A Globally Exponentially Stable Tracking Controller for Mechanical Systems with Friction Using Position Feedback

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Abstract: A solution to the problem of global exponential tracking without velocity measurement of mechanical systems *with friction* and possibly *unbounded* inertia matrix is given in the paper. The proposed controller is obtained combining a new full–information passivity–based controller with a new immersion and invariance observer. The resulting closed–loop system has, in some suitably defined coordinates, a port–Hamiltonian structure with a desired energy function and a uniformly positive definite damping matrix. In this way, global *exponential* tracking of position and velocity for all desired reference trajectories is ensured.

Keywords: Mechanical systems, output–feedback tracking, observers, stabilization

1. INTRODUCTION

A solution to the long standing open problem of construction, *without velocity measurements*, of a (smooth) controller for mechanical systems that ensures global *exponential* tracking of position and velocity for all desired reference trajectories was recently reported in (Romero et al., 2013). The controller is a certainty equivalent combination of (a slight variation of) the Immersion and Invariance (I&I) globally exponentially convergent speed observer of (Astolfi et al., 2010) with a, suitably tailored, static state–feedback passivity–based controller (PBC) (Ortega et al., 1998). The reader is referred to (Børhaug et al., 2006; Ortega et al., 1998; Zergeroglu et al., 2000), and references therein, for a review of the long literature on position feedback tracking for mechanical systems.

Although the result of (Romero et al., 2013) is, to the best of our knowledge, the strongest one available to date for this important problem, it suffers from two drawbacks. First, it is assumed that there is no friction present in the system. Second, if the inertia matrix is not bounded from above, which is the case of robots with prismatic joints, only asymptotic (but not exponential) convergence is ensured. The main contribution of this paper is to present a new controller that overcomes the aforementioned limitations. The controller consists of a new I&I observer and a new full–information PBC, which ensure that the closed–loop is uniformly globally *exponentially* stable (UGES) in spite of the presence of Coulomb friction and without the assumption of upper–bounded inertia matrix.

The new design differs from the scheme proposed in (Romero et al., 2013) in the following.

– A new damping injection term in the PBC that, in some suitable coordinates (similar to the ones used in (Romero et al., 2012)), results in a uniformly positive definite damping coefficient.
– The inclusion of a new friction compensation term that uses the estimate of the velocity.
– The redesign of the I&I observer of (Astolfi et al., 2010) to compensate for the additional error terms that appear due to friction.

The remaining of the paper is organized as follows. The main result is presented in Section 2. To enhance readability its proof is split into three parts, given in three sections. The design of a full–state feedback PBC with friction compensation and a new damping injection term is given in Section 3. In Section 4 the I&I observer of (Astolfi et al., 2010) is redesigned to take into account the presence of friction. Finally, in Section 5 we analyze the overall closed–loop system to complete the proof of the main result. The paper is wrapped–up with some concluding remarks and open research problems in Section 6.

Notation. To avoid cluttering the notation, throughout the paper κ and α are generic positive constants. For $x \in \mathbb{R}^n$, $S \in \mathbb{R}^{n\times n}$, $S = S^\top > 0$, we denote the Euclidean norm $|x|^2 := x^\top x$, and the weighted–norm $\|x\|_S^2 := x^\top S x$. Given a function $f : \mathbb{R}^n \to \mathbb{R}$ we define the differential operators

$$\nabla f := \left(\frac{\partial f}{\partial x}\right)^\top,\ \nabla^2 f := \left(\frac{\partial^2 f}{\partial x^2}\right)^\top,\ \nabla_{x_i} f := \left(\frac{\partial f}{\partial x_i}\right)^\top,$$

where $x_i \in \mathbb{R}^p$ is an element of the vector x. For a mapping $g : \mathbb{R}^n \to \mathbb{R}^m$, its Jacobian matrix is defined as

$$\nabla g := \begin{bmatrix} (\nabla g_1)^\top \\ \vdots \\ (\nabla g_m)^\top \end{bmatrix},$$

where $g_i : \mathbb{R}^n \to \mathbb{R}$ is the i-th element of g.

