Energy-Peak Evaluation of Nonlinear Control Systems Under Neglected Dynamics ⋆

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Abstract: The main objective in this paper is to investigate the robust performance degradation for a class of nonlinear systems due to some dynamics that are not taken into account during the controller design stage. This is usually the case in practical applications where a simplified (nonlinear) model is used to design the controller. Therefore, it is expected some performance degradation in the application of such a controller due to the presence of the neglected dynamics. With this purpose, some convex conditions for stability analysis and energy-peak evaluation of nonlinear control systems are given. It is supposed that the nonlinear functions present in the model are subject to bounded uncertainties and that both the simplified model and the neglected dynamics model are affected by polytopic uncertainties. The theoretical conditions providing stability and energy-peak bound on the regulated output of the system despite the presence of uncertainties associated with the nonlinear functions are obtained by means of a parameter dependent Lyapunov function. The proposal is illustrated by numerical examples.

1. INTRODUCTION

In order to design controllers for the nonlinear systems affine in the input, Nonlinear Dynamic Inversion (NDI) is a popular approach. Based on a plant inversion thanks to a nonlinear feedback loop, NDI techniques allow to obtain controllers, which adjust to the operating point of a given domain. Nevertheless, these techniques may be too conservative and fail to cover a large operating domain, even if they offer, in the case of aircraft control, a very nice alternative to standard gain-scheduling techniques or Linear-Parameter-Varying (LPV) control requiring cumbersome tuning procedures (since numerous local controllers have to be designed). Actually, the most appealing aspect of NDI is that the design procedure inherently provides a nonlinear multivariable controller Isidori [1995], Reiner et al. [1996], Papageorgiou and Glover [2005], Wang and Stengel [2005], Menon et al. [2008]. Note that perfect knowledge of the system dynamics (and potentially accurate sensing of output signals) has to be assumed. This last assumption is, of course, not true in many practical cases. To ensure robust performance properties, some refinements have to be introduced allowing to take into account uncertainties affecting the system: see, for example Menon et al. [2008], Esfandiari and Khalil [1992], Lavergne et al. [2005a], Franco et al. [2006].Another source of potential uncertainty resides in the fact that some dynamics are intentionally neglected during the control design: see, for example,. Aldhaheri and Khalil. [1996], Arcak et al. [2000], Arcak et al. [2001]. At the knowledge of the authors, few papers deals with the NDI and some performance evaluation, as energyy-peak evaluation: see, Herrmann et al. [2010], Biannic et al. [2012] in the anti-windup compensator design and $\mathcal{L}_2$-gain contexts. In the more general nonlinear context, see also the studies in Topcu and Packard [2009], Yang et al. [2011].

The current paper focuses on the stability and performance analysis for nonlinear systems controlled through an NDI scheme when affected by uncertainty and neglected dynamics. More especially, when dealing with nonlinear systems subject to *i)* bounded uncertainties on the nonlinear functions present in the model; *ii)* polytopic bounded additive neglected dynamics and *iii)* polytopic uncertainty on system's matrices, we want to provide a measure of the potential degradation of the energy-peak gain of the system. By considering the additive neglected dynamics in the same way as in Tarbouriech et al. [2008], we propose theoretical sufficient conditions allowing to guarantee stability and performance for the nonlinear closed-loop system. From these conditions a deterioration measure on the performance level can be considered and a algorithm is derived to compute this sort of measure. Then the results of analysis are extended to the case where polytopic uncertainties are affecting the system. The theoretical conditions providing stability and energy-peak bound on the regulated output of the system despite the presence of uncertainties associated with the nonlinear functions are obtained by means of a parameter dependent Lyapunov function.

The paper is organized as follows. Section 2 details the system under consideration and describe the problem we intend to solve. Section 3 is dedicated to develop the main results, first in a stability and performance analysis and secondly in a tentative of performance degradation measure. In Section 4 the case with uncertainties is addressed. Finally, Section 5 provides evaluation of the proposed conditions pointing out their interest but also their

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conservatism.

Notation. The Euclidean norm is represented by $\|\cdot\|$.For any vector x (matrix A), x^T (matrix A^T) denotes its transpose. For two symmetric matrices, A and B, $A > B$ ($A \geq B$) means that $A-B$ is positive definite (positive semi-definite). The block-diagonal matrix obtained from vectors or matrices $x_1, ..., x_n$ is denoted by $\texttt{Diag}\{x_1, \ldots, x_n\}$. Identity and null matrices will be denoted respectively by $\mathbf{I}$ and $\mathbf{0}$. The maximal and minimal eigenvalues of the matrix P are respectively given by $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$. In the case of partitioned symmetric matrices, the symbol $\star$ stands for symmetric blocks. For shortness, $A+A^T$ is indicated as $\texttt{Sym}\{A\}$. The dependence on time t is omitted throughout the paper to simplify the notation.

2. PRELIMINARIES AND PROBLEM DEFINITION

Consider that a nonlinear system with a simplified continuous-time model in the state space is given by

$$\dot{x} = Ax + B\gamma(x)(u - \alpha(x)) \tag{1}$$

$$y = Cx \tag{2}$$

$$z = C_z x + D_z u \tag{3}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control signal, $y \in \mathbb{R}^p$ is the model output, and $z \in \mathbb{R}^\ell$ is the regulated output. The nonlinear functions $\gamma(x) : \mathcal{D} \to \mathbb{R}^{m\times m}$ and $\alpha(x) : \mathcal{D} \to \mathbb{R}^m$, with the operating domain $\mathcal{D} \subseteq \mathbb{R}^n$ including the origin, are supposed to be affected by uncertainty as follows:

