Trajectory Tracking for a Group of Unicycle-type Robots using an Attitude Observer ⋆

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Abstract: The trajectory tracking problem for a group of unicycle-type robots is addressed and solved by means of a partial state feedback strategy based on the leader-followers scheme using an observer to estimate the orientation angle of each mobile robot. The control law is based on an extended kinematic model where the output function to be controlled is the mid-point of the wheels axis of each robot. This choice leads to an ill defined control law when the robot is at rest. To avoid such a singularity, a complementary control law is enabled when the linear velocity of each robot is close to zero. It is shown that the combination of a classical dynamic full information controller with an exponentially convergent vehicle attitude observer yields an asymptotically stable closed-loop system. Real-time experiments show the performance of the proposed control scheme.

Keywords: Multi-Agent Systems; Observer Design; Mobile Robotics; Trajectory Tracking.

1. INTRODUCTION

During the last 20 years, the coordination of multiple mobile robots has emerged as a new research area of mobile robotics and multi-agent systems theory. The applications include toxic residues cleaning, transportation and manipulation of large objects, searching and rescue, security, simulation of biological entities behaviors, etc Arai et al. [2002]. Current research issues include motion coordination, task assignment, communication, etc Fukunaga and Kahng [1997]. Applications of multi-robot coordination have been mostly developed for wheeled mobile robots, specifically unicycle-type robots, shown in Fig. 1. Despite the apparent simplicity of this device, controlling it yields to different kinds of challenges de Wit et al. [1996]. First, the system in Fig. 1 is underactuated and has nonholonomic constraints, i.e., the velocities of the system satisfy non integrable constraints. As stated in Brockett [1983] such nonholonomic systems cannot be stabilized by continuously differentiable, time invariant, state feedback controllers. Because of this restriction, some works consider the front points of robots as outputs to control avoiding singularities in the control law Desai et al. [2001]. However, the resulting control laws do not influence directly the orientation angles which remain as internal dynamics.
From a technological point of view, other kind of problems arise. For instance, the estimation of the position and attitude of a mobile robot is not a simple task Jakubiak et al. [2002], Noijen et al. [2005]. Even if the position is relatively easy to estimate, the estimation of the attitude is more involved. Different absolute and relative position estimation approaches have been proposed over the past decade. Amongst the absolute positioning methods, video cameras and ultrasonic emitter-receiver arrangements are frequently encountered. Concerning relative positioning methods, dead reckoning is widely used because of its simplicity. It provides an attitude estimation, but it is unsuitable for long distances due to errors associated with noise and slipping conditions Borenstein and Feng [1996].
In this paper, it is shown that the fusion of a full information variable structure dynamic state feedback controller with a globally exponentially stable attitude observer yields a locally asymptotically stable solution to the trajectory tracking problem for a group of unicycle-type robots. The variable structure control scheme allows to overcome the singularity that arises when the robots' longitudinal velocity is zero. The proposed attitude observer requires information about the robots' position only.
The paper is organized as follows: Section 2 presents the kinematic model of unicycles and the problem statement. Section 3 presents the control strategy that solves the formation tracking problem with convergence of the orientation angles. The main results of the paper are the attitude observer design and the stability analysis of the partial information dynamic state feedback controller. These results are presented in Section 4. Section 5 shows the performance of the proposed control strategy using an experimental setup. Finally, in Section 6 some conclusions are given.

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2. KINEMATIC MODEL AND PROBLEM STATEMENT

Denote by $N = \{R_1, \ldots, R_n\}$ a set of n unicycle-type robots moving in the plane. The kinematic model of each robot R_i, as shown in Fig. 1, is given by

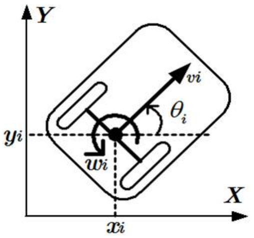

Fig. 1. Kinematic model of unicycles

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ w_i \end{bmatrix}, \quad i = 1, \ldots, n, \tag{1}$$

where $[x_i, y_i]^T$ are the coordinates of the mid-point of the wheels axis, θ_i is the orientation of the robot with respect to the $X-$axis, v_i is the longitudinal velocity of the mid-point of the wheels axis and w_i is the angular velocity. The outputs of the system (1) are chosen as $z_i = [x_i, y_i]^T$, $i = 1, \ldots, n$ and their dynamics are given by

$$\dot{z}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \tilde{A}_i(\theta_i) \begin{bmatrix} v_i \\ w_i \end{bmatrix}, \tilde{A}_i(\theta_i) = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \end{bmatrix}, \tag{2}$$

where $\tilde{A}_i(\theta_i)$ is the so-called decoupling matrix of every robot R_i. The decoupling matrix is singular for every value of θ_i. In order to overcome this problem, consider the following dynamic extension for the kinematic model.

