Force control of a class of standard mechanical systems in the port-Hamiltonian framework

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Abstract: This work is devoted to a force control strategy of a class of standard mechanical systems in the port-Hamiltonian framework. First, a coordinate transformation is applied to equivalently describe the original port-Hamiltonian system in a port-Hamiltonian form which has a constant mass-inertia matrix in the Hamiltonian. Then, we show how to derive an extended port-Hamiltonian system with structure preservation which can be used for force control purposes. Furthermore, we prove that the closed-loop system is asymptotically stable via a Lyapunov candidate function. Finally, experiments results are provided to show the advantages of the force control strategy in presence of external forces.

Keywords: port-Hamiltonian systems, force feedback, stability analysis, force control, mechanical systems.

1. INTRODUCTION

Skilled manipulation is required when a mechanical system, e.g. a robot, is in contact with the environment. In the robotics field, the number of possible tasks to perform is increased when the information about the dynamics of the contact with the environment is available. The interaction robot-environment is intentional in industrial applications such as grinding, polishing, cutting, excavating and non-industrial such as domotics and health care purposes Canudas et al. (1996); Gorinevsky et al. (1997). Implementation of all these tasks requires force feedback and force control. It becomes possible to feed back force of the manipulator links by installing force sensors. The force control in robot manipulators is thoroughly discussed in Canudas et al. (1996); Gorinevsky et al. (1997); Murray et al. (1994); Siciliano and Kathib (2008); Spong et al. (2006) in the Euler-Lagrange framework. Contrary to the Euler-Lagrange strategies, it is the aim of this paper to propose a dynamic extension for a class of mechanical system and based on the port-Hamiltonian formulation Duindam et al. (2009); Maschke and van der Schaft (1992) for force feedback and force control purposes. Port-Hamiltonian systems include a large family of physical nonlinear systems, and since the port-Hamiltonian framework is an efficient way to describe the environment, the physical systems, and the interactions between them, the dynamics of nonlinear controllers have a more suitable interpretation.

A class of standard mechanical systems with force feedback and zero external forces is introduced previously in Munoz-Arias et al. (2012). The preliminary results are based on an extension on the system coordinates in order to include a type of integral action over the force sensor measurements. Furthermore, in Munoz-Arias et al. (2012) we have considered mechanical systems with a constant mass inertia matrix, which simplifies the change of coordinates. The present research is an extension of the port-Hamiltonian framework to obtain force control instead of position control in presence of external forces in the input of the system.

The main result of this paper relies on a new strategy for force control for a class of standard mechanical systems in the port-Hamiltonian framework, thus exploiting the fact that many new systems are equipped with force sensors. The main strategy adopted here is first to apply the results of Viola et al. (2007) via a coordinate transformations of Fujimoto and Sugie (2001) to equivalently describe the original port-Hamiltonian system in a port-Hamiltonian form which has a constant mass-inertia matrix in the Hamiltonian. We then realize an extended port-Hamiltonian system in order to include a type of force feedback with structure preservation. Furthermore, we provide a Lyapunov candidate function of the closed-loop system in order to prove asymptotic stability in the desired constant force. The main advantage of the port-Hamiltonian formulation with force feedback modeling is that we obtain a robust force control strategy with a clear physical interpretation.

The paper is organized as follows. In Section 2, we provide a general background in the port-Hamiltonian framework, especially for a class of standard mechanical systems. We then introduce in Section 3 the dynamics of the new state, and the change of variables to obtain force feedback. In Section 3, we also show how the change of variables yields a port-Hamiltonian framework without losing its structure, and a new Hamiltonian that qualifies as a Lyapunov function. Based on the extended port-Hamiltonian system with force feedback, we obtain asymptotic stability in a desired force via a force control law in presence of external forces in Section 4. Finally, experiments are given in Section 5 to motivate our results for force control, and Section 6 provides concluding remarks.

2. PRELIMINARIES

We briefly recap the definition, properties and advantages of modeling and control with the port-Hamiltonian formalism. First, we give a brief summary about systems modeling with actuation of additional external forces. Then, we applied the results of Viola et al. (2007) to equivalently describe the original port-Hamiltonian system in a port-Hamiltonian form which has a constant mass-inertia matrix via the coordinates transformations of Fujimoto and Sugie (2001).

2.1 port-Hamiltonian Systems

The port-Hamiltonian framework is based on the description of systems in terms of energy variables, their interconnection and dissipation structures, and power ports. Port-Hamiltonian systems include a large family of physical nonlinear systems. The transfer of energy between the physical system and the environment is given through energy elements, dissipation elements and power preserving ports Duindam et al. (2009); Maschke and van der Schaft (1992).

A time-invariant port-Hamiltonian system, introduced by Maschke and van der Schaft (1992), is described by

$$\Sigma = \begin{cases} \dot{x} = [J(x) - R(x)] \dfrac{\partial H}{\partial x}(x) + g(x) w \\ \\ y = g(x)^\top \dfrac{\partial H}{\partial x}(x) \end{cases} \tag{1}$$

with $x \in \mathbb{R}^{\mathcal{N}}$ the states of the system, the skew-symmetric interconnection matrix $J(x) \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}$, the symmetric, positive-semidefinite damping matrix $R(x) \in \mathbb{R}^{\mathcal{N}\times\mathcal{N}}$, and the Hamiltonian $H(x) \in \mathbb{R}$. The matrix $g(x) \in \mathbb{R}^{\mathcal{N}\times\mathcal{M}}$ weights the action of the control inputs $w \in \mathbb{R}^{\mathcal{M}}$ on the system, and $w, y \in \mathbb{R}^{\mathcal{M}}$ with $\mathcal{M} \leq \mathcal{N}$, form a power port pair. We now restrict the analysis to the class of standard mechanical systems.