$$\gamma(x) = \gamma_n(x) + \Delta_{\gamma(x)} \tag{4}$$

$$\alpha(x) = \alpha_n(x) + \Delta_{\alpha(x)} \tag{5}$$

The terms $\gamma_n(x)$ and $\alpha_n(x)$ are the nominal terms supposed perfectly known and available for control purpose, whereas the terms $\Delta\gamma(x)$ and $\Delta\alpha(x)$ contain all the uncertainties. Furthermore the classical assumption on the regularity of $\gamma_n(x)$ is done, namely $\gamma_n^{-1}(x)$ exists $\forall x \in \mathcal{D}$. In this case, the control signal u calculated as in Khalil [2002]:

$$u = \gamma_n^{-1}(x)v + \alpha_n(x), \tag{6}$$

with v a control signal issued from a linear controller, achieves the input-state linearization of the model (1)-(2) and leads to the following dynamical equation:

$$\dot{x} = Ax + Bv + B\mathbf{\Delta}(x, v) \tag{7}$$

In (7), the term $\mathbf{\Delta}(x, v)$ contains all the uncertain terms issued of both $\gamma(x)$ and $\alpha(x)$: $\mathbf{\Delta}(x, v) = \Delta_{\gamma(x)}(\gamma_n^{-1}(x)v - \Delta_{\alpha(x)}) - \gamma_n(x)\Delta_{\alpha(x)}$. The assumption on $\mathbf{\Delta}(x, v)$ will be specified later.

Suppose that a linear DOF controller has been designed for model (7) as:

$$\dot{x}_c = A_c x_c + B_c y_t \tag{8}$$

$$v = C_c x_c + D_c y_t \tag{9}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state, y_t is the input of the controller. In the practical application, y_t is composed by the output of the simplified model (2) and the output of the neglected dynamics, which was not taken into account in the design of the DOF controller. Such a dynamics is assumed to be additive and is described by

$$\dot{x}_f = A_f x_f + B_f u \tag{10}$$

$$y_f = C_f x_f \tag{11}$$

where $x_f \in \mathbb{R}^{n_f}$ is the state vector and $y_f \in \mathbb{R}^p$ is the output of the unmodeled dynamics. Hence, the physical interconnection between the DOF controller (8)-(9) and system (7) yields $y_t = y + y_f$.

It is clear that model (1)-(7), initially used to design the controller, is a simplified representation of the real nonlinear system to be controlled. By considering the connection between model, controller and neglected dynamics a complete description can be done in the state-space by defining an augmented vector $X = \begin{bmatrix} x^T & x_c^T & x_f^T \end{bmatrix}^T \in \mathbb{R}^{n+n_c+n_f}$. From this definition one gets $v = \tilde{C}X$ and therefore the term $\mathbf{\Delta}(x, v)$ can be defined in function of X and simply denoted $\mathbf{\Delta}(X)$. The complete closed-loop system reads then:

$$\dot{X} = \left(\tilde{A} + \tilde{B}\gamma_n^{-1}(x)\tilde{C}\right)X + \tilde{B}\alpha_n(x) + \mathsf{B}\mathbf{\Delta}(X) \tag{12}$$

$$z = \left(\tilde{C}_z + D_z\gamma_n^{-1}(x)\tilde{C}\right)X + D_z\alpha_n(x) \tag{13}$$

with

$$\tilde{A} = \begin{bmatrix} A + BD_cC & BC_c & BD_cC_f \\ B_cC & A_c & B_cC_f \\ \mathbf{0} & \mathbf{0} & A_f \end{bmatrix}, \tilde{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ B_f \end{bmatrix}, \tag{14}$$

$$\mathsf{B} = \begin{bmatrix} B^T & \mathbf{0} & \mathbf{0} \end{bmatrix}^T, \tag{15}$$

$$\tilde{C} = [D_cC \ \ C_c \ \ D_cC_f], \tilde{C}_z = [C_z \ \ \mathbf{0} \ \ \mathbf{0}]. \tag{16}$$

In this case, the augmented state evolves in a domain $\mathcal{D}_X \subseteq \mathbb{R}^{n+n_c+n_f}$, such that $\mathcal{D}_X = \mathcal{D}\times\mathbb{R}^{n_c}\times\mathbb{R}^{n_f}$. Observe that, eventually the matrices associated to model (1)-(3) and to the neglected dynamics (10)-(11) may be described as belonging to a polytope where only the vertices are known. This possible representation is exploited in Section 4. Another source of uncertainty for the considered closed-loop system is issued from the nonlinear functions $\gamma(x)$ and $\alpha(x)$ as expressed in (4) and (5). The following assumptions are taken in this paper:

A1 Function $\alpha_n(x)$ is known and belongs to a family of functions Ψ_α given by

$$\begin{aligned}\Psi_\alpha = \{\alpha_n(x) \in \mathbb{R}^m : \alpha_n(x)^T\alpha_n(x) &\leq \delta_1 x^T x \\ &= \delta_1 X^T E E^T X, \ \forall X \in \mathcal{D}_X\},\end{aligned} \tag{17}$$

with $0 < \delta_1 < \infty$ and $E^T = [\mathbf{I} \ \mathbf{0} \ \mathbf{0}]$.

A2 Function $\gamma_n^{-1}(x)$ is known and belongs to a family of functions Ψ_γ given by

$$\begin{aligned}\Psi_\gamma = \{\gamma(x) \in \mathbb{R}^m :& \\ &\gamma_n^{-T}(x)\gamma_n^{-1}(x) \leq \delta_2\mathbf{I}, \ \forall x \in \mathcal{D}\},\end{aligned} \tag{18}$$

and $0 < \delta_2 < \infty$.

A3 Function $\mathbf{\Delta}$ is not known and verifies the condition of state bounded

$$\|\mathbf{\Delta}(X)\|^2 = \mathbf{\Delta}(X)^T\mathbf{\Delta}(X) \leq \delta_3 X^T X, \tag{19}$$

with $X \in \mathcal{D}_X$ and $0 < \delta_3 < \infty$.

