Data Driven Tuning of Inventory Controllers

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***Abstract*— A systematic method for criterion based tuning of inventory controllers based on data-driven Iterative Feedback Tuning is presented. This tuning method circumvent problems with modeling bias. The process model used for the design of the inventory control is utilized in the tuning as an approximation to reduce time required on experiments. The method is illustrated in an application with a multivariable inventory control implementation on a four tank system.**

I. INTRODUCTION

The purpose of this paper is to present a systematic method for tuning inventory controllers with Iterative Feedback Tuning. This data-driven tuning approach optimizes the actual closed loop performance hence circumventing problems due to modelling bias, that is part of any model for a real system, which would affect the control design. Furthermore, the process model will be utilized in the tuning algorithm in order to decrease time for plant experiments.

Inventory process control is based on passivity theory which states that the dynamical behavior of a system can be classified in terms of the conservation, dissipation and transport of positive extensive thermodynamic properties of the system. In passive systems, the stored amount of this property in any given time interval, is always lower or at most equal to the amount delivered to the system during the same time [17]. The theory is closely connected to optimization of just in time production of supply chains. In the work by Ydstie and coworkers, passivity theory was first applied on process systems and a formal connection was established between the macroscopic thermodynamics of process systems and passivity theory of nonlinear control [19]. In continuation [4] utilized the structure of first principle models in formulation of a nonlinear control law which has the form of output feedback linearization for which [2] has proven closed loop stability through fulfillment of the passivity inequality for minimum phase systems and certain classes of nonlinear minimum phase systems. Inventory control has proven a useful methodology to synthesize a complex control law with a simple transfer function in the feedback and have been tested for a number of applications [3], [4].

The problem of tuning the parameters in the feedback loop in the inventory control law is an area which has not received much attention. [3] states that classic tuning rules for linear systems can be applied in case where a perfect model of the system is available and all inventories are used for control, in which case perfect feedback linearization is achieved. Tuning of systems where a biased process model has been used in the design of the inventory controller will be addressed in this paper. The approach which will be presented uses the process model in the control design but a data driven method for tuning performance of the closed loop in order to compensate for modelling errors. Iterative Feedback Tuning, presented in [5] for linear SISO systems, is an applicable methodology which have since been matured and developed [8] and tested in a number of papers [7], [12], [13].

This paper is organized with a short introduction on the formulation of the control law for an inventory controller for a SISO system in the following section. A SISO formulation is used to ease notation but the remaining part of paper will focus on MIMO formulation due to the nature of the case study. Section III contains a formulation and problem statement for criterion based controller tuning which is followed by section IV explaining Iterative Feedback Tuning. A case study on tuning a multivariable inventory controller implemented on a pilot scale of the quadruple tank process as given in section V before the concluding remarks.

II. INVENTORY CONTROL

An inventory, υ, is represented by a physical extensive property and its general balance is given by

$$\begin{pmatrix} Accumul. \\ of \quad \upsilon \end{pmatrix} = \underbrace{\begin{pmatrix} Input \quad flow \\ of \quad \upsilon \end{pmatrix} - \begin{pmatrix} Output \quad flow \\ of \quad \upsilon \end{pmatrix}}_{\phi(d,x,u)} + \underbrace{\begin{pmatrix} Generation \\ of \quad \upsilon \end{pmatrix} - \begin{pmatrix} Consump. \\ of \quad \upsilon \end{pmatrix}}_{p(x)} \tag{1}$$

where a distinction is made between $\phi(d,x,u)$ and $p(x)$ which represent transport to the system and production in the system respectively. x,u and d is the state, the input and the disturbances for the associated general nonlinear dynamical system

$$\dot{x} = f(x) + g(d,x,u) \qquad x(0) = x_0 \tag{2a}$$
$$y = h(x) \tag{2b}$$

The function $f(\cdot)$ describes the internal state evolution due to generation or consumption, the function $g(\cdot)$ describes the external contribution to the state evolution and $h(\cdot)$ maps the state to the output. Let υ be an arbitrary inventory associated with the dynamic system (2), then the dynamic behavior is