Consider a class of standard mechanical systems of n degrees of freedom (DoF) as in (1), e.g., an n-DoF rigid robot manipulator. Consider furthermore the addition of an external force vector. The resulting system is then given by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -I_{n\times n} & -D(q,p) \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(q,p)}{\partial q} \\ \dfrac{\partial H(q,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ G(q) \end{bmatrix} u + \begin{bmatrix} 0_{n\times n} \\ B(q) \end{bmatrix} f_e$$
$$y = G(q)^\top \frac{\partial H(q,p)}{\partial p} \tag{2}$$

with the vector of generalized configuration coordinates $q \in \mathbb{R}^n$, the vector of generalized momenta $p \in \mathbb{R}^n$, the identity matrix $I_{n\times n}$, the damping matrix $D(q,p) \in \mathbb{R}^{n\times n}$, $D(q,p) = D(q,p)^\top \geq 0$, $y \in \mathbb{R}^n$ the output vector, $u \in \mathbb{R}^n$ the input vector, $f_e \in \mathbb{R}^n$ the vector of external forces, $\mathcal{N} = 2n$, matrix $B \in \mathbb{R}^{n\times n}$, and the input matrix $G(q) \in \mathbb{R}^{n\times n}$ everywhere invertible, i.e., the port-Hamiltonian system is *fully actuated*. The Hamiltonian of the system is equal to the sum of kinetic and potential energy,

$$H(q,p) = \frac{1}{2} p^\top M^{-1}(q) p + V(q) \tag{3}$$

where $M(q) = M^\top(q) > 0$ is the $n \times n$ inertia (generalized mass) matrix and $V(q)$ is the potential energy.

We consider the port-Hamiltonian system (2) as a class of standard mechanical systems with external forces.

2.2 Nonconstant to constant mass-inertia matrix transformation

Consider a class of standard mechanical systems in the port-Hamiltonian framework with a nonconstant mass-inertia matrix $M(q)$ as in (2). The aim of this section is to transform the original system (2) into a port-Hamiltonian formulation with a constant mass-inertia matrix via a generalized canonical transformation of Fujimoto and Sugie (2001). This simplifies the coordinate transformation in order to realize force feedback in the port-Hamiltonian framework with structure preservation. Furthermore, we use the results of this canonical transformation for a force control strategy with an external force vector. The proposed change of variables to deal with a nonconstant mass inertia matrix is first proposed by Viola et al. (2007).

Consider a time-invariant system (2) with nonconstant $M(q)$, and a change of variables $\bar{x} = \Phi(x) = \Phi(q,p)$ as

$$\bar{x} = \Phi(x) = \begin{pmatrix} \bar{q} \\ \bar{p} \end{pmatrix} = \begin{pmatrix} q \\ T(q)^{-1} p \end{pmatrix} \tag{4}$$

where $T(q)$ is a lower triangular matrix such that

$$M(q) = T(q)^T T(q) = T(\bar{q})^T T(\bar{q}) \tag{5}$$

Consider now the Hamiltonian $H(q,p)$ as in (3), using (4), we realize $\bar{H}(\bar{x}) = H\left(\Phi^{-1}(\bar{x})\right)$ as

$$\bar{H}(\bar{x}) = \frac{1}{2} \bar{p}^\top \bar{p} + V(\bar{q}) \tag{6}$$

The new form of the interconnection and dissipation matrices of the port-Hamiltonian system are realized via the change of variables (4), the mass-inertia matrix decomposition (5), and the new Hamiltonian (6), which is proposed by Viola et al. (2007).

Consider the system (2), and assume that $G(q)$ is invertible. Consider furthermore the change of variables $\Phi(q,p)$ as in (4), the $M(q)$ decomposition as in (5), and the Hamiltonian $\bar{H}(\bar{x})$ as in (6). The resulting forced port-Hamiltonian system is then given by

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{bmatrix} = \begin{bmatrix} 0 & T(\bar{q})^{-\top} \\ -T(\bar{q})^{-1} & \bar{J}_2(\bar{q},\bar{p}) - \bar{D}(\bar{q},\bar{p}) \end{bmatrix} \begin{bmatrix} \dfrac{\partial \bar{H}(\bar{q},\bar{p})}{\partial \bar{q}} \\ \dfrac{\partial \bar{H}(\bar{q},\bar{p})}{\partial \bar{p}} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{G}(\bar{q}) \end{bmatrix} u + \begin{bmatrix} 0 \\ \bar{B}(\bar{q}) \end{bmatrix} f_e$$
$$y = \bar{G}(\bar{q})^\top \frac{\partial \bar{H}(\bar{q},\bar{p})}{\partial \bar{p}} \tag{7}$$

where the skew-symmetric matrix $\bar{J}_2(\bar{q},\bar{p})$ takes the form

$$\bar{J}_2(\bar{q},\bar{p}) = \frac{\partial (T(\bar{q})\bar{p})}{\partial \bar{q}} T(\bar{q})^{-\top} - T(\bar{q})^{-1} \frac{\partial (T(\bar{q})\bar{p})}{\partial \bar{q}}^\top \tag{8}$$

Furthermore, the positive matrix $\bar{D}(\bar{q},\bar{p})$, and the input matrices $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$, are described by

$$\bar{D}(\bar{q}) = T(\bar{q})^{-1} D\left(\Phi^{-1}(\bar{q},\bar{p})\right) T(\bar{q})^{-\top} \tag{9}$$

$$\bar{G}(\bar{q}) = T(\bar{q})^{-1} G(\bar{q}) \tag{10}$$

$$\bar{B}(\bar{q}) = T(\bar{q})^{-1} B(\bar{q}) \tag{11}$$

respectively. Via the transformation (4), we then obtain a class of mechanical systems with a constant mass inertia matrix in the Hamiltonian function as in (6), which equivalently describes the original system (2) with nonconstant mass-inertia matrix. We use the results for our force feedback framework in the next section.