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\xi}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \xi_i \cos\theta_i \\ \xi_i \sin\theta_i \\ u_i \\ w_i \end{bmatrix} \tag{3}$$

where $\xi_i = v_i$ is a new state and u_i is a new control signal. The dynamics of the output variables z_i, $i = 1, \ldots, n$ with respect to the extended system (3) now is given by

$$\ddot{z}_i = A_i(\theta_i, \xi_i) \begin{bmatrix} u_i \\ w_i \end{bmatrix}, A_i(\theta_i, \xi_i) = \begin{bmatrix} \cos\theta_i & -\xi_i \sin\theta_i \\ \sin\theta_i & \xi_i \cos\theta_i \end{bmatrix}. \tag{4}$$

Note that $\det[A_i(\theta_i, \xi_i)] = \xi_i$. Therefore, the new decoupling matrix $A_i(\theta_i, \xi_i)$ is non-singular whenever $\xi_i \neq 0$. Considering this restriction, it is possible to design a strategy to control the output functions z_i. The singularity $\xi_i = 0$ will be dealt with in Section 3.

Based on the leader-followers scheme, consider R_n as the group leader and the rest as followers. Let $z_i^* = [x_i^*, y_i^*]^T$ be the desired position of R_i in a particular formation pattern. In this work, we define

$$\begin{aligned} z_i^* &= z_{i+1} + c_{(i+1)i}, \quad i = 1, \ldots, n-1 \\ z_n^* &= m(t) \end{aligned} \tag{5}$$

where $c_{(i+1)i} = [p_{(i+1)i}, r_{(i+1)i}]^T \in \Re^2$, $i = 1, \ldots, n-1$, denotes the desired relative position of R_i with respect to R_{i+1} and $m(t) = [x_d(t), y_d(t)]^T$ is a twice continuously differentiable function that corresponds to the desired trajectory of the leader.

Problem statement (Marching with orientation): The control goal is to design a control law

$$\begin{bmatrix} u_i(t) \\ w_i(t) \end{bmatrix} = f_i(z_i(t), z_{i+1}(t)), \quad i = 1, \ldots, n-1$$

for every follower robot and

$$\begin{bmatrix} u_n(t) \\ w_n(t) \end{bmatrix} = f_n(z_n(t), m(t))$$

for the leader robot such that

- $\lim_{t\to\infty} (z_i(t) - z_i^*(t)) = 0$, $i = 1, \ldots, n-1$,
- $\lim_{t\to\infty} (z_n(t) - m(t)) = 0$,
- $\lim_{t\to\infty} (\theta_i(t) - \theta_j(t)) = 0$, $\forall i \neq j$

Fig. 2 displays the position of the robots when they satisfy the desired trajectory tracking and formation pattern. The goal of the leader is to follow the path of marching whereas the goal of the followers is to maintain a desired pattern formation. Note that the marching control also requires that the orientation angles converge to the same value.

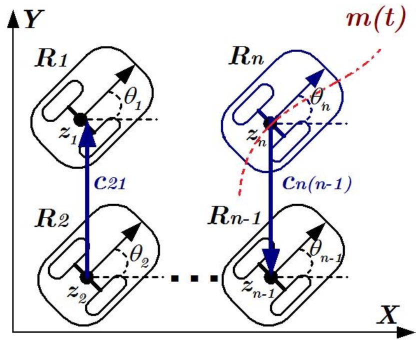

Fig. 2. Marching with orientation control of unicycles

3. CONTROL STRATEGY

In this section, a marching strategy is presented for the group of unicycles. After that, a complementary control law is designed to commute to when the state trajectory of the robots approaches a singularity. The results of this Section have been presented in González-Sierra et al. [2011] and González-Sierra et al. [2013]. They are briefly recalled here for the reader's convenience. We define the following marching control law:

$$\eta_1 : \begin{cases} \begin{bmatrix} u_i \\ w_i \end{bmatrix} = A_i^{-1}(\theta_i, \xi_i)\left[\ddot{m} - k_1\dot{e}_i - k_0 e_i\right] \\ \\ \begin{bmatrix} u_n \\ w_n \end{bmatrix} = A_n^{-1}(\theta_n, \xi_n)\left[\ddot{m} - c_1\dot{e}_n - c_0 e_n\right] \end{cases} \tag{6}$$

where k_0, k_1, c_0, c_1 are scalar design parameters and

$$\begin{aligned} e_i &= z_i - z_i^*, \quad i = 1, \ldots, n-1 \\ e_n &= z_n - m(t) \end{aligned}$$

are the error coordinates.