Note that Assumptions **A1**-**A3** correspond to geometric conditions on nonlinearities $\alpha_n(x)$, $\gamma_n(x)$ and $\mathbf{\Delta}(X)$. Hence, $\alpha_n(x)$ belongs to a conic sector, whereas the inverse of $\gamma_n(x)$ is bounded. The term $\mathbf{\Delta}(X)$ satisfies also a conic sector condition.

The problem we intend to solve can be summarized as follows:

Problem 1. Considering the closed-loop system (12)-(16) under Assumptions **A1**-**A3** the following statements are verified:

(1) when $\mathbf{\Delta}(X) = \mathbf{0}$, i.e. $\Delta_{\gamma(x)} = \mathbf{0}$ and $\Delta_{\alpha(x)} = \mathbf{0}$, the asymptotic stability of the closed-loop system is ensured for any $X \in \mathcal{D}_X$.

(2) when $\mathbf{\Delta}(X) \neq \mathbf{0}$, i.e. $\Delta_{\gamma(x)} \neq \mathbf{0}$ and/or $\Delta_{\alpha(x)} \neq \mathbf{0}$, it is ensured that:
   (a) the trajectories of the closed-loop system remain bounded for any $X \in \mathcal{D}_X$;
   (b) the energy of the regulated output signal z is bounded:
$$\int_0^\infty z^T z dt \leq \delta_4 < \infty \tag{20}$$
   with $X \in \mathcal{D}_X$ and $0 < \delta_4 < \infty$.

An associated problem of practical interest is to search for the minimal value of δ_4 such that items (1) and (2) of Problem 1 hold.

Before developing the main conditions to address Problem 1, let us give the following useful lemmas.

Lemma 1. (de Oliveira and Skelton [2001]). Let $\omega \in \mathbb{R}^n$, $\mathcal{Q} = \mathcal{Q}^T \in \mathbb{R}^n$ and $\mathcal{B} \in \mathbb{R}^{m\times n}$ such that $\mathtt{rank}(\mathcal{B}) < n$. The following statements are equivalent:

i) $\omega^T \mathcal{Q} \omega < \mathbf{0}$, $\forall \omega \ : \ \mathcal{B}\omega = \mathbf{0}$, $\omega \neq \mathbf{0}$
ii) $\exists\, \mathcal{X} \in \mathbb{R}^{n\times m} : \mathcal{Q} + \mathtt{Sym}\{\mathcal{X}\mathcal{B}\} < \mathbf{0}$

Lemma 2. For any matrices $Q \in \mathbb{R}^{m\times n}$, $U \in \mathbb{R}^{q\times n}$ and $\Delta \in \mathbb{R}^{m\times q}$ such that $\|\Delta\|^2 \leq \delta^2$ it is verified that
$$\mathtt{Sym}\left\{U^T \Delta^T Q\right\} \leq \delta(U^T U + Q^T Q) \tag{21}$$

3. MAIN RESULTS

3.1 Stability and performance analysis

For solving Problem 1 through Lyapunov theory, we consider a quadratic Lyapunov candidate function $V(X) = X^T P X$, with $P = P^T > \mathbf{0}$.It is worth to say that the proposed procedures involves the solution of a finite set of linear matrix inequalities that do not depend explicitly on the nonlinear functions $\gamma(x)$ and $\alpha(x)$ that are under the geometric restrictions assumed in **A1** and **A2** as well as function $\mathbf{\Delta}$ verifies **A3**.

Theorem 1. Suppose that Assumptions **A1-A3** hold for finite scalars δ_1, δ_2, and $\delta_3 \in \mathbb{R}_+$. If there exist a symmetric positive definite matrix $P \in \mathbb{R}^{(n+n_c+n_f)\times(n+n_c+n_f)}$, a matrix $\mathcal{X} \in \mathbb{R}^{(2(n+n_c+n_f+m)+\ell)\times(n+n_c+n_f+\ell)}$, and positive real scalars τ_1, τ_2, δ_0, ϵ_1 such that
$$\Theta \equiv \left[\begin{array}{c|c} \theta & \mathcal{X}\begin{bmatrix}\tilde{B}\\ D_z\end{bmatrix} \quad \begin{bmatrix}\tilde{C}^T\\ \mathbf{0}\end{bmatrix} \\ \hline \star & -\mu\mathbf{I}\end{array}\right] < \mathbf{0} \tag{22}$$
$$S(P,\epsilon_1) = \{X \in \mathbb{R}^{n+n_c+n_f}; X^T P X \leq \epsilon_1\} \subseteq \mathcal{D}_X, \tag{23}$$
with $\eta_1 = \tau_1\delta_1$, $\eta_2 = \tau_2\delta_3$, $\mu = \sqrt{\delta_2}^{-1}$, and
$$\theta = \mathtt{Diag}\left\{\begin{bmatrix}\eta_1 E E^T + \eta_2\mathbf{I} & P\\ \star & \mathbf{0}\end{bmatrix}, -\tau_1\mathbf{I}, \delta_0^{-1}\mathbf{I}, -\tau_2\mathbf{I}\right\} + \mathtt{Sym}\left\{\mathcal{X}\mathcal{B}_0\right\} \tag{24}$$
$$\mathcal{B}_0 = \begin{bmatrix}\tilde{A} & -\mathbf{I} & \tilde{B} & \mathbf{0} & \mathsf{B}\\ \tilde{C}_z & \mathbf{0} & D_z & -\mathbf{I} & \mathbf{0}\end{bmatrix} \tag{25}$$
then the closed-loop system (12) verifies:

(1) When $\mathbf{\Delta}(X) = \mathbf{0}$ (i.e. $\Delta_{\gamma(x)} = 0$ and $\Delta_{\alpha(x)} = 0$), the closed-loop system (12) subject to assumptions **A1**, **A2** is asymptotically stable for $X \in S(P,\epsilon_1)$.
(2) When $\mathbf{\Delta}(X) \neq \mathbf{0}$ (i.e. $\Delta_{\gamma(x)} \neq \mathbf{0}$ and/or $\Delta_{\alpha(x)} \neq \mathbf{0}$), the trajectories of the closed-loop system (12) subject to assumptions **A1-A3** do not leave the set $S(P,\epsilon_1)$ given in (23), for any $X(0) \in S(P,\epsilon_1)$. In other words, system (12) is robustly stable under assumptions **A1-A3** in the set $S(P,\epsilon_1) \subseteq \mathcal{D}_X$.
(3) The constant $\delta_4 = \delta_0\epsilon_1$ bounds the energy of the regulated output z.