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given by

$$\frac{d\upsilon(x)}{dt} = \underbrace{\frac{d\upsilon(x)}{dx} f(x)}_{L_f\upsilon} + \underbrace{\frac{d\upsilon(x)}{dx} g(d,x,u)}_{L_g\upsilon} \tag{3}$$

Where the terms $L_f\upsilon$ and $L_g\upsilon$ are directional derivatives. Equation (1) shows how ϕ and p are represented in the conservation law balance. Consequently, inventory systems can be written in the same form as the dynamic system (2).

$$\frac{d\upsilon}{dt} = p(x) + \phi(d,x,u) \tag{4a}$$

$$\upsilon = w(x) \tag{4b}$$

which is the notation used for inventory control [4]. The term p denotes the production rate of the inventory and let p^* represent a stationary value of the production rate. The term ϕ is the supply rate for the system. The connection between these and the directional derivatives are given as

$$p(x) = L_f\upsilon(x) + p^*, \qquad p^* = p(0) \tag{5}$$

$$\phi(d,x,u) = L_g\upsilon(x) - p^* \tag{6}$$

The inventory controller with proportional action on the feedback $e(t) = (\upsilon(t) - \upsilon^{set}(t))$ or with on-off control is given by control laws (7) and (8) respectively.

$$\phi(d,x,u) + p(x) = -K_c e(t) \tag{7}$$

$$\phi(d,x,u) + p(x) = \begin{cases} \delta & if \quad e(t) < -\varepsilon \\ 0 & if \quad -\varepsilon \le e(t) \le \varepsilon \\ -\delta & if \quad e(t) > \varepsilon \end{cases} \tag{8}$$

In case a perfect model has been used for the inventory, a proportional controller will be sufficient and efficient in rejecting disturbances and tracking set points given a proper value of the proportional gain. In case the model is biased, which is the case for all real model based control implementations, it may be necessary to include integral action in the feedback control. This formulation is given in (9). Likewise, derivative action could be a part of the feedback, but given a reasonable process model the feed forward part of the inventory control renders such action unnecessary.

$$\phi(d,x,u) + p(x) = -K_c \left(e(t) + \frac{1}{\tau_I}\int_0^t e(\tau)d\tau \right) \tag{9}$$

It is seen that the inventory formulation can yield a complex and nonlinear controller depending on the model for the inventory. The problem of tuning the inventory controller is then to select proper parameters for the feedback part of the controller, which will provide sufficient closed loop performance.

III. CRITERION BASED CONTROLLER TUNING

Given a description of a closed loop system where the controller, $\mathbf{C}(\rho)$ is acting on the multivariable discrete linear time invariant system $\mathbf{G}$, the transfer functions are given as:

$$\begin{aligned}\mathbf{y}(\rho) &= (\mathbf{1}+\mathbf{C}(\rho)\mathbf{G})^{-1}\mathbf{C}(\rho)\mathbf{G}\mathbf{r} + (\mathbf{1}+\mathbf{C}(\rho)\mathbf{G})^{-1}\mathbf{v} \\ &= \mathbf{T}(\rho)\mathbf{r} + \mathbf{S}(\rho)\mathbf{v} \end{aligned} \tag{10a}$$

$$\begin{aligned}\mathbf{u}(\rho) &= (\mathbf{1}+\mathbf{C}(\rho)\mathbf{G})^{-1}\mathbf{C}(\rho)\mathbf{r} - (\mathbf{1}+\mathbf{C}(\rho)\mathbf{G})^{-1}\mathbf{C}(\rho)\mathbf{v} \\ &= \mathbf{S}(\rho)\mathbf{C}(\rho)\mathbf{r} - \mathbf{S}(\rho)\mathbf{C}(\rho)\mathbf{v} \end{aligned} \tag{10b}$$

where $\mathbf{r}$ is the reference value for the measurements $\mathbf{y}(\rho)$, $\mathbf{u}(\rho)$ is the actuator variable and $\mathbf{v}$ is a noise signal for the system which is presented in deviation variables. $\mathbf{S}(\rho)$ and $\mathbf{T}(\rho)$ are the sensitivity and the complementary sensitivity functions respectively. Given a desired reference model for the closed loop $\mathbf{T}^d$, the desired response from the loop is given as $\mathbf{y}^d = \mathbf{T}^d\mathbf{r}$. The performance criterion can then be formulated as a typical quadratic cost function