In (6), the control law of every follower robot R_i requires the marching path acceleration, the position and the velocity of the robot R_{i+1}. The control law for the leader robot requires position, velocity and acceleration of the marching path.

Remark 1. Note that the leader is the only robot with complete information about the position, velocity and acceleration of the desired marching path. The follower robots do not require to process complete information about the path of marching and the positions of all agents. Therefore, the proposed control law constitutes a decentralized approach. In related works, for instance Yamaguchi [2003], all agents must know the target position of the leader, the marching trajectory and more than one desired relative positions with respect to other robots.

Theorem 1. Consider the system (3) and the control law (6). Suppose that $\xi_i \neq 0\ \forall\ t \geq 0$, k_0, k_1, c_0, $c_1 > 0$. Then, in the closed-loop system (3)-(6), the follower robots converge to the desired formation, i.e. $\lim_{t\to\infty}(z_i - z_i^*) = 0$, $i = 1, ..., n-1$ whereas R_n converge to the marching trajectory i.e. $\lim_{t\to\infty}(z_n(t) - m(t)) = 0$. Moreover it holds that $\lim_{t\to\infty}(\theta_i(t) - \theta_j(t)) = 0$, $\forall i \neq j$.

3.1 Commutation Scheme

Recall that the control law η_1, is ill-defined when $\xi_i = 0$. To cope with this drawback, consider the output function

$$h_i = \begin{bmatrix} \xi_i \\ \theta_i \end{bmatrix}, \quad i = 1, \ldots, n. \tag{7}$$

The dynamics of these variables is simply given by

$$\begin{bmatrix} \dot{\xi}_i \\ \dot{\theta}_i \end{bmatrix} = M \begin{bmatrix} u_i \\ w_i \end{bmatrix}, i = 1, \ldots, n, M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where M is the non-singular decoupling matrix. Therefore, it is possible to define the next feedback

$$\eta_2 : \begin{bmatrix} u_i \\ w_i \end{bmatrix} = \dot{h}_d - q_0(h_i - h_d), \quad i = 1, \ldots, n \tag{8}$$

where $h_d = [\xi_d, \theta_d]^T$ and q_0 =diag(q_{01}, q_{02}) is a design parameter. Now we propose a commutation strategy between controller (6) and (8) to avoid the singularity at $\xi_i = 0$. The rule of commutation can be established by

$$\eta = \begin{cases} \eta_1, & \text{when } |\xi_i| \geq \delta \\ \eta_2, & \text{when } |\xi_i| < \delta \end{cases} \tag{9}$$

with $\delta > 0$ as the commutation threshold. Finally, it is necessary to define the desired trajectory of h_d to apply the control law (8) according to

$$\xi_d = \dot{x}_d \cos\theta_d + \dot{y}_d \sin\theta_d, \quad \theta_d = \arctan\left(\frac{\dot{y}_d}{\dot{x}_d}\right)$$

The control law (8) does not guarantee the tracking of the marching control. It only attempts to preserve the convergence of the orientation angles during the commutation interval.

Theorem 2. Consider the switched closed-loop system (3)-(8) using the control law (6), and the alternative control law (8) and suppose $q_0 > 0$ Then, it holds that $\lim_{t\to\infty}(h_i - h_d) = 0$, $i = 1, ..., n$ and consequently $\lim_{t\to\infty}(\theta_i - \theta_j) = 0$, $\forall i \neq j$.