2. MAIN RESULT

In the paper we consider n–degrees of freedom, fully-actuated, mechanical systems with Coulomb friction described in port–Hamiltonian (pH) form by

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -\mathfrak{D}(q) \end{bmatrix} \nabla H(q,\mathbf{p}) + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \qquad (1)$$

with total energy function $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$H(q,\mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M^{-1}(q)\mathbf{p} + V(q),$$

where $q, \mathbf{p} \in \mathbb{R}^n$ are the generalized positions and momenta, respectively, $u \in \mathbb{R}^n$ is the control input, $\mathfrak{D}(q) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the Coulomb friction and satisfies

$$\mathfrak{D}(q) = \mathfrak{D}^\top(q) \geq d_{\min} I_n \geq 0,$$

the inertia matrix $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ verifies the (uniform in q) bound

$$M(q) = M^\top(q) \geq m_{\min} I_n > 0,$$

and $V : \mathbb{R}^n \to \mathbb{R}$ is the potential energy function.

The following observations regarding the model are in order.

(i) The mechanical system includes the presence of friction forces, whose compensation is far from obvious in the present scenario of absence of velocity measurement.
(ii) The standard assumption of upper–bounded inertia matrix is conspicuous by its absence. This assumption rules out many interesting mechanical systems, including robots with prismatic joints.
(iii) It is assumed that all parameters, including the Coulomb friction matrix, are *known*. See Section 6 for some remarks regarding the possible inclusion of parameter estimation.

Proposition 1. Consider the mechanical system (1). For all twice differentiable, bounded, reference trajectories $(q_d(t), \mathbf{p}_d(t)) \in \mathbb{R}^n \times \mathbb{R}^n$, there exists a dynamic position–feedback controller that ensures UGES of the closed–loop system. More precisely, there exist two (smooth) mappings

$$\mathbf{F} : \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{3n+1}$$
$$\mathbf{H} : \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$$

such that, for all initial conditions

$$(q(t_0), \mathbf{p}(t_0), \varpi(t_0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}$$

the system (1) in closed–loop with

$$\dot{\varpi} = \mathbf{F}(\varpi, q, t)$$
$$u = \mathbf{H}(\varpi, q, t)$$

verifies

$$\left\| \begin{bmatrix} q(t) - q_d(t) \\ \mathbf{p}(t) - \mathbf{p}_d(t) \\ \varpi(t) \end{bmatrix} \right\| \leq \kappa \exp^{-\alpha(t-t_0)} \left\| \begin{bmatrix} q(t_0) - q_d(t_0) \\ \mathbf{p}(t_0) - \mathbf{p}_d(t_0) \\ \varpi(t_0) \end{bmatrix} \right\|,$$

for some constants $\alpha, \kappa > 0$ (independent of t_0) and all $t \geq t_0 \geq 0$.

Remark 1. As indicated in the proposition, the dimension of the state of the controller ϖ is $3n+1$. As will be shown below, the last component is always non–negative, hence the definition of the domain of the mappings $\mathbf{F}$ and $\mathbf{H}$, and the constraint on the initial conditions.

3. A NEW FULL–STATE FEEDBACK PBC

Similarly to (Romero et al., 2013), two changes of coordinates are used in the design of the full–state feedback PBC. First, the change of coordinates in momenta proposed in (Venkatraman et al., 2010) is used to assign a constant inertia matrix to the energy function. To compensate for the presence of friction and relax the assumption of upper–bounded inertia the second change of coordinates and the state–feedback PBC of (Romero et al., 2013) are modified.

3.1 A suitable pH representation

As shown in (Venkatraman et al., 2010), the change of coordinates

$$(q,p) \mapsto (q, T(q)\mathbf{p}),$$

with $T : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ the symmetric, positive definite, uniquely defined, square root of the inverse inertia matrix (see Theorem 1 in Section 5.4 of (Lancaster et al., 1985)), that is

$$M^{-1}(q) = T^2(q),$$

transforms (1) into

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & T(q) \\ -T(q) & S(q,p) - D(q) \end{bmatrix} \nabla W + \begin{bmatrix} 0 \\ I_n \end{bmatrix} v, \qquad (2)$$

with

$D(q) := T(q)\mathfrak{D}(q)T(q)$ and $v := T(q)u$ the new control signal, new Hamiltonian function $W : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$W(q,p) = \frac{1}{2}|p|^2 + V(q),$$

and the gyroscopic forces matrix $S : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$