3. FORCE FEEDBACK VIA DYNAMIC EXTENSION

In this section, a dynamic extension and a coordinate transformation are introduced for the port-Hamiltonian system (7). The dynamics of the new state and the coordinate transformation are realized for force feedback purposes. The dynamics of the new state depends on the sum of the internal and external forces of the mechanical system. The internal forces are given by a set of kinetic, potential, and energy dissipation elements. The external forces are exerted from the environment and are modeled as generalized force vectors which are preliminarily presented in the port-Hamiltonian framework (2). The force feedback is included via a change of variable and a new Hamiltonian function. Furthermore, the dynamic extension is included through a coordinate transformation in order to preserve the structure of the transformed port-Hamiltonian system. The present work is inspired by the results of Dirksz and Scherpen (2011, 2012).

We assume that the system (2) has force sensors that measure the internal and external forces given by

$$f(q,p) = -\frac{\partial H(q,p)}{\partial q} - D(q,p)\frac{\partial H(q,p)}{\partial p} + B(q) f_e \tag{12}$$

with $H(q,p)$ as in (3), and where $f(q,p) \in \mathbb{R}^n$. We propose now a new state $\hat{z}$ as the dynamic extension of the port-Hamiltonian framework. The extension is realized in order to include the internal and external forces while preserving the form of the interconnection and the dissipation matrices, i.e., to preserve the port-Hamiltonian structure. Consider (12), and define the dynamics of the new state $\hat{z}$ as a function of the forces in the form of

$$\dot{\hat{z}} = -Y^\top T(q)^{-1} f(q,p) \tag{13}$$

with a constant gain matrix Y over the internal and external forces, where the symmetric part of Y is positive definite, i.e., $Y + Y^\top > 0$, $Y \in \mathbb{R}^{n\times n}$, and $T(q)$ given by the matrix decomposition (5). Matrix Y is defined later on. Consider now the system (7), we rewrite then the dynamics of $\dot{\hat{z}}$ as in (13) as (for simplicity of notation, we leave out here the arguments of $T(\bar{q})$, $\bar{D}(\bar{q},\bar{p})$, $\bar{J}_2(\bar{q},\bar{p})$, and $\bar{B}(\bar{q})$)

$$\dot{\hat{z}} = Y^\top \left(-T^{-1}\frac{\partial \bar{H}(\bar{q},\bar{p})}{\partial \bar{q}} - (\bar{D} - \bar{J}_2)\frac{\partial \bar{H}(\bar{q},\bar{p})}{\partial \bar{p}} + \bar{B} f_e\right) \tag{14}$$

with $\bar{H}(\bar{q},\bar{p})$ as in (6), $\bar{J}_2(\bar{q},\bar{p})$ as in (8), $\bar{D}(\bar{q},\bar{p})$ as in (9), and $\bar{B}(\bar{q})$ as in (11). Given the force sensor readings (12), and the dynamic extension (13), we have constant desired forces given by

$$f_d = Y^\top T(q_z)\dot{z}_d \tag{15}$$

where $f_d \in \mathbb{R}^n$, with $\dot{z}_d \in \mathbb{R}^n$ a constant that depends on the desired forces (15), and the position vector $q_z \in \mathbb{R}^n$ given by the solution of the equation

$$-Y^\top T(q_z)^{-1}\left(\frac{\partial \bar{H}(q_z)}{\partial \bar{q}_z} + \bar{B}(q_z) f_e\right) - \dot{z}_d = 0 \tag{16}$$

Given (7), we have a new state called $\hat{z}$ as in (14), and a function of a type of integral action over the desired forces, i.e., z_d, with z_d two times differentiable. Define then the adapted momenta as

$$\hat{p} = \bar{p} - A(\hat{z} - z_d) \tag{17}$$

with a constant diagonal matrix $A > 0, A \in \mathbb{R}^{n\times n}$. We feed back the force by application of the input

$$u = \bar{G}(\bar{q})^{-1} A\left(\dot{\hat{z}} - \dot{\hat{z}}_d\right) + v \tag{18}$$

in the system (7), and a new input v, which then realizes a new port-Hamiltonian system under the condition that $\bar{G}(\bar{q})$, as in (10), is invertible. Consider finally a resulting Hamiltonian $H_z(\hat{x})$ with $\bar{q} = \hat{q} = q$ given by

$$H_z(\hat{x}) = \frac{1}{2}\hat{p}^\top \hat{p} + \frac{1}{2}(\hat{z} - z_d)^\top K_z^{-1}(\hat{z} - z_d) + V(\hat{q}) \tag{19}$$

with a tuning parameter $K_z = K_z^\top > 0$, and $\hat{x} = (\hat{q}, \hat{p}, \hat{z})$.