Proof: Consider system (12)-(16) and the quadratic Lyapunov candidate function $V(X) = X^T P X$, with $\mathbf{0} < P = P^T \in \mathbb{R}^{(n+n_c+n_f)\times(n+n_c+n_f)}$, yielding $\dot{V}(X) = \dot{X}^T P X + X^T P \dot{X}$. In order to solve Problem 1, we want to verify $\dot{V}(X) + \frac{1}{\delta_0} z^T z < 0$. By using S-procedure (for example, see Boyd et al. [1994]) Assumptions **A1** and **A3** can be incorporated yielding
$$\mathcal{L}(X) = \dot{V}(X) - \tau_2\left[\mathbf{\Delta}(X)^T\mathbf{\Delta}(X) - \delta_3 X^T X\right] + \frac{1}{\delta_0} z^T z - \tau_1\left[\alpha_n(x)^T\alpha_n(x) - \delta_1 X^T E E^T X\right] < 0, \tag{26}$$
with $\tau_1 > 0$, $\tau_2 > 0$. Defining $\omega = \begin{bmatrix}X^T & \dot{X}^T & \alpha_n^T & z^T & \mathbf{\Delta}^T\end{bmatrix}^T$, $\mathcal{Q} = \mathtt{Diag}\left\{\begin{bmatrix}\eta_1 E E^T + \eta_2\mathbf{I} & P\\ \star & \mathbf{0}\end{bmatrix}, -\tau_1\mathbf{I}, \frac{1}{\delta_0}\mathbf{I}, -\tau_2\mathbf{I}\right\}$ with η_1 and η_2 given after (23), $\mathcal{B} = \mathcal{B}_0 + \mathcal{B}_\gamma$, where $\mathcal{B}_0$ is given in (25), and
$$\mathcal{B}_\gamma = \begin{bmatrix}\tilde{B}\\ D_z\end{bmatrix}\gamma_n^{-1}(x)\begin{bmatrix}\tilde{C} & \mathbf{0}\end{bmatrix}, \tag{27}$$
It is then possible to use Lemma 1 to obtain $\theta + \mathtt{Sym}\{\mathcal{X}\mathcal{B}_\gamma\} < \mathbf{0}$ where θ is given in (24). Besides, the term $\mathtt{Sym}\{\mathcal{X}\mathcal{B}_\gamma\} = \mathtt{Sym}\left\{\mathcal{X}\begin{bmatrix}\tilde{B}\\ D_z\end{bmatrix}\gamma_n^{-1}(x)\begin{bmatrix}\tilde{C} & \mathbf{0}\end{bmatrix}\right\}$ can be over bounded by using Lemma 2 as
$$\mathtt{Sym}\{\mathcal{X}\mathcal{B}_\gamma\} \leq \left[\mathcal{X}\begin{bmatrix}\tilde{B}\\ D_z\end{bmatrix} \ \begin{bmatrix}\tilde{C} & \mathbf{0}\end{bmatrix}^T\right]\sqrt{\delta_2}\mathbf{I}\left[\mathcal{X}\begin{bmatrix}\tilde{B}\\ D_z\end{bmatrix} \ \begin{bmatrix}\tilde{C} & \mathbf{0}\end{bmatrix}^T\right]^T$$
Then, by Schur complement one gets Θ in (22) with $\mu = \sqrt{\delta_2}^{-1}$. Thus, with $P = P^T > \mathbf{0}$, the negativity of $\mathcal{L}(X)$ is assured. Hence item 1 is verified. By integrating the relation (26) it follows:
$$V(X(T)) - V(X(0)) + \frac{1}{\delta_0}\int_0^T z^T z dt < \mathbf{0} \tag{28}$$
Then, the item 2 is verified for any $X(\mathbf{0}) \in S(P,\epsilon_1)$. Moreover it also follows that
$$\int_0^T z^T z dt \leq \delta_0 V(X(0)) \leq \delta_0\epsilon_1 \tag{29}$$
meaning that the energy of the regulated output signal z is bounded by $\delta_4 = \delta_0\epsilon_1$. ■

A simplified condition can be obtained whenever $D_z = \mathbf{0}$ in (3). This case is handled in the next theorem.