$$\begin{aligned} S(q,p) &:= \nabla(T\mathbf{p})T - T\nabla^\top(T\mathbf{p})|_{\mathbf{p}=T^{-1}p}, \\ &= \sum_{i=1}^n \Big[\big[\nabla_{q_i}(T)T^{-1}p\big](Te_i)^\top - \\ &\qquad -(Te_i)\big[\nabla_{q_i}(T)T^{-1}p\big]^\top \Big], \qquad (3) \end{aligned}$$

with $e_i \in \mathbb{R}^n$ the i–th Euclidean basis vector of $\mathbb{R}^n$. Clearly,

$$S(q,p) = -S^\top(q,p).$$

See (Astolfi et al., 2010; Venkatraman et al., 2010) for its relationship with the Coriolis and centrifugal forces matrix of the Euler–Lagrange model.

3.2 The new PBC and its pH error system

Proposition 2. Consider the pH system (2). Define the mapping $v^\star : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$

$$\begin{aligned} v^\star(q,p,t) = {} & Dp - \frac{d}{dt}(T^{-1})R_1(q - q_d(t)) + \dot{p}_d - \\ & - R_2(p - p_d(t)) - T\Big[(q - q_d(t)) - \nabla V\Big] + \\ & + \Big[S - R_2\Big]T^{-1}R_1(q - q_d(t)) - \\ & - T^{-1}R_1T(p - p_d(t)) - Sp_d(t) \qquad (4) \end{aligned}$$

where

$$p_d := T^{-1}(q)\dot{q}_d, \qquad (5)$$

and $R_1, R_2 \in \mathbb{R}^{n\times n}$ are free symmetric positive definite gain matrices.

(i) The closed–loop dynamics obtained setting

$$v = v^\star(q,p,t)$$

expressed in the coordinates

$$
\begin{aligned}
w_1 &= \tilde{q} \\
w_2 &= T^{-1} R_1 \tilde{q} + \tilde{p},
\end{aligned} \tag{6}
$$

where

$$\tilde{q} := q - q_d,\ \tilde{p} := p - p_d,$$

takes the pH form

$$\dot{w} = \begin{bmatrix} -R_1 & T(q) \\ -T(q) & S(q,p) - R_2 \end{bmatrix} \nabla H_w \tag{7}$$

with Hamiltonian function $H_w : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$

$$H_w(w) = \frac{1}{2}|w_2|^2 + \frac{1}{2}|w_1|^2 \tag{8}$$

(ii) The zero equilibrium point of (7) is UGES with Lyapunov function $H_w(w)$. Consequently, $(\tilde{q}(t), \tilde{p}(t)) \to 0$ exponentially fast.

Proof. Taking the time derivative of the change of coordinates given in (6) and using the control law (4) yields the closed–loop (7), establishing the claim (i). Now, taking the time derivative of (8), along the system's trajectories, it follows

$$\dot{H}_w = -\|w_1\|^2_{R_1} - \|w_2\|^2_{R_2} \leq -\delta H_w, \tag{9}$$

where

$$\delta := 2 \min\{\lambda_{\min}(R_1),\ \lambda_{\min}(R_2)\} > 0. \tag{10}$$

The main modifications to the PBC of (Romero et al., 2013) introduced here are the first and second right hand terms of (4). While the interest of the first term—that achieves friction compensation—is clear, the use of $\frac{d}{dt}(T^{-1})$ in the second one is far from obvious. Its motivation is to impose to the $(1,1)$ block of the damping matrix of the closed–loop pH system (7) the positive *definite* matrix R_1. To achieve this end it is necessary also to include in the coordinate change (6) the matrix T^{-1}. Without these modifications, we get in the $(1,1)$ block the matrix T, which is only positive *semidefinite* if the inertia matrix is unbounded. See equations (10) and (13) of (Romero et al., 2013).

4. A NEW EXPONENTIALLY CONVERGENT I&I MOMENTA OBSERVER

In (Romero et al., 2013) the exponentially convergent speed I&I observer reported in (Astolfi et al., 2010) was modified to estimate directly the momenta p. In this section an additional modification is introduced to take into account the presence of friction. Since the proof closely mimics the ones given in (Romero et al., 2013; Astolfi et al., 2010) it is only sketched below.