The new form of the interconnection and dissipation matrices of the port-Hamiltonian system are realized via the adapted momentum (17), the dynamic extension (14), and the new input (18), i.e.,

Proposition 1. Consider the system (2). Consider furthermore the change of variables $\Phi(q,p)$ as in (4), the $M(q)$ decomposition as in (5), and the Hamiltonian $\bar{H}(\bar{x})$ as in (6) that realizes the system (7). Finally, consider the dynamics of the new state $\hat{z}$, the change of coordinate $\hat{p}$, with $\bar{q} = \hat{q} = q$, and the control input u as in (14), (17), and (18), respectively. The resulting extended forced port-Hamiltonian system is then given by (for simplicity of notation, we leave out here the arguments of $\tilde{D}(\hat{q},\hat{p})$)

$$\begin{bmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \\ \dot{\hat{z}} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & T(\hat{q})^{-\top} & T(\hat{q})^{-\top} Y \\ -T(\hat{q})^{-1} & -\tilde{D} & -\tilde{D}Y \\ -Y^\top T(\hat{q})^{-1} & -Y^\top \tilde{D} & -Y^\top \tilde{D} Y \end{bmatrix} \begin{bmatrix} \frac{\partial H_z(\hat{x})}{\partial \hat{q}} \\ \frac{\partial H_z(\hat{x})}{\partial \hat{p}} \\ \frac{\partial H_z(\hat{x})}{\partial \hat{z}} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ \bar{G}(\hat{q}) \\ 0_{n\times n} \end{bmatrix} v + \begin{bmatrix} 0_{n\times n} \\ \bar{B}(\hat{q}) \\ Y^\top \bar{B}(\hat{q}) \end{bmatrix} f_e \tag{20}$$

with $\tilde{D}(\hat{q},\hat{p}) = -\bar{J}_2(\hat{q},\hat{p}) + \bar{D}(\hat{q},\hat{p})$, $Y = AK_z$, and with a new Hamiltonian function (19). The passive output of the transformed system (20) is

$$\hat{y} = \bar{G}(\hat{q})^\top \frac{\partial H_z(\hat{x})}{\partial \hat{p}} = \bar{G}(\hat{q})^\top \hat{p} \tag{21}$$

where $\hat{x} = (\hat{q}, \hat{p}, \hat{z})^\top$, and it follows that the new skew-symmetric interconnection matrix $\hat{J}(\hat{x})$, and the new symmetric, positive-semidefinite damping matrix $\hat{R}(\hat{x})$ of the port-Hamiltonian system (20) are

$$\hat{J}(\hat{x}) = \begin{bmatrix} 0_{n\times n} & T(\hat{q})^{-\top} & T(\hat{q})^{-\top} Y \\ -T(\hat{q})^{-1} & 0_{n\times n} & 0_{n\times n} \\ -Y^\top T(\hat{q})^{-1} & 0_{n\times n} & 0_{n\times n} \end{bmatrix} \tag{22}$$

$$\hat{R}(\hat{x}) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \tilde{D}(\hat{q},\hat{p}) & \tilde{D}(\hat{q},\hat{p}) Y \\ 0_{n\times n} & Y^\top \tilde{D}(\hat{q},\hat{p}) & Y^\top \tilde{D}(\hat{q},\hat{p}) Y \end{bmatrix} \tag{23}$$

Proof. When we use the new Hamiltonian $H_z(\hat{x})$ as in (19), the adapted momentum $\hat{p}$ as in (17), the new state $\hat{z}$ as in (14), the new input u as in (18), along with the port-Hamiltonian system (7), we obtain straightforwardly the form of the matrices (22) and (23) with the output (21). We can verify that the new dissipation matrix (23) is symmetric, positive-semidefinite via the Schur complement.

We have realized an extended port-Hamiltonian system with force feedback and structure preservation. The extended system for a nonconstant inertia mass matrix is obtained via a generalized canonical transformation. The next section takes advantage of the extended system (20) in order to obtain force control in the presence of an external force vector.

4. FORCE CONTROL

Based on the forced mechanical systems in the port-Hamiltonian framework (7), we now want to attain asymptotic stability in a desired force vector f_d as in (15). We realize an extended port-Hamiltonian system (20) via output feedback of the forces. Then, we assume that the lower and upper bounds of matrices $T(\hat{q})$, $\tilde{D}(\hat{q},\hat{p})$, and $\bar{G}(\hat{q})$ satisfy,

$$t_1 I \leq T(\hat{q}) \leq t_2 I \tag{24}$$

$$d_1 I \leq \tilde{D}(\hat{q},\hat{p}) \leq d_2 I \tag{25}$$

$$g_1 I \leq \bar{G}(\hat{q}) \leq g_2 I \tag{26}$$

with t_1, t_2, d_1, d_2, g_1, and g_2 positive constants. Based on the conditions (24), (25), and (26), we define a force control law of the system (7), i.e.,

