^2+\Delta^2} + \frac{\Delta^2 z_2^2}{e^2+\Delta^2}\right) \qquad \text{(B.1)}$$

Choose $V_2 = \frac{1}{4}kk_{e1}e^2$ where $k_{e1} > 0$ and take derivative along the solution of (33a):

$$\dot{V}_2 = -\frac{kk_{e1}\Gamma}{2}u_d e^2 - \frac{k}{2}k_{e1}\Gamma\left(e^2 z_1 - e z_2\Delta\right) \qquad \text{(B.2)}$$

Select $V = V_1 + V_2$ as a positive definite, radially unbounded Lyapunov function candidate, and take derivative with respect to time. In light of the fact that the second line of (B.1) is non-positive, $\dot{V}$ is bounded by:

$$\dot{V} \le -\frac{1}{2}k^2\bar{z}^{\mathrm{T}}\bar{z} - \frac{kk_{e1}\Gamma u_d}{2}e^2 + \frac{k\Omega_1\Pi u_d}{2}(e^2|z_1| + |e||z_2|\Delta)$$

As $0 < \Delta\Gamma \le 1$ for all e, the next inequalities hold

$$-\Gamma \le -\Delta\Pi \quad \Rightarrow \quad \Delta\Pi \le \Gamma \qquad \text{(B.3)}$$

Therefore, we obtain a bound on $\dot{V}$ as

$$\dot{V} \le -\frac{1}{2}k^2\bar{z}^{\mathrm{T}}\bar{z} - \frac{kk_{e1}\Delta\Pi u_d}{2}e^2$$
$$+ \frac{k\Omega_1\Pi u_d}{2}e^2|z_1| + \frac{k\Omega_1\Gamma u_d}{2}|e||z_2| \qquad \text{(B.4)}$$

One can write it as

$$\dot{V} \le -\frac{1}{2}k^2\bar{z}^{\mathrm{T}}\bar{z} - \frac{kk_{e1}\Delta\Pi}{3\times 2}u_d e^2 - \frac{kk_{e1}\Delta\Pi}{3\times 2}u_d e^2$$
$$+ \frac{k\Omega_1\Pi u_d}{2}e^2|z_1| - \frac{kk_{e1}\Delta\Pi}{3\times 2}u_d e^2 + \frac{k\Omega_1\Gamma u_d}{2}|e||z_2|$$
$$= -\frac{1}{2}k^2\bar{z}^{\mathrm{T}}\bar{z} - \frac{kk_{e1}\Delta\Pi}{3\times 2}u_d e^2 - \frac{ku_d}{2}\Pi e^2(\frac{k_{e1}\Delta}{3} - \Omega_1|z_1|)$$
$$- \frac{ku_d}{2}(\frac{k_{e1}\Delta e^2\Gamma^2}{3} - \Omega_1|z_2||e|\Gamma)$$

and complete the squares. As $\Pi e^2 = \frac{e^2}{e^2+\Delta^2} < 1$, we obtain

$$\dot{V} \le -\frac{kk_{e1}u_d\Delta}{3\times 2}\frac{e^2}{e^2+\Delta^2} - \frac{k}{2}(k - \frac{3u_d\Omega_1^2}{4k_{e1}\Delta})(z_1^2 + z_2^2)$$

Under assumption **T1.2**, $\dot{V} < 0$ for nonzero z_1, z_2 and e, which proves the unforced system (33) is GAS at zero.