$$F(\rho) = \frac{1}{2N}\mathrm{E}\left[\sum_{t=1}^{N}(\mathbf{y}_t(\rho) - \mathbf{y}_t^d)^2\right] \tag{11}$$

where $\mathrm{E}[\cdot]$ denotes the expectation with respect to a weakly stationary disturbance, since the measurement $\mathbf{y}(\rho)$ is affected by the process and measurement noise. The formulation in (11) gives minimal variance control. Penalty on the control position or its increments can also be part of such a performance criterion as well. The optimal controller will be the set of controller parameters, ρ, that minimizes the cost function.

$$\rho^{opt} = \min_{\rho} F(\rho) \tag{12}$$

Given a convex cost function, this minimization is equivalent to solving

$$0 = \mathbf{J}(\rho) = \frac{\partial F}{\partial \rho} = \frac{1}{N}\mathrm{E}\left[\sum_{t=1}^{N}(\mathbf{y}_t(\rho) - \mathbf{y}_t^d)^T \frac{\partial \mathbf{y}_t}{\partial \rho}\right] \tag{13}$$

This equation can be solved iteratively by the following scheme

$$\rho_{i+1} = \rho_i - \gamma_i \mathbf{R}_i^{-1}\mathbf{J}(\rho_i) \tag{14}$$

where $\mathbf{R}$ is some positive definite matrix. In case $\mathbf{R} = \mathbf{I}$ the algorithm steps in the steepest decent direction. In case $\mathbf{R} = \mathbf{H}(\rho) = \partial^2 F/\partial\rho^2$ or an approximation to the Hessian, the Newton or Gauss-Newton algorithm appears. γ_i determines the step length and the choice of $\mathbf{R}$ and γ will thus affect the convergence properties of the method [5], [15].

The problem involved with the optimization of performance through this scheme is that the actual process model often is unknown. That implies that the sensitivity functions, $\mathbf{T}$ and $\mathbf{S}$, are unknown and it is therefore not possible to calculate $\partial \mathbf{y}/\partial\rho$ and thus $\mathbf{J}(\rho)$. Iterative Feedback Tuning solves this problem, and offers a purely data driven algorithm. With respect to tuning of inventory controllers with imperfect process models, the true sensitivity function is unknown. This constitutes a problem since it is the performance of the actual loop that is subject to optimization, and hence motivates application of the Iterative Feedback Tuning.

IV. ITERATIVE FEEDBACK TUNING

The key contribution in Iterative Feedback Tuning is that it supplies an unbiased estimate of the cost function gradient without estimating a plant model, $\hat{\mathbf{G}}$, given that the noise $\mathbf{v}$ is a zero mean, weakly stationary random signal [7]. Using an estimate of the Jacobian in (14) instead of the analytical Jacobian, as a stochastic approximation method, will still make the algorithm converge to a local minimizer, provided that the estimate is unbiased, the Jacobian, $\mathbf{J}(\rho)$, is a monotonically increasing function and the sequence of γ_i fulfills condition (15) [16].

$$\sum_{i=1}^{\infty} \gamma_i^2 < \infty, \quad \sum_{i=1}^{\infty} \gamma_i = \infty \tag{15}$$

This condition is fulfilled by having $\gamma_i = a/i$ where a is some constant. This method however has a convergence rate which is too slow for most industrial purposes [14]. In cases where the variance of the Jacobian approaches zero due to a large number of data points classic Gauss-Newton optimization with $\gamma_i = 1$, may be used instead to speed up convergence.