Theorem 2. Consider a forced port-Hamiltonian system (7), and the assumptions (24), (25), and (26). Let $\hat{z}$ be the dynamics of the new state as in (14) with nonzero external forces f_e, and a passive output $\hat{y}$ as in (21). Then, the control input

$$u = \bar{G}(\hat{q})^{-1}\left(T(\hat{q})^{-1}\frac{\partial \bar{H}(\hat{x})}{\partial \hat{q}} + A\left(\dot{\hat{z}} - \dot{z}_d\right)\right) - C\hat{y} \tag{27}$$

with $C > 0$, s.t., $c_1 I \leq C \leq c_2 I$, and c_1, c_2, positive constants, asymptotically stabilizes the system (20) with zero steady-state error at $\hat{x} = x^* = (q_z, 0, z_d)$, z_d as the type of integral action over the desired force, q_z as in (16), where $Y = AK_z$, s.t., $\kappa_1 I \leq K_z \leq \kappa_2 I$, and $\gamma_1 I \leq Y \leq \gamma_2 I$, with κ_1, κ_2, γ_1, and γ_2, positive constants, and $\hat{H}$ as in (6). The closed-loop system becomes (for simplicity of notation, we leave out here the arguments of $\tilde{D}(\hat{q},\hat{p})$, and $\bar{G}(\hat{q})$)

$$\begin{bmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \\ \dot{\hat{z}} \end{bmatrix} = \begin{bmatrix} 0 & T(\hat{q})^{-\top} & T(\hat{q})^{-\top} Y \\ -T(\hat{q})^{-1} & -\tilde{D} - \bar{G}C\bar{G}^{\top} & -\tilde{D}Y \\ -Y^{\top} T(\hat{q})^{-1} & -Y^{\top}\tilde{D} & -Y^{\top}\tilde{D}Y \end{bmatrix} \begin{bmatrix} \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{q}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} \end{bmatrix} \tag{28}$$

with $\tilde{D}(\hat{q},\hat{p}) = -\bar{J}_2(\hat{q},\hat{p}) + \bar{D}(\hat{q},\hat{p})$, a Hamiltonian

$$\hat{H}(\hat{x}) = \frac{1}{2}\hat{p}^{\top}\hat{p} + \frac{1}{2}(\hat{z} - z_d)^{\top} K_z^{-1}(\hat{z} - z_d) \tag{29}$$

a skew-symmetric interconnection matrix $\hat{J}(\hat{x})$ as in (22), and a symmetric, positive-semidefinite damping matrix $\tilde{R}(\hat{x})$ as

$$\tilde{R}(\hat{x}) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \tilde{D}(\hat{q},\hat{p}) + \bar{G}(\hat{q})C\bar{G}(\hat{q})^{\top} & \tilde{D}(\hat{q},\hat{p})Y \\ 0_{n\times n} & Y^{\top}\tilde{D}(\hat{q},\hat{p}) & Y^{\top}\tilde{D}(\hat{q},\hat{p})Y \end{bmatrix} \tag{30}$$

.

Proof. If we apply the control law (27) to the system dynamics (7) with an adapted dynamic extension (14), and a change of variables (17), the closed-loop system becomes (28) with Hamiltonian (29), a damping matrix $\tilde{R}(\hat{x})$ as in (30), and via Schur complement we can verify that (30) is symmetric, positive-semidefinite.

Denote by $\underline{\lambda}(\mathscr{S}) = s_1$, and $\overline{\lambda}(\mathscr{S}) = s_2$, the upper, and lower bounds of the norm of a positive semidefinite matrix S, i.e., $s_1 I \leq \mathscr{S} \leq s_2 I$. Consider then a candidate Lyapunov function

$$\mathscr{H}(\hat{x}) = \hat{H}(\hat{x}) + \varepsilon\hat{p}^{\top}(\hat{z} - z_d) \tag{31}$$

with a constant $\varepsilon > 0$. Notice that the function (31) can be written in a matrix form as

$$\mathscr{H}(\hat{x}) = \frac{1}{2}\begin{bmatrix} (\hat{z} - z_d) \\ \hat{p} \end{bmatrix}^{\top} \begin{bmatrix} K_z^{-1} & \varepsilon \\ \varepsilon & I \end{bmatrix} \begin{bmatrix} (\hat{z} - z_d) \\ \hat{p} \end{bmatrix} \tag{32}$$

Then, the function (32) satisfies

$$\mathscr{H}(\hat{x}) \geq \frac{1}{2}\begin{bmatrix} \|(\hat{z} - z_d)\| \\ \|\hat{p}\| \end{bmatrix}^{\top} \underbrace{\begin{bmatrix} \underline{\lambda}(K_z^{-1}) & \varepsilon \\ \varepsilon & I \end{bmatrix}}_{P_1} \begin{bmatrix} \|(\hat{z} - z_d)\| \\ \|\hat{p}\| \end{bmatrix} \tag{33}$$

and from the definition of K_z, matrix P_1 is positive definite if $\underline{\lambda}(K_z^{-1}) - \varepsilon^2 > 0$, i.e.,

$$\sqrt{\frac{1}{\kappa_1}} > \varepsilon \tag{34}$$

Now, we want to prove that $\dot{\mathscr{H}}(\hat{x}) \leq 0$, along the trajectories of (7). First, we write $\dot{\mathscr{H}}(\hat{x})$ as

$$\dot{\mathscr{H}}(\hat{x}) = \frac{\partial \mathscr{H}(\hat{x})}{\partial \hat{q}}^{\top}\dot{\hat{q}} + \frac{\partial \mathscr{H}(\hat{x})}{\partial \hat{p}}^{\top}\dot{\hat{p}} + \frac{\partial \mathscr{H}(\hat{x})}{\partial \hat{z}}^{\top}\dot{\hat{z}} \tag{35}$$

