Control of Nonlinear Systems Using Multiple Model Black-Box Identification ⋆

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Abstract: The paper is devoted to development of control algorithms for nonlinear parametrically uncertain systems. Original system dynamics is approximated by a set of local NARX models combined by a special mixing rule. Algorithm for local models' parameters estimation and structure adjustment has been developed. Proposed approach allows straightforward designing of the combined feedforward/feedback controller.

Keywords: Identification, NARX model, switching algorithms, feedforward control.

1. MOTIVATION

For many technical systems an attempt of designing physical-based model leads to analytical description of high complexity. These models reproduce the real plant dynamics rather precisely, but have inherent disadvantage. Identification and model-based control design in this case are very complicated tasks. While analytical solution could be just impossible to derive, even numerical procedures lead to massive calculations that pushes back real-time implementation.

An example where aforementioned problems arise could be air-to-fuel ratio control of internal combustion engines as well as motion control of industrial manipulators. Of course, this is not a complete list, but tasks of current interest of authors.

Even the mean value internal combustion engine model is quite complicated for analysis and control development (see e.g. Gerasimov et al. (2010); Turin and Geering (1993)). Such processes as air flow through the intake manifold, fuel injection, combustion energy to torque conversion should be described based on physical laws that leads to a system of nonlinear differential equations. For these equations majority of the coefficients are unknown and a number of the state variables are either not measurable or can be measured with an unknown variable time delay. Moreover, some processes such as fuel evaporation in the collector are hard to model at all.

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Another example is a dynamical model of a flexible link 6-DOF industrial robotic arm derived using Lagrange or Newton-Euler formulations(see e.g. Tomei (1991)). Even though we have to deal with the system of at least 12 nonlinear equations, since to increase its accuracy friction (see e.g. Al-Bender et al. (2005); Canudas de Wit et al. (1995)) and gearbox stiffness (see e.g. Ruderman et al. (2009)) should be included in consideration. In most cases not all robot parameters are defined precisely. On the other hand, it is hard to use complete physical-based robot model for identification purposes.

One of the natural ways to deal with nonlinear systems is linearisation, but frequently this approach does not work properly. For example, a single linear model could be not enough to approximate original dynamics with a desired accuracy.

An alternative approach is to use a set of local models of "suitable" structure combined by a switching rule. The piecewise affine and mixed logical dynamical systems are among the most popular forms of such hybrid approximations (see e.g. Bemporad (2004); Heemels et al. (2001)).

Special attention is paid to analysis of robustness properties of the resulting system due to switching effects. By convention we can define two types of switching algorithms such as hierarchical and fuzzy ones. According to this classification well known dwell-time and hysteresis switching algorithms belong to hierarchical category (see e.g. Efimov (2006); Liberzon (2003)). In both cases, at the instant of switching a jump in control usually occurs, and the response to it deteriorates the quality of control. To avoid it a fuzzy switching or mixing algorithms could be used. In many situations, using of fuzzy-logic methods for nonlinear systems approximation gives acceptable results for applications (see Takagi and Sugeno (1985)).

So, the goal of the paper is to find trade-off between the accuracy and complexity of nonlinear systems' models that allow us designing of controllers that can be effectively implemented in practice.

While the proposing approach leads to the models structurally allied to the piecewise affine systems, nevertheless there are special features. At first, NARX models are used for the local approximation. Secondly, system' mode determination is based on the values of the special characteristic variables only, so there is no need to measure or estimate all state and input variables. Finally, an original fuzzy switching approach has been developed that allows us to avoid jumps in transients.

2. LOCAL MODELS IDENTIFICATION

Our first task is to obtain accurate models of a complex nonlinear system. Suppose that the overall system dynamics can be divided into separate modes and for each of these modes a corresponding local approximating model can be defined.

Without loss of generality we can define all models in discrete time, while this might be more convenient representation for future implementation using digital controllers.

We have chosen nonlinear autoregression with external inputs (NARX) as the basis for local models. This is a rather flexible structure. On the one hand, such model allows one to bring in effects of nonlinear dynamics. On the other hand, it is well studied concerning design of control algorithms.