Theorem 2. ($D_z = \mathbf{0}$). Suppose that Assumptions **A1-A3** hold for finite scalars δ_1, δ_2, $\delta_3 \in \mathbb{R}_+$ and that $D_z = \mathbf{0}$ in (3). If there exist a symmetric positive definite matrix $P \in \mathbb{R}^{(n+n_c+n_f)\times(n+n_c+n_f)}$, a matrix $\bar{\mathcal{X}} \in \mathbb{R}^{2(n+n_c+n_f+m)\times(n+n_c+n_f)}$, and positive real scalars τ_1, τ_2, δ_0, and ϵ_1 such that (23) is satisfied and
$$\bar{\Theta} \equiv \left[\begin{array}{c|ccc} \bar{\theta} & \begin{bmatrix}\tilde{C}_z^T\\ \mathbf{0}\end{bmatrix} & \bar{\mathcal{X}}\tilde{B} & \begin{bmatrix}\tilde{C}^T\\ \mathbf{0}\end{bmatrix} \\ \hline \star & -\delta_0\mathbf{I} & \mathbf{0} & \mathbf{0}\\ & \star & -\mu\mathbf{I} & \mathbf{0}\\ & \star & \star & -\mu\mathbf{I}\end{array}\right] < \mathbf{0} \tag{30}$$
with $\eta_1 = \tau_1\delta_1$, $\eta_2 = \tau_2\delta_3$, $\mu = \sqrt{\delta_2}^{-1}$, and
$$\bar{\theta} = \mathtt{Diag}\left\{\begin{bmatrix}\eta_1 E E^T + \eta_2\mathbf{I} & P\\ \star & \mathbf{0}\end{bmatrix}, -\tau_1\mathbf{I}, -\tau_2\mathbf{I}\right\} + \mathtt{Sym}\left\{\bar{\mathcal{X}}\bar{\mathcal{B}}_0\right\} \tag{31}$$

$$\bar{\mathcal{B}}_0 = \begin{bmatrix}\tilde{A} & -\mathbf{I} & \tilde{B} & \mathsf{B}\end{bmatrix} \tag{32}$$

then conditions (1)-(3) in Theorem 1 are verified.

Proof: The proof follows a similar way to the one of Theorem 1, by replacing $z = [C_z\ \mathbf{0}]\,X$ in $\mathcal{L}(X)$ given in (26), assuming $\omega = \begin{bmatrix}X^T & \dot{X}^T & \alpha_n^T & \mathbf{\Delta}^T\end{bmatrix}^T$, $\mathcal{B} = \bar{\mathcal{B}}_0 + \bar{\mathcal{B}}_\gamma$, with $\bar{\mathcal{B}}_0$ given in (32), and $\bar{\mathcal{B}}_\gamma = \tilde{B}\gamma_n^{-1}(x)\begin{bmatrix}\tilde{C} & \mathbf{0}\end{bmatrix}$, $\mathcal{Q} = \mathtt{Diag}\left\{\begin{bmatrix}\eta_1 EE^T + \delta_0^{-1}\tilde{C}_z^T\tilde{C}_z & P\\ \star & \mathbf{0}\end{bmatrix}, -\tau_1\mathbf{I}, -\tau_2\mathbf{I}\right\}$, using Lemma 2, and applying Schur's complement to get (30). ∎

It is important to note that when the domain $\mathcal{D}$ is the whole state space $\Re^n$ (therefore $\mathcal{D}_X = \mathbb{R}^{n+n_c+n_f}$), then Theorems 1 and 2 can be written in a global context (i.e. valid for any $X \in \mathbb{R}^{n+n_c+n_f}$), removing conditions (23).

Note that conditions (22) and (30) are linear on the decision variables on δ_0^{-1} and δ_0, respectively. This allows to found $\delta_4 = \delta_0\epsilon_1$ by solving:

$$\left.\begin{array}{cc}\displaystyle\min_{\mathcal{X},P,\tau_1,\tau_2,\eta_1,\eta_2,\mu} & f(\delta_0)\\ \text{subject to:} & (22)\ \textbf{or}\ (30), (23)\\ & P = P^T > \mathbf{0}, \epsilon_1,\\ & \tau_i > 0, \eta_i > 0, i \in \{1,2\}\end{array}\right\}\mathcal{P}_{\delta_4}. \tag{33}$$

with $f(\delta_0) = -\delta_0^{-1}$ for (22) and $f(\delta_0) = \delta_0$ for (30), yielding $\delta_4 = \delta_0\epsilon_1$. As mentioned above, in the global context the optimization problem $\mathcal{P}_{\delta_4}$ can be considered as well by removing relation (23) and the variable ϵ_1. Moreover, it can be interesting to consider δ_1, δ_2, and δ_3 as decision variables. In this case some convex optimization problems derived from $\mathcal{P}_{\delta_4}$ could be formulated.

3.2 Performance degradation

Naturally, other optimization problems can be obtained by replacing adequately the objective function in (33). A case of special interest consists in determining the deterioration on δ_4 as some neglected dynamics affect the real system. This can be achieved by using the results of Theorems 1 and 2 and by introducing a deterioration measure. Then we can introduce such a deterioration measure through a positive scalar γ, satisfying $1 \leq \gamma < \infty$, and then by replacing δ_0^{-1} by $\gamma\delta_0^{-1*}$ in conditions (22) or (30). The value δ_0^{-1*} is the performance level resulting from the closed-loop system without neglected dynamics.

With this objective, consider (8)-(9) with $y_t = y$ and $\hat{X} = \begin{bmatrix}x^T & x_c^T\end{bmatrix}^T \in \mathbb{R}^{n+n_c}$. The simplified (i.e. without neglected dynamics) closed-loop system then reads:

$$\dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}\mathbf{\Delta}(\hat{X}) \tag{34}$$

$$z = \left(\hat{C}_z + D_z\gamma_n^{-1}(x)\hat{C}\right)\hat{X} + D_z\alpha_n(x) \tag{35}$$

with

$$\hat{A} = \begin{bmatrix}A + BD_cC & BC_c\\ B_cC & A_c\end{bmatrix}, \hat{B} = \begin{bmatrix}B\\ \mathbf{0}\end{bmatrix}, \tag{36}$$

$$\hat{C} = [D_cC\ \ C_c], \hat{C}_z = [C_z\ \ \mathbf{0}]. \tag{37}$$

In this case, the augmented state evolves in a domain $\mathcal{D}_{\hat{X}} \subseteq \mathbb{R}^{n+n_c}$, such that $\mathcal{D}_{\hat{X}} = \mathcal{D} \times \mathbb{R}^{n_c}$. Assumptions **A1-A3** are taken into account replacing X, $\mathcal{D}_X$, and E respectively by $\hat{X}$, $\mathcal{D}_{\hat{X}}$, and $\hat{E} = [\mathbf{I}\ \mathbf{0}]^T$.