4. ATTITUDE OBSERVER

In this section we present the attitude observer developed, first in Rodríguez-Cortés and Aranda-Bricaire [2007], Velasco-Villa et al. [2012] for single mobile robot and extended this results to a group of unicycle-type robots in González-Sierra et al. [2012]. To begin with, we need to establish the following standing assumption:

$$\lim_{t\to\infty}\int_0^t \xi_i \tan^{-1}(\xi_i)\, dt = \infty \quad i = 1, \ldots, n. \tag{10}$$

and define the estimation errors, as follows

$$\begin{aligned} s_{i1} &= \cos(\theta_i) - \gamma_{i1} - \Gamma x_i \tan^{-1}(\xi_i) \\ s_{i2} &= \sin(\theta_i) - \gamma_{i2} - \Gamma y_i \tan^{-1}(\xi_i) \quad i = 1, \ldots, n. \end{aligned} \tag{11}$$

Proposition 1. Consider the unicycle mobile robot described by equations (3). Consider now the system

$$\begin{aligned} \dot{\gamma}_{i1} &= -\left[\gamma_{i2} + \Gamma y_i \tan^{-1}(\xi_i)\right] w_i \\ &\quad -\Gamma\xi_i \tan^{-1}(\xi_i)\left[\gamma_{i1} + \Gamma x_i \tan^{-1}(\xi_i)\right] - \frac{\Gamma x_i u_i}{1+\xi_i^2} \\ \dot{\gamma}_{i2} &= \left[\gamma_{i1} + \Gamma x_i \tan^{-1}(\xi_i)\right] w_i \\ &\quad -\Gamma\xi_i \tan^{-1}(\xi_i)\left[\gamma_{i2} + \Gamma y_i \tan^{-1}(\xi_i)\right] - \frac{\Gamma y_i u_i}{1+\xi_i^2} \end{aligned} \tag{12}$$

where u_i, w_i and ξ_i are known signals. Assuming that (10) holds, then there exists a positive constant Γ such that for any initial condition $[\gamma_{i1}(0), \gamma_{i2}(0)]^T$ the following holds for $i = 1, \ldots, n$

$$\begin{aligned} &\lim_{t\to\infty}\left[\cos\theta_i - \gamma_{i1} - \Gamma x_i \tan^{-1}(\xi_i)\right] = 0, \\ &\lim_{t\to\infty}\left[\sin\theta_i - \gamma_{i2} - \Gamma y_i \tan^{-1}(\xi_i)\right] = 0 \end{aligned} \tag{13}$$

Proof González-Sierra et al. [2012].

Note that the dynamics of the observer (12) depends only on known signals. In turn, if (13) holds, then $\cos\theta_i$, $\sin\theta_i$ and θ_i can be approximated by

$$\begin{aligned} \cos\theta_i &\approx \bar{c}_{\theta_i} = \gamma_{i1} + \Gamma x_i \arctan(\xi_i) \\ \sin\theta_i &\approx \bar{s}_{\theta_i} = \gamma_{i2} + \Gamma y_i \arctan(\xi_i) \\ \theta_i &\approx \bar{\theta}_i = \arctan\left(\frac{\gamma_{i2} + \Gamma y_i \arctan(\xi_i)}{\gamma_{i1} + \Gamma x_i \arctan(\xi_i)}\right) \end{aligned}$$

4.1 Dynamic Patial State Feedback

In this subsection we present an asymptotically stabilizing dynamic partial state feedback for the group of unicycle-type robot. This is obtained combining the full information controller (6) with the estimation algorithm presented in Proposition 1.

Proposition 2. Consider the unicycle mobile robot dynamics (3) in closed loop with the dynamic controller

$$\sigma_1 : \begin{cases} \begin{bmatrix} u_i \\ w_i \end{bmatrix} = \bar{A}_i \begin{bmatrix} \ddot{x}_d - k_1(\xi_i\bar{c}_{\theta i} - \xi_{i+1}\bar{c}_{\theta i+1}) - k_0 e_{i1} \\ \ddot{y}_d - k_1(\xi_i\bar{s}_{\theta i} - \xi_{i+1}\bar{s}_{\theta i+1}) - k_0 e_{i2} \end{bmatrix} \\ \qquad i = 1, \ldots, n-1 \\ \begin{bmatrix} u_n \\ w_n \end{bmatrix} = \bar{A}_n \begin{bmatrix} \ddot{x}_d - k_1(\xi_n\bar{c}_{\theta n} - \dot{x}_d) - k_0 e_{n1} \\ \ddot{y}_d - k_1(\xi_n\bar{s}_{\theta n} - \dot{y}_d) - k_0 e_{n2} \end{bmatrix} \end{cases} \tag{14}$$