By differentiation of equation (10) it can be shown that

$$\frac{\partial \mathbf{y}}{\partial \rho} = \mathbf{S}(\rho)\mathbf{G}\frac{\partial \mathbf{C}}{\partial \rho}(\mathbf{r} - \mathbf{y}(\rho)) \tag{16}$$

The data needed for estimation of the gradient $\mathbf{J}(\rho)$ can therefore be generated from two types of closed loop experiments on the system. First the system is run in nominal mode which reflects the normal operation for which good performance is desired, and the sequence $\mathbf{y}_1$ is collected. Secondly a set of special experiments are performed in order to get information of $\partial\mathbf{y}/\partial\rho$. Here the reference is set to zero and the signal $\mathbf{e} = \mathbf{r} - \mathbf{y}_1$ filtered through $\partial\mathbf{C}/\partial\rho_i$ is added to the control signal in order to get an estimate of $\partial\mathbf{y}/\partial\rho_i$ cf. (16). This type of experiment has to be performed as many times as the number of parameters in ρ in the controller [6]. For SISO systems the number of necessary experiments are reduced to one, since scalar linear operators commute.

$$\frac{\partial y}{\partial \rho} = S(\rho)G\frac{\partial C}{\partial \rho}(r - y) = C(\rho)^{-1}\frac{\partial C}{\partial \rho}S(\rho)GC(\rho)(r - y(\rho)) \tag{17}$$

that implies that the gradient estimate can be formed by filtering y_2 through $C(\rho)^{-1}\frac{\partial C}{\partial \rho}$ when the reference signal in the gradient experiments is $r_2 = r_1 - y_1$ hence only one gradient experiment is since the filtering is not performed prior to the experiment. [10] suggests this strategy for MIMO system as an approximation and provides sufficient conditions for local convergence in the vicinity of the optimum. In case this approximation causes the algorithm not to step in a descent direction, due to the error caused by non commuting matrices, the full method have to be applied.

The requirements on the controller are that the controller transfer function itself and $\frac{\partial C}{\partial \rho}$ or $C(\rho)^{-1}\frac{\partial C}{\partial \rho}$ are proper stable filters. This is the case for tuning proportional and integral action in the feedback for inventory control. Tuning of the feed forward part can be performed too if this requirement is fulfilled.

In order to reduce the time spent on experiments in each iteration in Iterative Feedback Tuning of inventory controllers, a process model can be utilized. The first experiment which reflects the normal operation, for which good closed loop performance is desired, has to be performed on the actual system. The gradient experiments where data from the first experiments are used can then be performed by simulation. This will produce a biased but noise free estimate of the gradient of the output and hence $\mathbf{J}(\rho)$. Even though this approximation is biased, convergence may be faster since the gradient will be deterministic while the gradient estimate from classic Iterative Feedback Tuning may be affected by a poor signal to noise ratio and hence poor convergence properties [9].

V. CASE STUDY - FOUR TANK SYSTEM

The quadruple tank process in Fig. 1 has received attention because it shows interesting multivariable characteristics which permit illustration and analysis of different control concepts. In spite of its simple model (18) derived by mass balances and the Bernoulli's flow equation, it exhibits both minimum phase and non-minimum phase behavior [11]. Water can be directed in different ways to the tanks dependent on the position of the three-way valves ϑ_i and the flow rate from the reservoir can be manipulated through a centrifugal pump. A pilot plant scale of this process is available at CAPEC, Dept. of Chem. Eng. for testing control structures for which the physical parameters are presented in table I.

$$\frac{dV_1}{dt} = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \vartheta_1 F_1 \tag{18a}$$
$$\frac{dV_2}{dt} = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \vartheta_2 F_2 \tag{18b}$$
$$\frac{dV_3}{dt} = -a_3\sqrt{2gh_3} + (1 - \vartheta_2)F_2 \tag{18c}$$
$$\frac{dV_4}{dt} = -a_4\sqrt{2gh_4} + (1 - \vartheta_1)F_1 \tag{18d}$$

The model (18) is formulated in terms of the inventory being

TABLE I

PHYSICAL PARAMETERS FOR THE FOUR TANK PILOT PLANT

Symbol	Value	Units	Parameter
a_i	1.23	cm^2	Area of the outlet pipes
A_i	380	cm^2	Transversal area for each tank
g	981	cm/s^2	The acceleration of gravity

the liquid volume in each tank. The actual measurement from the process is the liquid level. This process therefore has a very simple transformation between the underlying dynamical system and the model in terms of inventories.

$$V_i = A_i h_i, \qquad i \in \{1,2,3,4\} \tag{19}$$

In [1] a centralized multivariable inventory control law for this system has been derived based on the model (18). The static model has been validated on steady state plant data and linear correlations, with a squared Pearson correlation coefficient of 0.999, have been fitted for h_i vs. F_j^2 in order to