In general, NARX model can be represented as

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{j=0}^{p} \boldsymbol{b_j^T} \boldsymbol{d}(k-j), \tag{1}$$

where k is the number of step, a_i and $\boldsymbol{b}_j$ are regression coefficients and $\boldsymbol{d}(k)$ is the vector of external inputs.

In our case, coefficients a_i and $\boldsymbol{b}_j$ as well as orders of the polynomials $n \geq 1$ and $p \geq n-1$ are supposed to be a priori unknown.

Additional question is how to set the vector of external inputs. Let there be R physically measurable signals $x_1, x_2, \dots x_R$, including control signals, and the maximal degree of their occurrence s_{max} is given. Then, in the general case the input vector can be constructed according to the rule $[x_1 \dots x_R \quad x_1^2 \quad x_1 x_2 \dots x_R^2 \dots x_R^{s_{max}}]$.

To keep model linear in control (affine), all entrances of the corresponding variable in powers higher than one should be eliminated in $\boldsymbol{d}(k)$. Thus, we can split $\boldsymbol{d}(k)$ in two parts

$$\boldsymbol{d}(k) = \left[\boldsymbol{d}_1^T(k) \quad \boldsymbol{d}_2^T(k)\right]^T, \tag{2}$$

where $\boldsymbol{d}_1(k) = \tilde{\boldsymbol{d}}(k) u(k)$, $\tilde{\boldsymbol{d}}(k) = \left[1 \quad \tilde{d}_2(k) \quad \dots \quad \tilde{d}_l(k)\right]$, and $u(k)$ is the control signal.

Remark 1. Analysis of the physical-based model of the system should be useful for the external input vector assignment. For example, maximum powers for each input signal can be chosen according to the structure of this model. It can help to avoid overparametrization problem.

Since the structure of the local model has been defined, we can move to estimation of unknown parameters.

Transform (1) to the following form

$$y(k) = \theta^T \phi(k), \tag{3}$$

where $\theta = [a_1 \dots a_n \quad \boldsymbol{b}_0^T \dots \boldsymbol{b}_p^T]^T$,
$\phi(k) = [y(k-1) \dots y(k-n) \quad \boldsymbol{d}^T(k) \dots \boldsymbol{d}^T(k-p)]^T$.

Suppose that after conducted active experiment we have arrays of N measurements for input $\boldsymbol{\Phi} = [\phi(1) \dots \phi(N)]$ and output $\boldsymbol{Y} = [y(1) \dots y(N)]^T$ variables of (3). Thus, one of the well-known estimation algorithms can be implemented (see e.g. Ljung (1987); Eykhoff (1974)). For example, least squares approach gives the following solution

$$\hat{\theta} = (\boldsymbol{\Phi}^T \boldsymbol{\Phi})^{-1} \boldsymbol{\Phi}^T \boldsymbol{Y}, \tag{4}$$

where $\hat{\theta}$ is the estimation of θ.

The following indices can be introduced to estimate accuracy of approximation:

- **root-mean-square error** $J = \sqrt{N^{-1} \sum_{k=1}^{N} e^2(k)}$, where $e(k) = y^*(k) - y(k)$, $y^*(k)$ is the output of the plant, and $y(k)$ is the output of the approximating model;
- **maximum error** $e_m = \max_{1 \leq k \leq N} |e(k)|$.

Complex approximation index can be defined as

$$\kappa = \frac{\kappa_1}{J} + \frac{\kappa_2}{e_m} + \kappa_3 \eta, \tag{5}$$

where $\kappa_1 > 0$, $\kappa_2 > 0$, and $\kappa_3 > 0$ are constant weighting coefficients, $\eta = \min_{1,n}(1 + ln(1 - |\lambda_i|))$ is the stability margin of (1), where λ_i is the root of characterstic polynomial of (1). The objective is to find coefficients of the model (1), which maximize the value of κ.

Finally, the following algorithm has been designed for black-box identification of the local models.