Suppose that (10) holds and that $\xi_i(t) \neq 0 \; \forall t \geq 0$, with

$$\bar{A}_i = \begin{bmatrix} \bar{c}_{\theta i} & \bar{s}_{\theta i} \\ \frac{-\bar{s}_{\theta i}}{\xi_i} & \frac{\bar{c}_{\theta i}}{\xi_i} \end{bmatrix} \quad \bar{A}_n = \begin{bmatrix} \bar{c}_{\theta n} & \bar{s}_{\theta n} \\ \frac{-\bar{s}_{\theta n}}{\xi_n} & \frac{\bar{c}_{\theta n}}{\xi_n} \end{bmatrix}$$

where γ_{i1} and γ_{i2} are computed from equation (12); $\bar{c}_{\theta i}$, $\bar{s}_{\theta i}$ and $\bar{\theta}_i$ are the estimated variables. Then all tracking errors of the closed-loop system are bounded and are such that

$$\lim_{t\to\infty} e_i = 0, \quad \lim_{t\to\infty} \dot{e}_i = 0, \quad i = 1, \ldots, n$$

Proof To begin with, let us define

$$\zeta = \begin{bmatrix} \zeta_{11} \\ \zeta_{12} \\ \zeta_{21} \\ \zeta_{22} \\ \vdots \\ \zeta_{(2n-1)1} \\ \zeta_{(2n-1)2} \\ \zeta_{(2n)1} \\ \zeta_{(2n)2} \end{bmatrix} = \begin{bmatrix} e_{11} \\ e_{12} \\ \dot{e}_{11} \\ \dot{e}_{12} \\ \vdots \\ e_{n1} \\ e_{n2} \\ \dot{e}_{n1} \\ \dot{e}_{n2} \end{bmatrix} \tag{15}$$

Note now that the closed-loop dynamics (3)-(14) expressed in terms of the (ζ, s) coordinates becomes

$$\dot{\zeta} = K\zeta + \Psi(\zeta, s) \tag{16}$$

where

$$K = \begin{bmatrix} \bar{K} & \tilde{K} & 0_4 & \ldots & 0_4 & 0_4 \\ 0_4 & \bar{K} & \tilde{K} & \ldots & 0_4 & 0_4 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_4 & 0_4 & 0_4 & \ldots & \bar{K} & \tilde{K} \\ 0_4 & 0_4 & 0_4 & \ldots & 0_4 & \bar{K} \end{bmatrix}$$

$$\bar{K} = \begin{bmatrix} 0_2 & I_2 \\ -K_0 & -K_1 \end{bmatrix}, \quad \tilde{K} = \begin{bmatrix} 0_2 & 0_2 \\ K_0 & K_1 \end{bmatrix}$$

0_4 is a 4×4 zero matrix, I_2 is the 2×2 identity matrix of , 0_2 is a 2×2 zero matrix, $K_0 = \mathrm{diag}\{k_0, k_0\}$, $K_1 = \mathrm{diag}\{k_1, k_1\}$ and

$$\Psi(\zeta, s) = \begin{bmatrix} 0 \\ \Psi_1(\zeta, s) \\ 0 \\ \Psi_2(\zeta, s) \\ 0 \\ \vdots \\ \Psi_{n-2}(\zeta, s) \\ 0 \\ \Psi_{n-1}(\zeta, s) \\ 0 \\ \Psi_n(\zeta, s) \end{bmatrix} \tag{17}$$

where

$$\begin{aligned} \Psi_i(\zeta, s) &= \bar{\phi}_i - \bar{\phi}_{i+1} + \varphi_{u_i} K_1 s_i - 2K_1 \varphi_{u_{i+1}} s_{i+1} \\ &\quad + \varphi_{u_{i+2}} K_1 s_{i+2}, \quad i = 1, \ldots, n-2 \\ \Psi_{n-1}(\zeta, s) &= \bar{\phi}_{n-1} - \bar{\phi}_n + \varphi_{u_{n-1}} K_1 s_{n-1} - 2K_1 \varphi_{u_n} s_n \\ \Psi_n(\zeta, s) &= \bar{\phi}_n + \varphi_{u_n} K_1 s_n, \end{aligned}$$

$$\begin{aligned} \bar{\phi}_i &= \phi_i \left(\ddot{m} + K_1 \varphi_{u_i} s_i - K_1 \varphi_{u_{i+1}} s_{i+1}\right) - \phi_i K_0 \zeta_{2i-1} \\ &\quad - \phi_i K_1 \zeta_{2i}, \quad i = 1, \ldots, n-1 \\ \bar{\phi}_n &= \phi_n (\ddot{m} + K_1 \varphi_{u_n} s_n) - \phi_n K_0 \zeta_{2n-1} - \phi_{n-1} K_1 \zeta_{2n} \end{aligned}$$