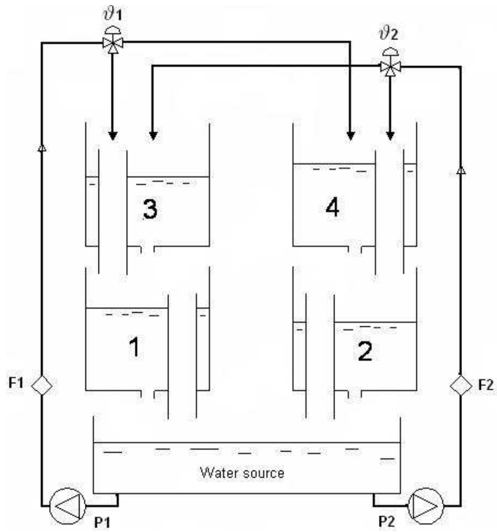

Fig. 1. Schematic diagram of the quadruple tank process.

increase the model accuracy for the flow expressions, which in steady state are

$$h_i = \frac{1}{2ga_i^2}F_j^2 \tag{20}$$

The valve characteristics were investigated around the desired operation point but the model does not include the nonlinear behavior of the three way valves. These investigations show that despite some effort in the modeling of a relative simple system, the feed forward action from the inventory controller is not sufficient and both proportional and integral action will be required in the feedback in order to eliminate offset from e.g. a step response.

The objective is to control the inventories for the two lower tanks i.e. no. 1 and 2 see Fig. 1. The manipulated variables are the two flow rates F_1 and F_2 and the three way valves are considered as disturbance variables and will remain in a constant position through out the test. From the tank model the controlled inventories are given as:

$$\phi_1(\vartheta,\mathbf{h},\mathbf{F}) = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \vartheta_1 F_1 \tag{21a}$$

$$\phi_2(\vartheta,\mathbf{h},\mathbf{F}) = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \vartheta_2 F_2 \tag{21b}$$

since the production term is zero for this process. Utilizing the static formulation of (18)

$$\phi_1(\vartheta,\mathbf{h},\mathbf{F}) = -a_1\sqrt{2gh_1} + (1-\vartheta_2)F_2 + \vartheta_1 F_1 \tag{22a}$$

$$\phi_2(\vartheta,\mathbf{h},\mathbf{F}) = -a_2\sqrt{2gh_2} + (1-\vartheta_1)F_1 + \vartheta_2 F_2 \tag{22b}$$

Choosing both proportional and integral action on $e_i = V_i(t) - V_i^{set}$ in the feedback loop, and isolating the manipulated variable gives the following multivariable control law.

$$\begin{bmatrix} \vartheta_1 & (1-\vartheta_2) \\ (1-\vartheta_2) & \vartheta_2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} -K_1\left(e_1(t) + \frac{1}{\tau_{I_1}}\int_0^t e_1(\tau)d\tau\right) + a_1\sqrt{2g(h_1)} \\ -K_2\left(e_2(t) + \frac{1}{\tau_{I_2}}\int_0^t e_2(\tau)d\tau\right) + a_2\sqrt{2g(h_2)} \end{bmatrix} \tag{23}$$

It is clear that this control law can not be solved for any arbitrary setting of the three way valves, since $\vartheta_1 + \vartheta_2 = 1$ renders this matrix singular and hence not invertible. [11] shows that $\vartheta_1 + \vartheta_2 < 1$ gives non-minimum phase behavior while the system is in minimum phase for $\vartheta_1 + \vartheta_2 > 1$.

Implementation of the inventory controller on the pilot plant can not be done directly since the flow rates are not free to be manipulated directly. A set of lower level SISO PI-controllers are implemented to adjust the speed of rotation for the centrifugal pumps in order to achieve the desired flow rates calculated from the inventory controller, which will act at a supervisory control layer for the underlying regulatory SISO loops. The control structure implemented on the tank system is depicted in Fig. 2. The tuning of the regulatory PI-control loops is performed based on IMC tuning rules and a first order model for the pump dynamics based on step response experiments. The parameters are $K_c = 0.8\,s^{-1}$ and $\tau_I = 8\,s$ for the loop controlling F_1. For the second loop controlling F_2 they are $K_c = 0.7\,s^{-1}$ and $\tau_I = 8\,s$. Both the control layers have been executed every 4 seconds. In practice this cascade structure is not effective if the underlying loops are not executed at least ten times faster than the outer loop [18].