(1) Set ranges $[n_{min}, n_{max}]$, $[p_{min}, p_{max}]$ (the intervals of admissible values for the orders of the polynomials n and p in (1) respectively) and value s_{max} and generate the corresponding "full" vector $\boldsymbol{d_f}(k)$. Set $p = p_{min}$ and $\kappa_{bs} = -\infty$.
(2) Set $n = n_{min}$.
(3) Set the number of elements of input vector $l = l_m in$
(4) Set the first l elements of the $\boldsymbol{d_f}(k)$ as the current external input $\boldsymbol{d}(k)$.
(5) Estimate $\hat{\theta}$.
(6) Calculate approximation index κ.
(7) If $\kappa > \kappa_{bs}$, set $\kappa_{bs} = \kappa_m$ and $\Omega_{bs} = \left\{\hat{\theta}, n, p, \boldsymbol{d}(k)\right\}$.
(8) If $l < \mathbf{dim}(\boldsymbol{d_f}(k))$, then set $l = l+1$, and go to step 4. Otherwise, go to the next step.
(9) If $n < n_{max}$, set $n = n+1$ and go to step 3. Otherwise, go to the next step.
(10) If $p < p_{max}$ set $p = p+1$ and go to step 2. Otherwise, go to the next step.
(11) Define Ω_{bs} as the best set of parameters for local approximating model and exit.

Remark 2. Described algorithm implements an exhaustive search to find the best possible set of parameters of the approximating model. More advanced methods of stochastic optimization may be used to speed up this process.

3. SWITCHING RULE

Since we described the procedure of local model black-box identification the following step is the switching rule introduction.

Two different approaches of decomposition by the dynamic modes can be specified. One is based on the entire system behaviour analysis. The second one uses for partitioning just values of particular state variables. We used the latter one. This approach is rather simple and always provides a unique solution. For internal combustion engines such characteristic variables could be torque or speed of rotation of a crankshaft. For robotic arm it could be generalized coordinates.

Suppose that characteristic variables that uniquely define every dynamical mode of the original system or zone are given and corresponding boundary values are known. Then for each time step we can calculate the so-called salience function. For one dimensional case its values can be defined in the following way

$$s_m(k) = \begin{cases} 1, & \text{if } Z_{m-1} \le x(k) \le Z_m, \\ 1 - \dfrac{x(k) - Z_m}{\Delta_Z}, & \text{if } Z_m \le x(k) \le Z_m + \Delta_Z, \\ 1 - \dfrac{Z_{m-1} - x(k)}{\Delta_Z}, & \text{if } Z_{m-1} - \Delta_Z \le x(k) \le Z_{m-1}, \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

where m is the number of dynamical mode, $x(k)$ is characteristic variable, Z_m, $m = \overline{1 : N_z - 1}$ are corresponding boundary values, N_z is the total number of modes, $\Delta_Z < \min_{m=\overline{2:N_z-1}}(Z_m - Z_{m-1})$ is the buffer zone between modes introduced to avoid instantaneous jumps during switching.

The graphical interpretation of salience function for the one-dimensional partitioning is shown in Fig. 1.

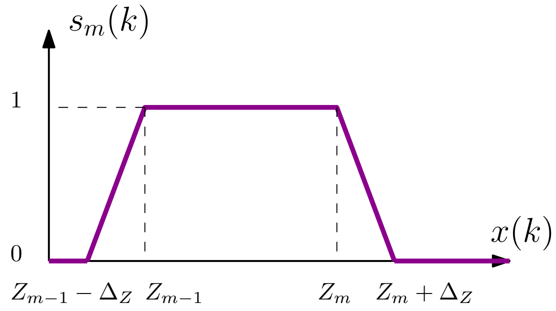

Fig. 1. Salience function distribution for one-dimensional partitioning

Remark 3. We can do deeper partitioning by increasing the number of characteristic variables. If we use r independent partitioning variables then the number of modes or zones will be $N_\Sigma = \prod_{i=1,r} N_{z,i}$. Flip side is that we need to provide enough experimental data filling every zone. Otherwise, we can't guarantee convergence of local model parameters' estimations.