and

$$\phi_i = \begin{bmatrix} -s_{i2}\varphi_{s_i} - s_{i1}\varphi_{c_i} & s_{i1}\varphi_{s_i} - s_{i2}\varphi_{c_i} \\ s_{i2}\varphi_{c_i} - s_{i1}\varphi_{s_i} & -s_{i2}\varphi_{s_i} - s_{i1}\varphi_{c_i} \end{bmatrix} \quad i = 1, ..., n$$

$$\varphi_{u_n} = \sqrt{\left(\zeta_{(2n)1} + \dot{x}_d\right)^2 + \left(\zeta_{(2n)2} + \dot{y}_d\right)^2}$$

$$\varphi_{u_i} = \sqrt{\left(\zeta_{(2i)1} + \varphi_{u_{i+1}}\varphi_{c_{i+1}}\right)^2 + \left(\zeta_{(2i)2} + \varphi_{u_{i+1}}\varphi_{s_{i+1}}\right)^2}$$

$$\varphi_{s_n} = \frac{\zeta_{(2n)2} + \dot{y}_d}{\varphi_{u_n}} \quad \varphi_{s_i} = \frac{\zeta_{(2i)2} + \varphi_{u_{i+1}}\varphi_{s_{i+1}}}{\varphi_{u_i}}$$

$$\varphi_{c_n} = \frac{\zeta_{(2n)1} + \dot{x}_d}{\varphi_{u_n}} \quad \varphi_{c_i} = \frac{\zeta_{(2i)1} + \varphi_{u_{i+1}}\varphi_{c_{i+1}}}{\varphi_{u_i}}$$

Since the dynamics (3) in closed-loop with (6) is asymptotically stable there exists a positive definite Lyapunov function

$$V_1 = \zeta^T P \zeta \tag{18}$$

such that

$$\dot{V}_1 = -\zeta^T Q \zeta$$

along (3)-(6) with P and Q positive definite matrices. Simple computations show that the time derivative of (18) along the trajectories of the perturbed system (16) is

$$\dot{V}_1 = -\zeta^T Q \zeta + \zeta^T P \Psi(\zeta, s) + \Psi^T(\zeta, s) P \zeta$$

The perturbation term (17) is bounded from above by,

$$\| \Psi(\zeta, s) \| \leq \bar{\Gamma}_{i1}(\| s \|) + \bar{\Gamma}_{i2}(\| s \|) \| \zeta \| \tag{19}$$

where $\bar{\Gamma}_{i1}$, $\bar{\Gamma}_{i2}$ are class-κ functions differentiable at $s = 0$ and are defined by

$$\bar{\Gamma}_{i1}(\| s \|) = \begin{bmatrix} \Gamma_{11}(\| s \|) \\ \Gamma_{21}(\| s \|) \\ \vdots \\ \Gamma_{(n-2)1}(\| s \|) \\ \Gamma_{(n-1)1}(\| s \|) \\ \Gamma_{n1}(\| s \|) \end{bmatrix}$$

$$\bar{\Gamma}_{i2}(\| s \|) = \begin{bmatrix} (6k_1 + 2k_0) \| s \| + 4k_1 \| s \|^2 \\ \vdots \\ (6k_1 + 2k_0) \| s \| + 4k_1 \| s \|^2 \\ (5k_1 + 2k_0) \| s \| + 3k_1 \| s \|^2 \\ (2k_1 + k_0) \| s \| + k_1 \| s \|^2 \end{bmatrix}$$

$$\begin{aligned} \Gamma_{i1}(\| s \|) &= (2 \| \ddot{m} \| + k_1 \nu) \| s \| \quad i = 1, \ldots, n-3 \\ \Gamma_{(n-2)1}(\| s \|) &= (2 \| \ddot{m} \| + k_1 \chi) \| s \| \\ \Gamma_{(n-1)1}(\| s \|) &= (2 \| \ddot{m} \| + k_1 \mu) \| s \| \\ \Gamma_{n1}(\| s \|) &= (\| \ddot{m} \| + k_1 \| s \| \| \dot{m} \| + k_1 \| \dot{m} \|) \| s \| \end{aligned}$$

with

$$\nu = \|s\| \left(\|\dot{z}_{i+1}\| + 2\|\dot{z}_{i+2}\| + \|\dot{z}_{i+3}\|\right)$$
$$+ \|\dot{z}_{i+1}\| + 2\|\dot{z}_{i+2}\| + \|\dot{z}_{i+3}\|$$
$$\chi = \|s\| \left(\|\dot{m}\| + \|\dot{z}_{n-1}\| + 2\|\dot{z}_{n}\|\right)$$
$$+ \|\dot{m}\| + \|\dot{z}_{n-1}\| + 2\|\dot{z}_{n}\|$$
$$\mu = 2\|s\| \left(\|\dot{m}\| + \|\dot{z}_{n}\|\right) + 2\|\dot{m}\| + \|\dot{z}_{n}\|$$