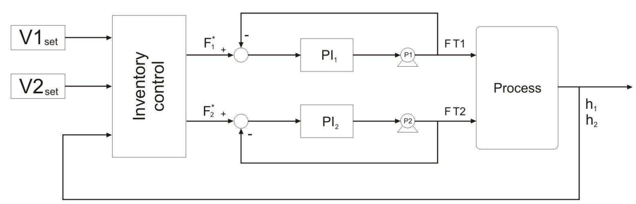

Fig. 2. Diagram for the implemented control structure on the four tank pilot plant.

Tuning

The design objective for the tuning is chosen as a servo problem i.e. tracking a desired trajectory. A step change is introduced simultaneously to the two lower tanks, operating at steady state at the nominal operating point. After one hour the reference is stepped back to the nominal value and the experiment ends after a total of two hours. For a sample time of 4 seconds this gives 1800 data points for each of the output measurements. The nominal operating point, which is in the non-minimum phase region, is defined by

$$\begin{bmatrix} h_1^{set} \\ h_2^{set} \end{bmatrix} = \begin{bmatrix} 24\,cm \\ 21\,cm \end{bmatrix}, \quad \begin{bmatrix} \vartheta_1 \\ \vartheta_2 \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.24 \end{bmatrix},$$

The step changes applied in the reference signal are a decrease $\Delta h_i^{set} = 2\,cm$ i.e. to a level of 22 and 19 cm in the two tanks respectively.

It is desired that the two outputs of the lower tanks perform as the following second order transfer function

$$T_i^d(s) = \frac{K_{T^d}}{\tau_{T^d}^2 s^2 + 2\tau_{T^d}\xi_{T^d}s + 1}, \quad i \in \{1,2\} \tag{24}$$

where $K_{T^d} = 1$, $\tau_{T^d} = 30$ and $\xi_{T^d} = 1.3$ in order to have a over damped system with DC-gain equal to one and rise time of approximately 150 seconds.

Initially the system is implemented with the following controller parameters $K_c = K_1 = K_2 = 0.0139\,s^{-1}$ and $\tau_I = \tau_{I_1} = \tau_{I_2} = 200\,s$. Performing the performance experiments on the pilot plant gave the responses in Fig. 3 for which the value of the cost function was evaluated to $F(\rho_0) = 0.0574$. The initial set of controller parameters results in an over shoot, and a slower response than desired.

Tuning of a controller in operation on a real process requires several repeated experiments and is therefore rather time-consuming. To save time and avoid noise, the gradient experiment are simulated and only the first experiment in each iteration is conducted as a plant experiment. This is reasonable since a very good process model is available. The gradient experiment is further more the SISO formulation of the gradient experiment, which also introduces an error in the gradient experiment. This is necessary since the data filters from the gradient of the controller causes problems, while filtering though $\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial \rho}$ does not. Despite these error sources, the tuning has been successfully performed. The results are presented in table II and the response using the final set of controller parameters are shown in figure 4. From

TABLE II

RESULT OF THE ITERATIVE CONTROLLER TUNING. FOR EACH CONTROLLER THE PARAMETERS ARE PRESENTED TOGETHER WITH THE VALUE OF THE PERFORMANCE COST FUNCTION BASED ON BOTH A NOISE FREE SIMULATION AND AN EXPERIMENT ON THE PILOT PLANT.