Hence, for the simplified closed-loop system (36)-(37), Theorem 1 can be rewritten as follows.

Theorem 3. (Without neglected dynamics). Suppose that Assumptions **A1-A3** hold for finite scalars δ_1, δ_2, and $\delta_3 \in \mathbb{R}_+$. If there exist a symmetric positive definite matrix $\hat{P} \in \mathbb{R}^{(n+n_c)\times(n+n_c)}$, a matrix $\hat{\mathcal{X}} \in \mathbb{R}^{(2(n+n_c+m)+\ell)\times(n+n_c+\ell)}$, and positive real scalars τ_1, τ_2, δ_0, ϵ_1 such that (23) is satisfied and

$$\hat{\Theta} \equiv \left[\begin{array}{c|c}\hat{\theta} & \hat{\mathcal{X}}\begin{bmatrix}\mathbf{0}\\ D_z\end{bmatrix} \quad \begin{bmatrix}\hat{C}^T\\ \mathbf{0}\end{bmatrix}\\ \hline \star & -\mu\mathbf{I}\end{array}\right] < \mathbf{0} \tag{38}$$

with $\eta_1 = \tau_1\delta_1$, $\eta_2 = \tau_2\delta_3$, $\mu = \sqrt{\delta_2}^{-1}$, and

$$\hat{\theta} = \mathtt{Diag}\left\{\begin{bmatrix}\eta_1\hat{E}\hat{E}^T + \eta_2\mathbf{I} & \hat{P}\\ \star & \mathbf{0}\end{bmatrix}, -\tau_1\mathbf{I}, \delta_0^{-1}\mathbf{I}, -\tau_2\mathbf{I}\right\} + \mathtt{Sym}\left\{\hat{\mathcal{X}}\hat{\mathcal{B}}_0\right\} \tag{39}$$

$$\hat{\mathcal{B}}_0 = \begin{bmatrix}\hat{A} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \hat{B}\\ \hat{C}_z & \mathbf{0} & D_z & -\mathbf{I} & \mathbf{0}\end{bmatrix} \tag{40}$$

then conditions (1)-(3) of Theorem 1 hold with the replacement of P, $\tilde{A}$, X, and $\mathcal{D}_X$ by $\hat{P}$, $\hat{A}$, $\hat{X}$, and $\mathcal{D}_{\hat{X}}$, respectively.

Thus a possible way to measure the performance degradation of the energy-peak on $\int_0^\infty z^T z dt$ is described as follows [1]:

(1) Solve $\mathcal{P}_{\delta_4}$ in (33) replacing (22) or (30) by (38). Store the optimal value δ_0^{-1*}.
(2) Solve the following convex optimization problem

$$\left.\begin{array}{cc}\displaystyle\min_{\mathcal{X},P,\tau_1,\tau_2,\eta_1,\eta_2} & \gamma\\ \text{subject to:} & (22)\ \textbf{or}\ (30),\\ & P = P^T > \mathbf{0}, \tau_i > 0,\\ & \eta_i > 0, i \in \{1,2\}\end{array}\right\}\mathcal{P}_\gamma, \tag{41}$$

where with (22) δ_0^{-1} is replaced by $\gamma\delta_0^{-1*}$ and it is imposed $\gamma \leq 1$, and with (30) δ_0 is replaced by $\gamma\delta_0^*$ and it is imposed $\gamma \geq 1$, δ_0^{-1*} (δ_0^*) being given.

Hence the resulting γ will represent a sort of measure of degradation on the energy-peak of the output of the closed-loop system. Closer is γ to 1, lower is the degradation of the performance level.

4. POLYTOPIC UNCERTAIN CASE

A relevant advantage of Theorems 1 and 2 is that, thanks to the use of Lemma 1, there is no product between matrices of (12)-(3) and the Lyapunov candidate matrix P in their respective LMIs. Such a property can be quite useful, once a case of practical interest concerns situations where matrices in (2), (7) and/or (10)-(11) belong to a polytope where only the vertices are known. Therefore, both sets of matrices (A, B, C, C_z, D_z) and (A_f, B_f, C_f) may be subjected to uncertainties in practical modeling cases. This allows to handle the polytopic-uncertainty version of (12)-(13) with matrices $\mathcal{S} \equiv (A, B, C, C_z, D_z, A_f, B_f, C_f)$ defined in a polytopic domain as follows:

$$\mathcal{S}(\xi) = \sum_{k=1}^{N}\xi_k\mathcal{S}_k; \quad \xi \in \Xi \tag{42}$$

with $\mathcal{S}_k = (A, B, C, C_z, D_z, A_f, B_f, C_f)_k$ standing for each known vertex of the concerned matrices, N is the number of vertices in the polytope, and

[1] To simplify we present the procedure in the global context, i.e., when the domain $\mathcal{D}$ is the whole state space $\Re^n$ (therefore $\mathcal{D}_X = \mathbb{R}^{n+n_c+n_f}$ and relation (23) and the variable ϵ_1 are removed).

$$\Xi \equiv \left\{ \xi \in \mathbb{R}^N; \sum_{k=1}^{N} \xi_k = 1; \xi_k \geq 0, k = 1, \ldots, N \right\}. \quad (43)$$