Since $\dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{q}} = 0$, $\dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} = \hat{p}$, and $K_z\dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} = (\hat{z} - z_d)$, and based on the closed-loop dynamics (28), we replace $\dot{\hat{q}}$, $\dot{\hat{p}}$, and $\dot{\hat{z}}$, in (35), i.e.,

$$\begin{aligned} \dot{\mathscr{H}}(\hat{x}) = & \\ & -\left(\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} + Y\frac{\partial \hat{H}(\hat{x})}{\partial \hat{z}}\right)^{\top} \tilde{D}(\hat{q},\hat{p}) \left(\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} + Y\frac{\partial \hat{H}(\hat{x})}{\partial \hat{z}}\right) \\ & -\varepsilon\left(Y\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} + K_z\frac{\partial \hat{H}(\hat{x})}{\partial \hat{z}}\right)^{\top} \tilde{D}(\hat{q},\hat{p}) \left(\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} + Y\frac{\partial \hat{H}(\hat{x})}{\partial \hat{z}}\right) \\ & -\left(\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} + \varepsilon K_z\frac{\partial \hat{H}(\hat{x})}{\partial \hat{z}}\right)^{\top} \bar{G}(\hat{q})C\bar{G}(\hat{q})^{\top}\frac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \end{aligned} \tag{36}$$

and furthermore, from (36), we obtain

$$\begin{aligned} \dot{\mathscr{H}}(\hat{x}) = & \\ & -\varepsilon\begin{bmatrix} \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} \end{bmatrix}^{\top} \begin{bmatrix} \tilde{S}(\hat{q},\hat{p}) & \frac{1}{\varepsilon}\tilde{S}(\hat{q},\hat{p})^{\top} \\ \frac{1}{\varepsilon}\tilde{S}(\hat{q},\hat{p}) & K_z\tilde{S}(\hat{q},\hat{p})^{\top} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} \end{bmatrix} \\ & -\varepsilon\begin{bmatrix} \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} \end{bmatrix}^{\top} \begin{bmatrix} \frac{1}{\varepsilon}\tilde{G}(\hat{q},\hat{p}) & \tilde{S}(\hat{q},\hat{p})Y \\ K_z\tilde{G}(\hat{q},\hat{p}) & \frac{1}{\varepsilon}\tilde{S}(\hat{q},\hat{p})Y \end{bmatrix} \begin{bmatrix} \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\hat{x})}{\partial \hat{z}} \end{bmatrix} \end{aligned} \tag{37}$$

where $\tilde{S}(\hat{q},\hat{p})$, and $\tilde{G}(\hat{q},\hat{p})$ are

$$\tilde{S}(\hat{q},\hat{p}) = Y^{\top}\tilde{D}(\hat{q},\hat{p}) \tag{38}$$

$$\tilde{G}(\hat{q},\hat{p}) = \tilde{D}(\hat{q},\hat{p}) + \bar{G}(\hat{q})C\bar{G}(\hat{q})^{\top} \tag{39}$$

respectively. We now write (37) in terms of the vectors $\hat{p}$, and $(\hat{z} - z_d)$, as

$$
\begin{aligned}
&\dot{\mathscr{H}}(\hat{x}) = \\
&-\varepsilon \begin{bmatrix} \hat{p} \\ (\hat{z}-z_d) \end{bmatrix}^{\top} \begin{bmatrix} \tilde{S}(\hat{q},\hat{p}) & \frac{1}{\varepsilon}\tilde{S}(\hat{q},\hat{p})^{\top} K_z^{-1} \\ \frac{1}{\varepsilon} K_z^{-1}\tilde{S}(\hat{q},\hat{p}) & \tilde{S}(\hat{q},\hat{p})^{\top} K_z^{-1} \end{bmatrix} \begin{bmatrix} \hat{p} \\ (\hat{z}-z_d) \end{bmatrix} \\
&-\varepsilon \begin{bmatrix} \hat{p} \\ (\hat{z}-z_d) \end{bmatrix}^{\top} \begin{bmatrix} \frac{1}{\varepsilon}\tilde{G}(\hat{q},\hat{p}) & \tilde{S}(\hat{q},\hat{p}) Y K_z^{-1} \\ \tilde{G}(\hat{q},\hat{p}) & \frac{1}{\varepsilon} K_z^{-1}\tilde{S}(\hat{q},\hat{p}) Y K_z^{-1} \end{bmatrix} \begin{bmatrix} \hat{p} \\ (\hat{z}-z_d) \end{bmatrix}
\end{aligned} \tag{40}
$$

The dynamics of $\dot{\mathscr{H}}(\hat{x})$ as in (37) satisfy (for simplicity of notation, we leave out here the arguments of $\tilde{S}(\hat{q},\hat{p})$, and $\tilde{G}(\hat{q},\hat{p})$)