Then we should normalize values of salience function

$$\bar{s}_m(k) = \frac{s_m(k)}{\sum_{i=1}^{N_z} s_i(k)}. \tag{7}$$

The next step is real-time adjustment of the salience function based on current system output measurements

$$\begin{cases} J_m^0(k) = e^{-\alpha\sqrt{M^{-1}\sum_{i=0}^{M}(y^*(k-i) - \hat{\theta}_m^T\phi_m(k-i))^2}}, \\ \tilde{s}_m(k) = \bar{s}_m(k) J_m^0(k), \end{cases} \tag{8}$$

where $\alpha > 0$ and M is the width of the sliding window, and $\hat{\theta}_m^T$ are defined in (4).

In some sense proposed salience function adjustment algorithm (8) is close to second level adaptation in multiple model adaptive control (see e.g. Anderson et al. (2001); Kuipers and Ioannou (2010)).

Finally, the output of the resulting model is the weighted sum of the outputs of the local models

$$y(k) = \sum_{m=1}^{N_z} \tilde{s}_m(k)\hat{\theta}_m^T\phi(k). \tag{9}$$

So, the obtained model (9) can be classified as non-stationary input-output linear system that for each dynamical mode or zone provides a stable local approximating model.

4. CONTROLLER DESIGN

Local models are constructed in a way to ensure its straightforward inversion. This advantage allows us to implement combined feedforward and feedback control algorithm (see Fig. 2).

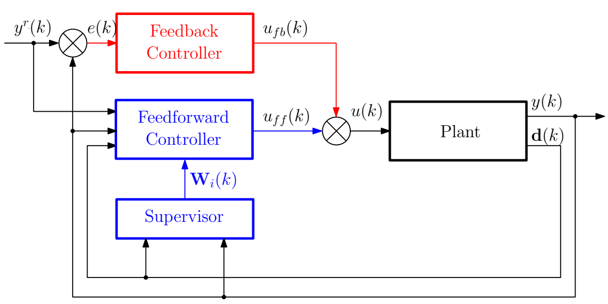

Fig. 2. Structure of the combined controller.

4.1 Feedforward Control

Feedforward control plays the main role in the proposed approach.

Using equation (2) we can rewrite (3) in the following form

$$\begin{aligned} y(k+1) = \sum_{i=0}^{n} \boldsymbol{W}_i^y y(k-i) + \sum_{l=-1}^{p} \boldsymbol{W}_{l+1}^1 \boldsymbol{f}_1(k-l) + \\ + \sum_{l=-1}^{p} [\boldsymbol{W}_{l+1}^u + \boldsymbol{W}_{l+1}^0 \boldsymbol{F}_0(k-l)] u(k-l), \end{aligned} \tag{10}$$

where $\boldsymbol{W}_i^y, \boldsymbol{W}_{l+1}^1, \boldsymbol{W}_{l+1}^u, \boldsymbol{W}_{l+1}^0$ are the values of time-dependent parameters, $y(k)$, $u(k)$, $\boldsymbol{f}_1(k)$, and $\boldsymbol{F}_0(k)$ are vectors of output, control signal, and additional inputs respectively.

In case of multiple models mixing (9) parameters of the resulting model can be calculated as weighted sums of corresponding parameters of local models

$$\begin{aligned} \boldsymbol{W}_i^y &= \sum_{m=1}^{N_z} \tilde{s}_m(i)\hat{\boldsymbol{W}}_{i,m}^y, \\ \boldsymbol{W}_i^1 &= \sum_{m=1}^{N_z} \tilde{s}_m(i)\hat{\boldsymbol{W}}_{i,m}^1, \\ \boldsymbol{W}_i^u &= \sum_{m=1}^{N_z} \tilde{s}_m(i)\hat{\boldsymbol{W}}_{i,m}^u, \\ \boldsymbol{W}_i^0 &= \sum_{m=1}^{N_z} \tilde{s}_m(i)\hat{\boldsymbol{W}}_{i,m}^0 \end{aligned} \tag{11}$$

After inversion of (10) relative to the control signal we obtain

$$\begin{aligned} u(k+l) = {} & [\boldsymbol{W}_0^u + \boldsymbol{W}_0^0\boldsymbol{F}_0(k+1)]^{-1}\{y^r(k+1)+ \\ & + \sum_{i=0}^{z} \boldsymbol{A}_i(y(k-i) - y^r(k-i)) \\ & - \sum_{i=0}^{n} \boldsymbol{W}_i^y y(k-i) - \sum_{l=-1}^{p} \boldsymbol{W}_{l+1}^1 \boldsymbol{f}_1(k-l) - \\ & - \sum_{l=0}^{p} [\boldsymbol{W}_{l+1}^u + \boldsymbol{W}_{l+1}^0\boldsymbol{F}_0(k-l)]u(k-l)\}, \end{aligned} \tag{12}$$

where y^r is the desired output and matrices $\boldsymbol{A}_i$ are Hurwitz.