Proposition 3. Consider the system (2), and assume v is such that trajectories exist for all $t \geq 0$. There exist smooth mappings [1]

$$
\begin{aligned}
\mathbf{A} &: \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{3n+1} \\
\mathbf{B} &: \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}^n
\end{aligned}
$$

such that the interconnection of (2) with

$$
\begin{aligned}
\dot{\mathrm{X}} &= \mathbf{A}(\mathrm{X}, q, v) \\
\hat{p} &= \mathbf{B}(\mathrm{X}, q),
\end{aligned}
$$

[1] Remark 1 applies *mutatis–mutandi* to the domain of the last component of the vector X and the range of the mappings **A** and **B**.

where $\mathrm{X} \in \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}$, $\hat{p} \in \mathbb{R}^n$, ensures

$$\lim_{t \to \infty} e^{\alpha t}[p(t) - \hat{p}(t)] = 0,$$

for some $\alpha > 0$, and for all initial conditions

$$(q(0), p(0), \mathrm{X}(0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}.$$

This implies that (11) is an exponentially convergent momenta observer for the mechanical system with friction (2).

Proof. The basic idea of I&I observers is to find a *measurable* mapping $\beta : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ such that the (so–called) off–the–manifold coordinate

$$z = \xi + \beta(q, \mathfrak{q}, \mathfrak{p}) - p, \tag{11}$$

asymptotically converges to zero, where $\xi, \mathfrak{q}, \mathfrak{p} \in \mathbb{R}^n$ are (part of) the observer state. If this is the case

$$\hat{p} := \xi + \beta(q, \mathfrak{q}, \mathfrak{p}) \tag{12}$$

is a consistent estimate of p. We, therefore, study the dynamic behavior of z and compute

$$
\begin{aligned}
\dot{z} = {} & \dot{\xi} + \nabla_q \beta \dot{q} + \nabla_{\mathfrak{q}} \beta \dot{\mathfrak{q}} + \nabla_{\mathfrak{p}} \beta \dot{\mathfrak{p}} - S(q,p)p + \\
& + T(q)\nabla V + D(q)p - v.
\end{aligned} \tag{13}
$$

In (Astolfi et al., 2010) it has been shown that the mapping S defined in (3) verifies the following properties:

(P.i) S is linear in the second argument, that is

$$S(q, \alpha_1 p + \alpha_2 \bar{p}) = \alpha_1 S(q,p) + \alpha_2 S(q, \bar{p}),$$

for all q, p, $\bar{p} \in \mathbb{R}^n$, and α_1, $\alpha_2 \in \mathbb{R}$.

(P.ii) There exists a mapping $\bar{S} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$ satisfying

$$S(q,p)\bar{p} = \bar{S}(q, \bar{p})p.$$

Hence, proposing

$$
\begin{aligned}
\dot{\xi} := {} & -\nabla_{\mathfrak{q}} \beta \dot{\mathfrak{q}} - \nabla_{\mathfrak{p}} \beta \dot{\mathfrak{p}} + S(q, \xi + \beta)(\xi + \beta) - \\
& - T(q)\nabla V + v - \nabla_q \beta T(q)(\xi + \beta) - D(q)(\xi + \beta),
\end{aligned} \tag{14}
$$

together with Properties (P.i) and (P.ii) yields

$$\dot{z} = [S(q,p) - D(q) + \bar{S}(q, \xi + \beta) - \nabla_q \beta T]z. \tag{15}$$

Notice the inclusion of the new term $-D(q)(\xi + \beta)$, which is absent in the I&I observer of (Romero et al., 2013). If the mapping β solves the partial differential equation (PDE)

$$\nabla_q \beta = [\psi I_n + \bar{S}(q, \xi + \beta)]T^{-1}(q),$$

the z–dynamics reduces to

$$\dot{z} = [S(q,p) - \psi I_n]z - D(q)z.$$

Since $D \geq 0$ and $S = -S^\top$ the system is (exponentially) asymptotically stable if ψ (that may be state–dependent) is positive. To avoid the solution of the PDE, which may not even exist, an alternative approach is proposed. First, define an *ideal* expression for $\nabla_q \beta$ as

$$[\psi I_n + \bar{S}(q, \xi + \beta)]T^{-1}(q) =: \mathcal{H}(q, \xi + \beta). \tag{16}$$

and, following (Liu et al., 2011), define β as

$$\beta(q, \mathfrak{q}, \mathfrak{p}) := \mathcal{H}(\mathfrak{q}, \mathfrak{p})q. \tag{17}$$