Then, thanks to the convexity of matrices in (14)-(16) in Ξ, they can be rewritten as (see Leite and Peres [2003] or Peres et al. [2003]):

$$M(\xi) = \sum_{k=1}^{N} \xi_k^2 M_{kk} + \sum_{k=1}^{N-1} \sum_{r=k+1}^{N} \xi_k \xi_r (M_{kr} + M_{rk}) \quad (44)$$

with $M \in \{\tilde{A}, \tilde{B}, \mathsf{B}, \tilde{C}, \tilde{C}_z, D_z\}$, where we have:

$$\tilde{A}_{kr} = \begin{bmatrix} A_k + B_k D_c C_r & B_k C_c & B_k D_c C_{fr} \\ B_c C_k & A_c & B_c C_{fk} \\ \mathbf{0} & \mathbf{0} & A_{fk} \end{bmatrix}, \quad (45)$$

$$\tilde{B}_{kr} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & B_{fk}^T \end{bmatrix}^T, \mathsf{B}_{kr} = \begin{bmatrix} B_k^T & \mathbf{0} & \mathbf{0} \end{bmatrix}^T, D_{zkr} = D_{zk} \quad (46)$$

$$\tilde{C}_{kr} = [D_c C_k \ \ C_c \ \ D_c C_{fk}], \tilde{C}_{zkr} = [C_{zk} \ \ \mathbf{0} \ \ \mathbf{0}] \quad (47)$$

where the subscripts k and r refer to the vertex number. To simplify we consider the global context, i.e., when the domain $\mathcal{D}$ is the whole state space $\Re^n$ (therefore $\mathcal{D}_X = \mathbb{R}^{n+n_c+n_f}$). The local context can be recovered by using results in Tarbouriech et al. [2011].

Theorem 4. Consider system (12) with matrices in a polytopic domain with (42)-(47) and subject to assumptions **A1-A3**. If there exist N symmetric positive definite matrices $P_k \in \mathbb{R}^{(n+n_c+n_f)\times(n+n_c+n_f)}$, a matrix $\mathcal{X} \in \mathbb{R}^{(2(n+n_c+n_f+m)+\ell)\times(n+n_c+n_f+\ell)}$, and positive real scalars τ_1, τ_2 and δ_0 such that

$$\Theta_{kk} < \mathbf{0}, \quad k = 1, \ldots, N; \quad (48)$$

$$\Theta_{rk} + \Theta_{kr} < \mathbf{0}, k = 1, \ldots, N-1; \ r = k+1, \ldots, N, \quad (49)$$

with $\eta_1 = \tau_1 \delta_1$, $\eta_2 = \tau_2 \delta_3$, $\mu = \sqrt{\delta_2}^{-1}$ and

$$\Theta_{kr} \equiv \left[\begin{array}{c|c} \theta_{kr} & \mathcal{X} \begin{bmatrix} \tilde{B}_k \\ D_{zk} \end{bmatrix} \ \begin{bmatrix} \tilde{C}_k^T \\ \mathbf{0} \end{bmatrix} \\ \hline \star & -\mu \mathbf{I} \end{array} \right] < \mathbf{0}, \quad (50)$$

$$\theta_{kr} = \mathtt{Diag} \left\{ \begin{bmatrix} \eta_1 E E^T + \eta_2 \mathbf{I} & P_k \\ \star & \mathbf{0} \end{bmatrix}, -\tau_1 \mathbf{I}, \delta_0^{-1} \mathbf{I}, -\tau_2 \mathbf{I} \right\} + \mathtt{Sym}\{\mathcal{X} \mathcal{B}_{0kr}\} \quad (51)$$

$$\mathcal{B}_{0kr} = \begin{bmatrix} \tilde{A}_{kr} & -\mathbf{I} & \tilde{B}_k & \mathbf{0} & \mathsf{B}_k \\ \tilde{C}_{zk} & \mathbf{0} & D_{zk} & -\mathbf{I} & \mathbf{0} \end{bmatrix} \quad (52)$$

then conditions 1-3 of Theorem 1 holds with P, $\tilde{A}$, $\tilde{B}$, B, and $\tilde{C}$ replaced by $P(\xi)$, $\tilde{A}(\xi)$, $\tilde{B}(\xi)$, $\mathsf{B}(\xi)$, and $\tilde{C}(\xi)$, respectively, and with all of them calculated as in (44).

Proof: The proof follows similar steps of the proof to Theorem 1 with matrices from the system and from the neglected dynamics, and matrix P replaced by their respective counterparts depending on ξ as in (42). It is then obtained a counter part of (22) with dependency on ξ, *i.e.*, $\Theta(\xi) < \mathbf{0}$. Noting that $\Theta(\xi)$ can rewritten as $\Theta(\xi) = \sum_{k=1}^{N} \xi_k^2 \Theta_{kk} + \sum_{k=1}^{N-1} \sum_{r=k+1}^{N} \xi_k \xi_r \Theta_{kr} < 0$, it is possible to obtain (48)-(52). For details, see Leite and Peres [2003] or Peres et al. [2003]. ∎

Note that, if $P_1 = \cdots = P_k = P$ in Theorem 4, a quadratic stability based condition is recovered. In this case, the parameter ξ can be even time-varying, but at the expense of some conservatism. Besides, the class of systems systems that can be described in a polytopic representation with time-invariant parameter is quite wide. The uncertain vector parameter ξ does not concern the nonlinear matrix functions $\gamma(x)$ and $\alpha(x)$, and makes $P(\xi)$ constant with respect to the uncertainties issued from functions $\gamma(x)$ and $\alpha(x)$. Therefore, the condition stated in Theorem 4 deals with two uncertainty types: the norm-bound based one — for $\gamma(x)$ and $\alpha(x)$ — and the polytopic based one.

Similar approach can be done to consider polytopic uncertainty with the conditions stated by Theorems 2 and 3. However, for sake of space, the respective conditions are not presented here.