4.2 Feedback Control

Feedback control plays a supporting role. Its auxiliary function is robustness increasing. For this purposes non-linear PID-controller of a special form has been used

$$\begin{aligned} u_{fb}(k) = {} & k_1 e(k) + k_2 \sum_{i=0}^{k} e(i) + k_3 \sum_{i=1}^{k} (e(i) - e(i-1)) \\ & + k_4 \mathtt{sign}(e(k)) + k_5 e^3(k), \end{aligned} \tag{13}$$

where $k_1, \ldots, k_5$ are chosen to guarantee stability of the closed-loop system.

The cubic term was added in (13) to provide faster response in error comparing the proportional term. At the same time an odd degree allows to keep sign of error. In turn, the sign function increases robustness of the closed-loop system against small noise in measurements.

4.3 Anti-Windup Modification

While in real systems the control signal is usually limited in magnitude, proposed controller (12) — (13) should be modified to take into account range of admissible control values. Existent approaches to anti-windup design do not consider the models in the form (10) (see e.g. Kapoor et al. (1998); Sofrony (2009)), when additional inputs do not play a role of disturbances, they really affect the system dynamics. To overcome this problem different anti-windup modification has been proposed.

Let the control $u_{min} \le u \le u_{max}$. Introduce the penalty function (see Fig. 3)

$$\phi(\Theta, u_{min}, u_{max}) = e^{-\beta(\Theta - u_{min})} + e^{-\beta(u_{max} - \Theta)}, \tag{14}$$

where $\beta > 0$, and its derivative

$$\dot{\phi}(\Theta, u_{min}, u_{max}) = \beta[e^{-\beta(u_{max} - \Theta)} - e^{-\beta(\Theta - u_{min})}]. \tag{15}$$

Now define the objective function

$$\begin{aligned} Q_k(\boldsymbol{W}, \Theta) = {} & 0,5\delta_k^T(\boldsymbol{W}, \Theta)\delta_k(\boldsymbol{W}, \Theta) \\ & + \phi(\Theta, u_{min}, u_{max}), \end{aligned} \tag{16}$$

where

$$\begin{aligned} \delta_k(\boldsymbol{W}, \Theta) = {} & \sum_{i=0}^{n} \boldsymbol{W}_i^y y(k-i) + \sum_{l=-1}^{p} \boldsymbol{W}_{l+1}^1 \boldsymbol{f}(k-l) \\ & + \sum_{l=0}^{p} [\boldsymbol{W}_{l+1}^u + \boldsymbol{W}_{l+1}^0 \boldsymbol{F}_{i-l}]u(k-l) \\ & + [\boldsymbol{W}_0^u + \boldsymbol{W}_0^0 \boldsymbol{F}_{i+l}]\Theta - y^r(k+1) \\ & - \sum_{i=0}^{z} \boldsymbol{A}_i(y(k-i) - y^r(k-i)). \end{aligned}$$

Then the admissible control can be found as the solution of the following optimization problem

$$u(k+1) = \min_{\Theta \in U} Q_k(\boldsymbol{W}, \Theta), \tag{17}$$

where $U = \{u \in R : u_{min} \le u \le u_{max}\}$.

To find an approximate solution of (17) one can use the simplest gradient descent optimization method with projection Ortega and Rheinboldt (2000)

$$\begin{aligned} & \Theta_0 = u(k), \\ & \Theta_{S+1} = \Theta_S - \Gamma\{[\boldsymbol{W}_0^u + \boldsymbol{W}_0^0 \boldsymbol{F}_{k+l}]\delta_k(\boldsymbol{W}, \Theta_S) \\ & \qquad + \dot{\phi}(\Theta_S, u_{min}, u_{max}\}, S = 0, K-1, \\ & u(k+1) = \Theta_K, \end{aligned} \tag{18}$$

where $K \ge 1$.