The above choice yields $\nabla_q \beta = \mathcal{H}(\mathfrak{q}, \mathfrak{p})$, which may be written as

$$\nabla_q \beta = \mathcal{H}(q, \xi + \beta) - [\mathcal{H}(q, \xi + \beta) - \mathcal{H}(\mathfrak{q}, \mathfrak{p})]. \tag{18}$$

Now, since the term in brackets in (18) is equal to zero if $\mathfrak{p} = \xi + \beta$ and $\mathfrak{q} = q$, we can compute mappings

$$\Delta_q, \Delta_p : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$$

verifying

$$\Delta_q(q, \mathfrak{p}, 0) = 0, \quad \Delta_p(q, \mathfrak{p}, 0) = 0, \tag{19}$$

and such that

$$\mathcal{H}(q, \xi + \beta) - \mathcal{H}(\mathfrak{q}, \mathfrak{p}) = \Delta_q(q, \mathfrak{q}, e_q) + \Delta_p(q, \mathfrak{p}, e_p), \tag{20}$$

where

$$e_q := \mathfrak{q} - q, \quad e_p := \mathfrak{p} - (\xi + \beta). \tag{21}$$

Substituting (16), (18) and (20) in (15), yields

$$\dot{z} = [S(q, p) - D - \psi I_n]z + (\Delta_q + \Delta_p)T(q)z.$$

The mappings Δ_q, Δ_p play the role of disturbances that are dominated with a dynamic scaling and a proper choice of the observer dynamics. For, define the dynamically scaled off–the–manifold coordinate

$$\eta = \frac{1}{r}z, \tag{22}$$

where r is a scaling factor to be defined. The dynamic behavior of η is given by

$$\dot{\eta} = (S - D - \psi I)\eta + (\Delta_q + \Delta_p)T(q)\eta - \frac{\dot{r}}{r}\eta. \tag{23}$$

Mimicking (Astolfi et al., 2010) select the dynamics of $\mathfrak{q}$, $\mathfrak{p}$ as

$$\begin{aligned}\dot{\mathfrak{q}} &= T(q)(\xi + \beta) - \psi_1 e_q &(24)\\ \dot{\mathfrak{p}} &= -T(q)\nabla V + v + S(q, \xi + \beta)(\xi + \beta)\\ &\quad -D(q)(\xi + \beta) - \psi_2 e_p &(25)\end{aligned}$$

where ψ_1, ψ_2 are some positive functions defined later. Using (24), together with (21), we get

$$\begin{aligned}\dot{e}_q &= T(q)\eta r - \psi_1 e_q\\ \dot{e}_p &= (\nabla_q \beta)T(q)\eta r - \psi_2 e_p. &(26)\end{aligned}$$

Moreover, select the dynamics of r as

$$\dot{r} = -\frac{\psi}{4}(r - 1) + \frac{r}{\psi}(\|\Delta_p T\|^2 + \|\Delta_q T\|^2), \; r(0) \geq 1, \tag{27}$$

with $\|\cdot\|$ the matrix induced 2–norm. Notice that the set $\{r \in \mathbb{R} : r \geq 1\}$ is invariant for the dynamics (27).

We show now that the (non–autonomous) *error* system (22), (26), (27)—with the coordinate $\tilde{r} = (r - 1)$ — has a UGES equilibrium at zero. For, define the proper Lyapunov function candidate.

$$V(\eta, e_q, e_p, \tilde{r}) := \frac{1}{2}[|\eta|^2 + |e_q|^2 + |e_p|^2 + \tilde{r}^2]. \tag{28}$$

Following the calculations done in (Astolfi et al., 2010) we obtain

$$\begin{aligned}\dot{V} \leq &-\left(\frac{\psi}{4} + \|D\| - 1\right)|\eta|^2 - \left(\psi_1 - \frac{1}{2}r^2\|T\|^2\right)|e_q|^2 -\\ &-\left(\psi_2 - \frac{1}{2}r^2\|\nabla_q\beta\|^2\|T\|^2\right)|e_p|^2 + \tilde{r}\dot{\tilde{r}}. &(29)\end{aligned}$$