5. NUMERICAL EXPERIMENTS

Consider the nonlinear system (1)-(2) described by the following data :

$$A = \begin{bmatrix} -1 & 0.3 \\ 0 & 0.5 \end{bmatrix}, \ B = \begin{bmatrix} 0.5 \\ -10 \end{bmatrix}, \ C = [1 \ 2]$$
$$\gamma_n(x) = 2.14 + 0.72\sin(x_2), \alpha_n(x) = \sqrt{|x_1 x_2|} \quad (53)$$

The controller (8)-(9) is described by the following matrices:

$$A_c = \begin{bmatrix} -4 & 1 \\ 0 & -8 \end{bmatrix}, B_c = \begin{bmatrix} -1 \\ -0.5 \end{bmatrix}, C_c = \begin{bmatrix} 0.3 \\ -2 \end{bmatrix}^T, D_c = 1. \quad (54)$$

As in Tarbouriech et al. [2008] it is supposed that this system is subject to an additive neglected dynamics, which is relative to a flexible mode with uncertain resonance frequency, which is described by (10)-(11) with matrices $A_f = 4A_{f0}$,

$$A_{f0} = \begin{bmatrix} 0 & 5 \\ -5 & -1 \end{bmatrix}, \ B_f = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \ C_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T. \quad (55)$$

The regulated output (3) is defined with $C_z = [1 \ 1]$ and $D_z = 1$. Note that for the given $\alpha_n(x)$ we have $\alpha_n^T(x)\alpha_n(x) = |x_1 x_2| \leq 0.5(x_1^2 + x_2^2)$ and thus we can choose $\delta_1 = 0.5$ in Assumption **A1**. From the given function $\gamma_n(x)$ we can verify that the maximal of $\gamma_n^{-T}(x)\gamma_n^{-1}(x)$ is 0.4959. Therefore, we choose $\delta_2 = 0.5$ in Assumption **A2**. These values of $\delta_1 = \delta_2 = 0.5$ are used in all numerical experiments that follows. Note that we apply our conditions in a global context since the domain $\mathcal{D}$ is the whole state space $\Re^n$ (therefore $\mathcal{D}_X = \mathbb{R}^{n+n_c+n_f}$).

Solving the optimization problem $\mathcal{P}_{\delta_4}$ with (38) (Theorem 3, no neglected dynamics) one gets $\delta_4^* = \delta_0^* = 0.5$. If the nominal neglected dynamics given in (10)-(11) with (55), is considered, the optimization problem given in the end of section 3.2 can be used with Theorem 1 yielding $\gamma = 2.4182$ or $\delta_4^* = 1.2091$, which clearly demonstrates the performance degradation due to the neglected dynamics. Furthermore, supposing that some uncertainty has been associated with the neglected dynamics, for example as in Tarbouriech et al. [2008] to consider flexible modes, such that its description can be done by a polytope with two vertices given as follows: $A_{f1} = 3A_{f0}$, $A_{f2} = 5A_{f0}$, $B_{f1} = B_{f2} = B_f$,$C_{f1} = C_{f2} = C_f$. In this case, optimization problem $\mathcal{P}_{\delta_4}$ with Theorem 1 and constant Lyapunov matrix (quadratic stability approach) it yields $\delta_4 = 1.9236$ or a $\gamma = 3.8472$ by the method described in Section 3.2. On the other hand, if a parameter dependent Lyapunov function is employed, the same optimization problem leads to $\delta_4 = 1.6647$ (respectively, $\gamma = 3.32942$).

Suppose that all system matrices are uncertain and that the neglected dynamics belongs to a polytope such that $A_{f1} = 3A_{f0}$, $A_{f2} = 5A_{f0}$ and the other vertices are obtained from $B_f(\nu) = B_f(1+0.03\nu)$, $C_f = C_f(1+0.03\nu)$, $\nu \in [-\rho, \ \rho]$. In the same way, suppose that matrices of the simplified system (1)-(2) can be expressed in function of ν as follows: $A(\nu) = A(1 + 0.1\nu)$, $B(\nu) = B(1 + 0.03\nu)$, $C(\nu) = C(1 + 0.03\nu)$, with A, B, and C taken from (53).

Also consider that the matrices of the regulated output z belong to a polytope with the same uncertain parameter of the neglected dynamics with vertices given by $C_{z1} = [1\ 1]$, $C_{z2} = [2\ 0.9]$, $D_{z1} = 1.1$, and $D_{z2} = 0.9$. This leads to a polytope with 2 vertices that depends on the parameter ρ. We have used the methodology described in Section 3.2 with (22) to search for the maximal value of ρ such that this closed-loop system with neglected dynamics can be certified as stable with $\delta_4 = \delta_0$ as the energy-peak performance index. Using a parameter dependent (PD) Lyapunov matrix, we have found $\rho_{\max,PD} = 0.9922$, with a deterioration measure of $\gamma_{PD} = 3.1386$ ($\delta_4 = 1.5693$). It has been noted that the value of δ_3 is significantly smaller as the value of δ_4 increases. Thus, these numerical tests illustrate how the performance index δ_4 can be affected by neglected dynamics and also how a parameter dependent Lyapunov function can be used to reduce the conservatism of the performance estimations.

6. CONCLUSION

In this paper we presented some new convex formulations for robust stability and performance analysis of nonlinear systems controlled by linearizing feedback loop. The nonlinear functions presented in the model are assumed to have a bounded uncertainty described in terms of geometric constraints. The proposed conditions take into account possible unmodeled dynamics neglected during the control synthesis stage. Such dynamics together with the linear matrices of the system can affected by polytopic uncertainty. A parameter dependent Lyapunov function is used to reduce the conservatism of the analysis, specially with respect to polytopic uncertainties. Nevertheless, this work constitutes preliminary results and lets some questions open. In particular, to improve robust performance properties, some refinements could be introduced in the process of re-tuning of the DOF controller, by example inspired by the robust multi-inversion technique in Lavergne et al. [2005b]. Also, other performance indexes such as the $\mathcal{L}_2$ index can be viewed as a possible extension.

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