The time derivative of (18) along (16) can be upper bounded as follows

$$\dot{V}_1 \leq -\lambda_{\min}(Q) \parallel \zeta \parallel^2$$
$$+2\lambda_{\max}(P) \parallel \zeta \parallel \left(\bar{\Gamma}_{i1}(\parallel s \parallel) + \bar{\Gamma}_{i2}(\parallel s \parallel) \parallel \zeta \parallel\right)$$

Because $\bar{\Gamma}_{i1} \parallel s \parallel$ and $\bar{\Gamma}_{i2} \parallel s \parallel$ converge to zero, from standard properties of cascaded systems (Proposition 4.11 of Kokotivić et al. [1997]) it can be concluded that position errors converge to zero.
Recall that the control law (14) is ill-defined when the longitudinal velocity $\xi_i = 0$. To overcome this difficulty, we propose a commutation scheme between the partial information dynamic control law (14) and a modified version of the alternative control law (8). More precisely, define

$$\bar{U} = \begin{cases} \sigma_1, & \text{when } \mid \xi_i \mid \geq \delta \\ \sigma_2, & \text{when } \mid \xi_i \mid < \delta \end{cases} \tag{20}$$

where σ_1 is equation (14) and

$$\sigma_2 : \begin{bmatrix} u_i \\ w_i \end{bmatrix} = \begin{bmatrix} \dot{\xi}_d - q_{01}(\xi - \xi_d) \\ \dot{\theta}_d - q_{02}(\bar{\theta}_i - \theta_d) \end{bmatrix} \tag{21}$$

5. REAL-TIME EXPERIMENTAL RESULTS

The Real-time experiments were carried out over an experimental setup composed of three unicycle-type mobile robots manufactured by MobileRobots Inc, model AmigoBot (Fig. 3), furnished on their top with different size ellipses for identification. The position and orientation of each robot is measured through a vision system composed of a video camera manufactured by JAI, model CM-030 able to provide 90 frames per second at a maximum resolution of 640×480 and a Pentium 4-based computer for image processing by means of an interface programmed in Visual C#, using the libraries designed by Common Vision Blox. This allows to discriminate the robots according to the ellipses' sizes and determine their position and orientation. A second computer calculates and sends the required control signal for each robot through Wi-Fi. The control law is calculated in Visual C++ using Aria libraries which are used to communicate with the robots. Both computers are linked through unidirectional RS-232. The parameters of the robots are: wheel radius $r = 6$ cm and length of wheels axis $L = 28$ cm. The workspace measures 2.4 x 1.8 m.

Fig. 4 shows the behavior on the plane of the closed-loop system (3)-(20), for $n = 3$. It is interesting to observe, at the beginning of the experiment, that the followers attempt to track the leader even if it is not converging to the desired trajectory . This is because the followers do not have information about the desired trajectory. The controller and observer parameters are: sampling period

Fig. 3. AmigoBot Robots

$T = 60$ ms, $k_1 = 3$, $k_0 = 2$, $\Gamma = 60$, $q_{01} = 2$, $q_{02} = 1$. The initial conditions for the observer states are $\gamma_{11}(0) = \gamma_{21}(0) = \gamma_{31}(0) = 1$, $\gamma_{12}(0) = \gamma_{22}(0) = \gamma_{32}(0) = 0$ and the commutation threshold was selected as $\delta = 0.02$. The desired formation pattern is a triangle described by $c_{32} = [-0.3, 0.8]^T$, $c_{21} = [-1, 0]^T$. The desired marching path is a Lemniscata of Bernoulli given by $m(t) = [1.2 + 0.4\cos(\omega t), -0.4 + 0.2\sin(2\omega t)]^T$ where $w = \dfrac{2\pi}{50}$. The initial conditions are, for the leader robot $[x_3, y_3, \xi_3, \theta_3] = [1.61, -0.34, 0, 0.012]$, and for the followers $[x_2, y_2, \xi_2, \theta_2] = [1.92, -1.12, 0, 0.05]$, $[x_1, y_1, \xi_1, \theta_1] = [0.92, -1.14, 0, -0.012]$. Fig. 5 and Fig. 6 show the estimated orientation angles and the estimation errors, respectively. Finally, Fig. 7 displays the controls u_i and w_i needed to achieve the trajectory tracking.