Clearly, if we set

$$\psi = 4(1 + \psi_3), \; \psi_1 = \frac{1}{2}r^2\|T\|^2 + \psi_4 \tag{30}$$

and

$$\psi_2 = \frac{1}{2}r^2\|\nabla_q\beta\|^2\|T\|^2 + \psi_5,$$

where ψ_3, ψ_4, ψ_5 are positive functions of the state defined below, one gets

$$\dot{V} \leq -(\|D\| + \psi_3)|\eta|^2 - \psi_4|e_q|^2 - \psi_5|e_p|^2 + \tilde{r}\dot{\tilde{r}}.$$

Let us look now at the last right hand term above

$$\tilde{r}\dot{\tilde{r}} = -\frac{\psi}{4}\tilde{r}^2 + \tilde{r}\frac{r}{\psi}(\|\Delta_p T\|^2 + \|\Delta_q T\|^2).$$

Now, (19) ensures the existence of mappings $\bar{\Delta}_q$, $\bar{\Delta}_p : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that

$$\begin{aligned}\|\Delta_q(q, \mathfrak{p}, e_q)\| &\leq \|\bar{\Delta}_q(q, \mathfrak{p}, e_q)\|\,|e_q|\\ \|\Delta_p(q, \mathfrak{p}, e_p)\| &\leq \|\bar{\Delta}_p(q, \mathfrak{p}, e_p)\|\,|e_p|.\end{aligned}$$

Hence

$$\|\Delta_p T\|^2 + \|\Delta_q T\|^2 \leq \|T\|^2|(\|\bar{\Delta}_p\|^2|e_p|^2 + |\bar{\Delta}_q\|^2|e_q|^2).$$

Finally, setting

$$\begin{aligned}\psi_3 &= \kappa\\ \psi_4 &= \frac{r\tilde{r}}{4(1 + \psi_3)}\|T\|^2\|\bar{\Delta}_q\|^2 + \kappa\\ \psi_5 &= \frac{r\tilde{r}}{4(1 + \psi_3)}\|T\|^2\|\bar{\Delta}_p\|^2 + \kappa,\end{aligned}$$

for some positive constant κ, yields

$$\dot{V} \leq -\Big[\|D\| + \kappa\Big]|\eta|^2 - \kappa\Big[|e_q|^2 + |e_p|^2 + \tilde{r}^2\Big] \leq -\alpha V. \tag{31}$$

This completes the proof of UGES of the equilibrium of the error system.

From (11), (12) and (22), boundedness of r and the exponential convergence of η we get that z and the estimation error $\hat{p} - p$ also converge to zero exponentially fast.

The proof is completed selecting the observer state as

$$\mathrm{X} := (\xi, \mathfrak{q}, \mathfrak{p}, \tilde{r}),$$

defining $\mathbf{A}(\mathrm{X}, q, v)$ from (14), (24) and (27) and $\mathbf{B}(\mathrm{X}, q)$ via (12).

5. PROOF OF PROPOSITION 1

The certainty equivalent version of the full–state feedback controller (4) of Proposition 1 is obtained replacing p by its estimate $\hat{p}$ generated with the observer of Section 4. Notice that (4) contains the terms $\dot{p}_d$ and $\frac{d}{dt}(T^{-1})$ that, as seen from (5), depends on the unknown $\dot{q}$. To define the certainty equivalent version of (4) we must compute

$$\begin{aligned}\dot{p}_d &= \Big[\nabla_q(T^{-1}\dot{q}_d)\Big]\dot{q} + T^{-1}\ddot{q}_d\\ &= \Big[\nabla_q(T^{-1}\dot{q}_d)\Big]Tp + T^{-1}\ddot{q}_d &(32)\end{aligned}$$

and

$$\frac{d}{dt}(T^{-1}) = \sum_{i=1}^{n}\Big[\nabla_{q_i}(T^{-1})\Big]e_i^\top T^{-1}p \tag{33}$$

Using (32) and (33) we get the implementable controller

$$v = -T(q)\Big[(q - q_d(t)) - \nabla V(q)\Big] - S(q,\hat{p})p_d(t) -$$
$$- \sum_{i=1}^{n} \Big[\nabla_{q_i}(T^{-1})\Big]\Big[R_1(q - q_d(t))\Big] e_i^\top T^{-1}\hat{p} + D(q)\hat{p} +$$
$$+ \Big[\nabla_q(T^{-1}\dot{q}_d)\Big]T\hat{p} + T^{-1}\ddot{q}_d - R_2(\hat{p} - p_d(t)) +$$
$$+ \Big[S(q,\hat{p}) - R_2\Big]T^{-1}(q)R_1(q - q_d(t)) -$$
$$- T^{-1}R_1T(\hat{p} - p_d(t)) \qquad (34)$$