At each step k, the current control calculated at the preceding step is used as the initial condition, and the algorithm makes K steps toward minimization of the objective functional (16). The result of optimization is used as the control at the $(k+1)$ step, and the procedure

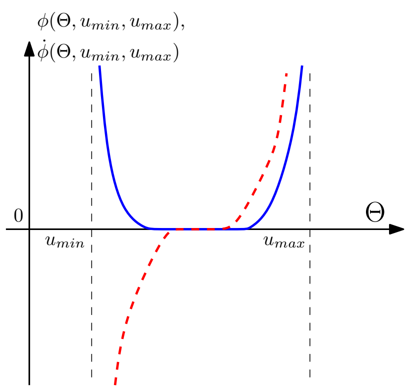

Fig. 3. Plots for $\phi(\theta, u_{min}, u_{max})$ (solid) and $\dot{\phi}(\theta, u_{min}, u_{max})$ (dashed).

is repeated. The number of steps K directly depends on the available computing resources.

5. VALIDATION RESULTS

Results of computer simulation are given in this section to illustrate workability of the proposed approach. To say a few words about the origin of the data we used for simulation. The used dataset is the same as it was obtained in GM report (2010). All data and variable values used for plotting are normalized for industrial security reasons.

Table 1 illustrates accuracy of the real ICE dynamics approximation based on the black-box identification of multiple local models with mixing. Results of computer simulation and experimental data have been compared for different partitioning parameters. In this example, Z_m is the engine torque.

Table 1.

$\{Z_m\}$, Nm	Δ_Z, Nm	J, d.u.	e_m, d.u.
[50 100 150 200]	10	0,0507	0,15891
[50 100 150 200]	15	0,0497	0,14973
[30 60 120 150 180]	10	0,0472	0,13987

As we can conclude from these results, accuracy of approximation strictly depends on partitioning parameters such as the distribution of boundary values of characteristic variable and the size of buffer zone. Boundary values should be assigned in accordance with the structure of the initial experimental data. Clustering algorithms can be implemented to automate this process. The wider buffer zone leads to the smaller root-mean-square error, but maximum error of approximation can be higher at the same time.

Table 2 compiles the comparative data of the computer simulation for estimation of the performance of the closed-loop system for the proposed multiple models mixing approach (6) — (9) and hierarchical switching between local models. During the simulation we tested different sets of controllers' parameters (12), (13), (18). Presented results have been obtained for $Z = [30\ \ 60\ \ 120\ \ 150\ \ 180]$, $\Delta_Z = 10$, $\alpha = 2500$, $M = 5$, $\beta = 100$, $\Gamma = 100$, $k_1 = 5$, $k_2 = 0,01$, $k_3 = 0,005$, $k_4 = k_5 = 0,1$.

Additional performance index has been introduced to estimate the energy of the control spent for the system stabilization

$$J_u(k) = \frac{1}{N}\sum_{m=0}^{N} u^2(m). \tag{19}$$

In the case of air-to-fuel ratio stabilization control signal is the pulse width of fuel injection.

Table 2.

Switching	J, d.u.	e_m, d.u.	J_u, ms
Hierarchical	0,145	0,332	19,709
Mixing	0,118	0,137	19,938

During experimental validation of proposed approach on a real vehicle we achieved maximum error of less than 25% and mean error about 5%. These values comply both with practical requirements and the performance of existing control systems based on manual calibration and gain-scheduling.

6. CONCLUSIONS

The paper presents an approach for control of parametrically uncertain nonlinear systems based on multiple models approximation. The structure of local approximating models and three-level algorithm for selecting the best one were introduced. Then the original partitioning and mixing algorithm with real-time self-tuning was described in details. Finally, the combined feedforward and feedback control algorithm was obtained.

Workability of the described approach has been proved both in the computer simulation and experiments with real internal combustion engines.

Development of automatic partitioning algorithms and design of advanced switching schemes maintaining system robustness can be mentioned as promising directions of future research.

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