$$
\begin{aligned}
&\dot{\mathscr{H}}(\hat{x}) \leq \\
&-\varepsilon \begin{bmatrix} \|\hat{p}\| \\ \|\hat{z}-z_d\| \end{bmatrix}^{\top} \underbrace{\begin{bmatrix} \underline{\lambda}(\tilde{S}) & \frac{1}{\varepsilon}\overline{\lambda}\left(\tilde{S}^{\top} K_z^{-1}\right) \\ \frac{1}{\varepsilon}\overline{\lambda}\left(K_z^{-1}\tilde{S}\right) & \underline{\lambda}\left(\tilde{S}^{\top} K_z^{-1}\right) \end{bmatrix}}_{Q_1} \begin{bmatrix} \|\hat{p}\| \\ \|\hat{z}-z_d\| \end{bmatrix} \\
&-\varepsilon \begin{bmatrix} \|\hat{p}\| \\ \|\hat{z}-z_d\| \end{bmatrix}^{\top} \underbrace{\begin{bmatrix} \frac{1}{\varepsilon}\underline{\lambda}(\tilde{G}) & \overline{\lambda}\left(\tilde{S} Y K_z^{-1}\right) \\ \overline{\lambda}(\tilde{G}) & \frac{1}{\varepsilon}\underline{\lambda}\left(K_z^{-1}\tilde{S} Y K_z^{-1}\right) \end{bmatrix}}_{Q_2} \begin{bmatrix} \|\hat{p}\| \\ \|\hat{z}-z_d\| \end{bmatrix}
\end{aligned} \tag{41}
$$

and Q_1, and Q_2 are matrices with the diagonal elements depending on the lower bounds and the off-diagonal elements on the upper bounds, which give conditions for ε such that (41) is negative definite. This results in

$$
\frac{\overline{\lambda}(\tilde{S})\,\overline{\lambda}\left(\tilde{S}^{\top} K_z^{-1}\right)}{\underline{\lambda}\left(K_z^{-1}\tilde{S}\right)\underline{\lambda}\left(\tilde{S}^{\top} K_z^{-1}\right)} > \varepsilon^2 \tag{42}
$$

$$
\frac{\underline{\lambda}(\tilde{G})\,\underline{\lambda}\left(\tilde{S} Y K_z^{-1}\right)}{\overline{\lambda}(\tilde{G})\,\overline{\lambda}\left(K_z^{-1}\tilde{S} Y K_z^{-1}\right)} > \varepsilon^2 \tag{43}
$$

and replacing $\tilde{S}(\hat{q},\hat{p})$ as in (38), and $\tilde{G}(\hat{q},\hat{p})$ as in (39), in (42), and (47), we obtain

$$
\frac{\overline{\lambda}\left(Y^{\top}\tilde{D}(\hat{q},\hat{p})\right)\overline{\lambda}\left(\tilde{D}(\hat{q},\hat{p}) Y K_z^{-1}\right)}{\underline{\lambda}\left(K_z^{-1} Y^{\top}\tilde{D}(\hat{q},\hat{p})\right)\underline{\lambda}\left(\tilde{D}(\hat{q},\hat{p}) Y K_z^{-1}\right)} > \varepsilon^2 \tag{44}
$$

$$
\frac{\underline{\lambda}\left(\tilde{D}(\hat{q},\hat{p}) + \bar{G}(\hat{q}) C \bar{G}(\hat{q})^{\top}\right)\underline{\lambda}\left(Y^{\top}\tilde{D}(\hat{q},\hat{p}) Y K_z^{-1}\right)}{\overline{\lambda}\left(\tilde{D}(\hat{q},\hat{p}) + \bar{G}(\hat{q}) C \bar{G}(\hat{q})^{\top}\right)\overline{\lambda}\left(Y^{\top}\tilde{D}(\hat{q},\hat{p}) Y K_z^{-1}\right)} > \varepsilon^2 \tag{45}
$$

respectively. The evaluation of the inequality (44), and (45), results in the conditions

$$
\frac{\gamma_2 d_2 \kappa_1}{\gamma_1 d_1 \sqrt{\kappa_2}} > \varepsilon \tag{46}
$$

$$
\frac{\gamma_1}{\gamma_2}\sqrt{\frac{\left(d_1 + c_1 g_1^2\right) d_1 \kappa_2}{\left(d_2 + c_2 g_2^2\right) d_2 \kappa_1}} > \varepsilon \tag{47}
$$

respectively. Furthermore, via the conditions (46), and (47), the time derivative of (31) along the trajectories of (28) is negative definite for a sufficiently small ε. Since, $\mathscr{H}(\hat{x}) \geq 0$, and $\dot{\mathscr{H}}(\hat{x}) \leq 0$, Lyapunov stability theory along with La Salle's Invariance Principle, implies asymptotic stability of system (7) in $\hat{x} = (\hat{q},\hat{p},\hat{z}) = (q_z, 0, z_d)$.

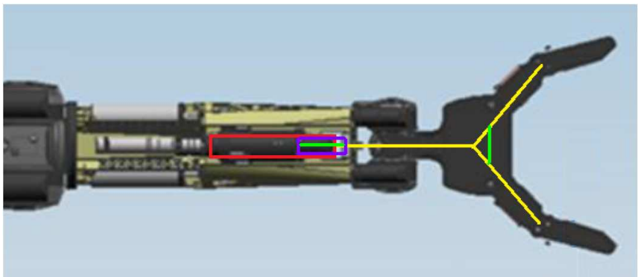

Fig. 1. Drawing of the Gripper of the PERA

Remark 3. It is clear the implementation of the control law (27) for a constant mass-inertia matrix. For a nonconstant mass-inertia matrix the control law is not clear how to do this because the external forces f_e are not known. A way to deal with this is to deduce the external forces from the sensor readings, i.e., the readings not be fed back as in the constant mass inertia case but f_e can be canceled when f_e are known.

We have realized a control law in order to obtain force control with force feedback. We now know that from the proposed integrator dynamics (14), we obtain structure preservation in the extended port-Hamiltonian system (20) which are useful for force control via force feedback. Furthermore, we have given a stability analysis of a standard mechanical system (7) via the control law (27). In Section 5, we finally motivate the present port-Hamiltonian approach with an example of a class of standard mechanical systems with constant mass-inertia matrix.