We invoke now the key property (P.i) of Section 4, namely that $S(q,\hat{p})$ is *linear* in $\hat{p}$. Consequently, since all other $\hat{p}$–dependent terms in (34) are linear, there exists mappings

$$\Psi : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n, \quad \Theta : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times n},$$

such that (34) can be written as

$$v = \Psi(q,t) + \Theta(q,t)\hat{p}.$$

Moreover, using (11) and (12) it can be expressed as

$$v = v^\star(q,p,t) + \Theta(q,t)z.$$

Replacing the latter in (2), and using (22), yields the perturbed pH system

$$\dot{w} = \begin{bmatrix} -R_1 & T(q) \\ -T(q) & S(q,p) - R_2 \end{bmatrix} \nabla H_w + \begin{bmatrix} 0 \\ \Theta(q,t) \end{bmatrix} r\eta, \qquad (35)$$

with the Hamiltonian function given by (8). The overall non–autonomous system (*e.g.*, closed–loop plant (35) plus observer (11)) is $5n + 1$–dimensional and has a state $(w_1, w_2, e_q, e_p, \eta, \tilde{r})$.

To establish the UGES claim consider the proper Lyapunov function

$$\mathcal{V}(w,\eta,e_q,e_p,\tilde{r}) = H_w(w) + V(\eta,e_q,e_p,\tilde{r}),$$

where the functions H_w and V are defined in (8) and (28), respectively. From the derivations of the previous two sections it is clear that the only troublesome term is the sign–indefinite cross product $w_2^\top \Theta(q,t)r\eta$, that appears in $\dot{H}_w$.

To dominate this term, consider the bound

$$w_2^\top \Theta(q,t)r\eta \leq \frac{1}{2}|w_2|^2 + \frac{r^2}{2}\|\Theta(q,t)\|^2|\eta|^2. \qquad (36)$$

From (9), (29) and (30) we see that there is the damping gain R_2 and the free gain function ψ_3, that can be used to dominate the cross–term. [2] More precisely, setting

$$R_2 = (\frac{1}{2} + \kappa)I_n$$

and $\psi_3 : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$

$$\psi_3(q,r,t) = \frac{r^2}{2}\|\Theta(q,t)\|^2 + \kappa,$$

yields $\dot{\mathcal{V}} \leq -\alpha\mathcal{V}$, establishing the UGES claim.

6. CONCLUSIONS AND FUTURE RESEARCH

In this paper we have extended the result of global exponential tracking of mechanical systems without velocity measurements of (Romero et al., 2013) to the case when the system has friction and the inertia matrix is not bounded from above.

The result assumes the existence—and knowledge—of friction, whose deleterious effect is compensated by the controller. This should be contrasted with (Zhang et al., 2000; Zergeroglu et al., 2000) where the presence of friction is necessary to construct the controller. Moreover, in (Zhang et al., 2000; Zergeroglu et al., 2000) the dissipation is assumed to be pervasive—i.e. with positive definite $\mathfrak{D}$—that is rarely the case in practical scenarios.

Current research is under way along several axes.

- Friction coefficients are usually highly uncertain, hence the interest on an adaptive version of the scheme that tries to estimate the matrix $\mathfrak{D}$. This is a challenging problem that involves the product on unknown parameters with unmeasurable states for which very few results are available in the literature.
- Another, simpler, open question is the analysis of the robustness of the design when the damping matrix is *not exactly known*. Preliminary calculations show that it is possible to prove convergence to a residual set, but no tuning parameters are available to reduce its size.
- The observer proposed in (Astolfi et al., 2010) is applicable for systems with non–holonomic constraints. How to formulate the position–feedback tracking problem in that case is still to be resolved.
- Simulation results of the proposed controller have shown the excellent behavior of the proposed scheme and will be reported elsewhere. Also, some preliminary experimental results of the observer are under way.

[2] For simplicity, in Proposition 3 ψ_3 is taken as constant.

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