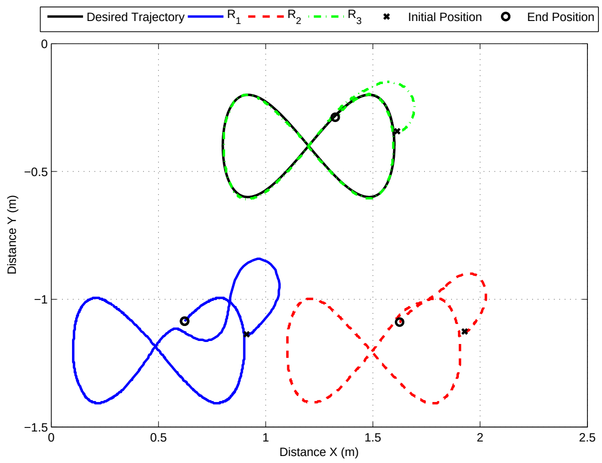

Fig. 4. Trajectory of the robots

6. CONCLUSIONS

The formation tracking problem for a group of unicycle-type robots has been addressed and solved in this work using partial information about other robots and marching trajectory. Assuming the knowledge of the position of the robot only, a bank of attitude observers is proposed to estimate the orientation of each vehicle. It is shown that the combination of a classical dynamic full information controller with an asymptotically convergent vehicle attitude observer yields an asymptotically stable closed-loop

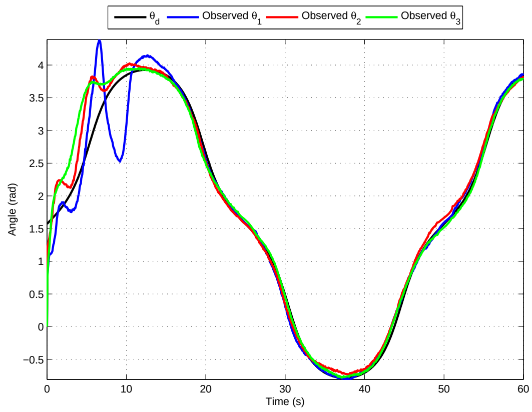

Fig. 5. Orientation angles

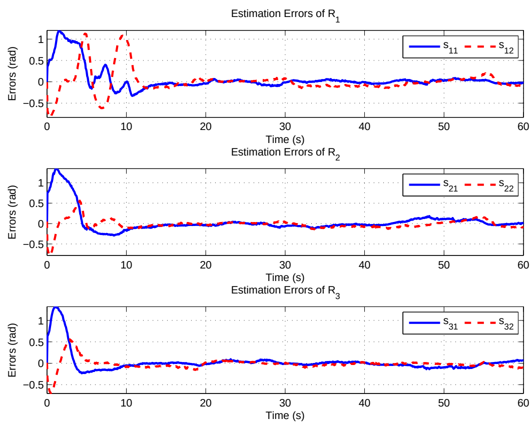

Fig. 6. Attitude estimation errors

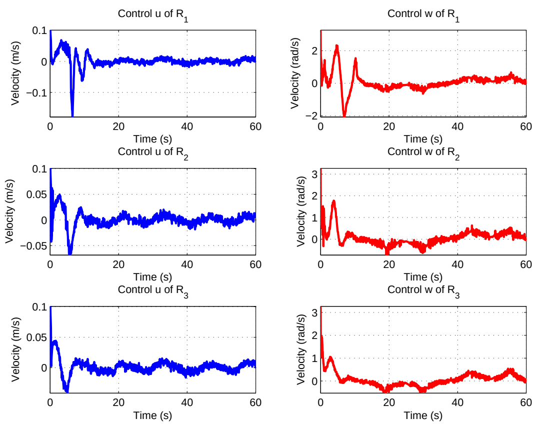

Fig. 7. Control signals for the robots

system. Since partial information dynamic controller steers the midpoint of the wheels axis of the robots, it is ill-defined when longitudinal velocities vanish. To overcome this obstruction, a commutation scheme based on the estimated attitude is proposed. From the experimental point of view, even though the sample period is relatively large, namely $T = 60$ ms, the performance of the attitude observer and the control law is rather satisfactory.

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