5. EXAMPLE

5.1 End-effector system

To gain more insight into the role of the forced port-Hamiltonian system (7), and the force control via force feedback, we consider a class of standard mechanical systems with a constant mass-inertia matrix. The system is given by the gripper (end-effector) of the Philips Experimental Robot Arm (PERA), Rijs et al. (2010). A drawing of the gripper is shown in Figure 1. The gripper consists of a shaft (red rectangle) actuated by the motor of the gripper which is attached to the fingers via cables (yellow lines). When the shaft moves left the gripper closes, and when it moves right the gripper opens. The two green lines depict the two springs actuating over the tips of the gripper. Furthermore, the gripper is controlled via scripts developed in Matlab® with a sampling time of $10ms$.

The model of the gripper in the port-Hamiltonian framework consists of a mass m_g, interconnected by a nonlinear spring of stiffness K_g, a rest-length c_g, and a linear damping $d_g > 0$. The states of the system are $x = (q,p)^{\top}$, where q is the displacement between the two tips of the gripper, and p is the generalized momenta of the system. The displacement of the two tips is directly proportional to the encoder of the motor. Finally, Coulomb friction forces, and the gravitational forces are neglected in the working space of the tips of the gripper.

The Hamiltonian of the system is

$$
H(q,p) = \frac{1}{2} m_g^{-1} p^2 + \frac{1}{2} k_g (q - c_g)^2 \tag{48}
$$

and the system is then described in the port-Hamiltonian framework as

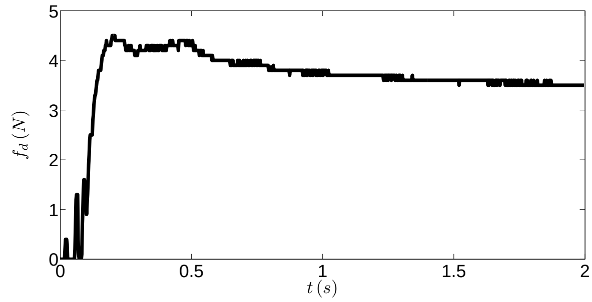

Fig. 2. Force control of the tips of the gripper of the PERA via the control law (27). Initial conditions $(q(0), p(0))^\top = (3,0)^\top$.

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & d \end{bmatrix} \begin{bmatrix} K_g(q-c_g) \\ m_g^{-1} p \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u + \begin{bmatrix} 0 \\ f_e \end{bmatrix} \quad (49)$$

with an input matrix $G = 1$ (fully actuated), and an external force $f_e \in \mathbb{R}$. The nonlinear spring of the gripper is defined as

$$K_g = \begin{cases} k_{g_1} & q - q_z \geq 0 \\ k_{g_2} & q - q_z < 0 \end{cases}$$

with positive constants k_{g_i}, and $i = 1,2$, and a resulting q_z as in (16). We implement the new input (18) and the change of variable (17) in the port-Hamiltonian system (49), and then we obtain an extended port-Hamiltonian system

$$\begin{bmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \\ \dot{\hat{z}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & X \\ -1 & -d_g & -d_g X \\ -X^\top & -X^\top d_g & -X^T d_g X \end{bmatrix} \begin{bmatrix} k_g(\hat{q}-c) \\ m_g^{-1}\hat{p} \\ K_z^{-1}\hat{z} \end{bmatrix} \quad (50)$$

$$+ \begin{bmatrix} 0 \\ v \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -X^T \end{bmatrix} f_e$$

where $\dot{\hat{q}} = \dot{q}$, and $X = m^{-1} A K_z$, with a new Hamiltonian function $H_z(\hat{x}) = H_z(\hat{q}, \hat{p}, \hat{z})$ as in (19), and a new output $\hat{y}$ as in (21).

In the next section we show, via an experiment with the system (49), the results of Section 4, in order to obtain a desired force.

5.2 *Experimental results*

For experiment purposes, we have grasped a *squash ball* which represents a nonzero external force vector. We then have a rest length $c_r = 0$ (gripper open), an inertia matrix $M = m = 0.2$, stiffness coefficients $k_{g_1} = 0.5$ and $k_{g_1} = 0.3$, a damping coefficient $d = 0.5$; constants $A = 1$, $K_p = 1$, and $C = 5$; an initial position $q(0) = 3cm$, and a desired force of $f_d = 3.5N$. Based on these parameters, we can see how the conditions (46), and (47) are given for a small ε. We now apply the control law (27) on the extended port-Hamiltonian system (50). Figure 2 shows the experiment results. We obtain the desired force f_d from a initial position $q(0)$ at $t = t_1 \geq 1.5s$ with a zero steady-state error.

6. CONCLUDING REMARKS

This paper is devoted to the development of a new strategy of force control via force feedback in the port-Hamiltonian framework. Our main motivation is given by the proposition of an alternative to the classical methods of force feedback and force control in the Euler-Lagrange formalism. We have shown that, given a force sensor output, we can realize force feedback for a class of mechanical systems in the port-Hamiltonian framework with structure preservation. A type of integral action over the force sensor output and a coordinate transformation are the main strategies to realize a force feedback. We also have given a force control law that consist of force and output feedback in presence of external forces. The closed-loop system is then asymptotically stable in a constant desired force. Future work includes developments for a nonzero, and nonconstant, desired forces.

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