Controller	$\mathbf{K}_1 \cdot 10^3$	τ_{I_1}	$\mathbf{K}_2 \cdot 10^3$	τ_{I_2}	$\mathbf{F}_{sim}$	$\mathbf{F}_{exp}$
C_0	13.9	200	13.9	200	0.0279	0.0574
C_1	15.2	171	19.5	505	0.0220	0.0502
C_2	18.6	237	19.8	1275	0.0128	0.0365
C_3	23.1	346	14.4	1401	0.0097	0.0384
C_4	26.5	483	12.3	1363	0.0100	0.0364

the values of the cost function, F, it can be concluded that the method does decrease the specified performance cost based on evaluation of the cost from pilot plant experiments and noise free simulations. The value of the cost function has dropped 37 % in 4 iterations based on the pilot plant experiments and from Fig. 4 it is clear that the tuning has reduced the over shoot substantially. It is seen that the control has become more aggressive which corresponds well with the minimal variance design of the cost function. From the development of the controller parameters it is clear that the dynamics of the two separate lower tanks is different which renders the parameters from these two loops deviate. The coupling between the tanks through the three way valves may also contribute to produce a complicated curvature of the cost function, which is indicated by the non monotonous development of the parameters.

VI. CONCLUSIONS

Criteria based tuning of inventory controllers has to rely on data driven methods due to modeling bias. Iterative Feedback Tuning has been shown to be an amenable method for tuning the feedback in inventory controllers, and the process model from the design of the inventory control can be used to simplify the steps in the iteration by simulating the gradient experiments. This approximation will give bias to the gradient estimate but the estimate will be noise free. Tuning of a multivariable inventory controller implemented on a four tank system show a clear improvement in performance in only 4 iterations.

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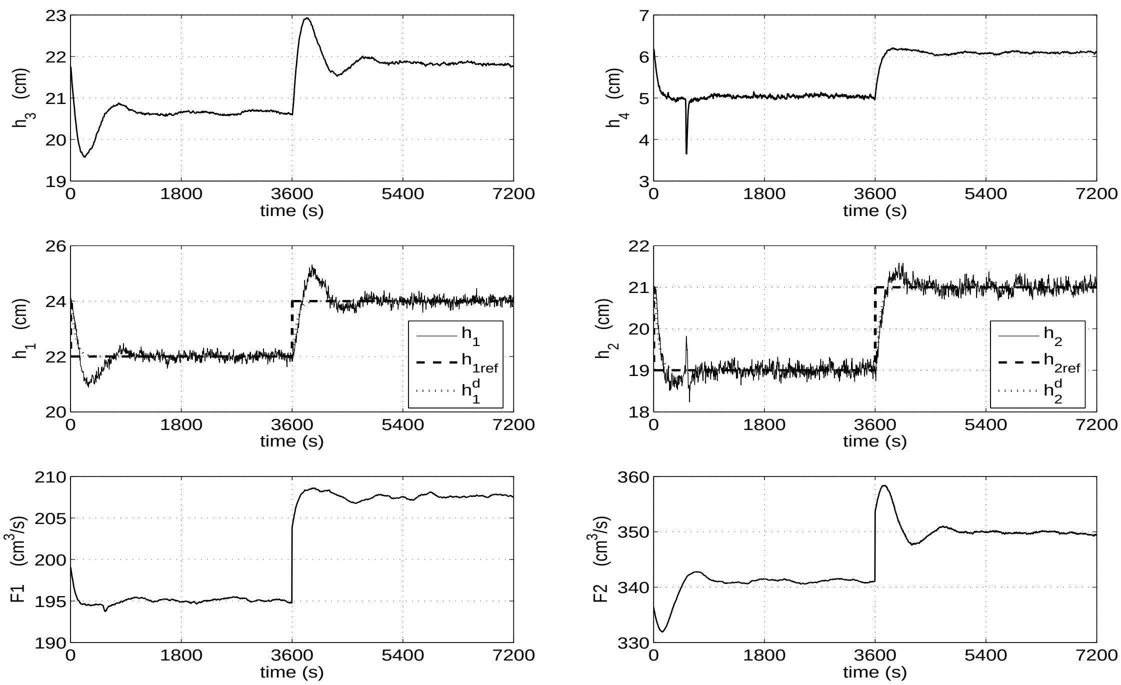

Fig. 3. Dynamic response of the pilot plant to +2 *cm*, simultaneous step changes in the reference to the two lower tanks. The responses are shown for the liquid level in all four tanks together with the desired response on the lower tanks. Furthermore the responses in the manipulated variable from the inventory controller are given. The implementation of the controller is based on the initial set of controller parameters.

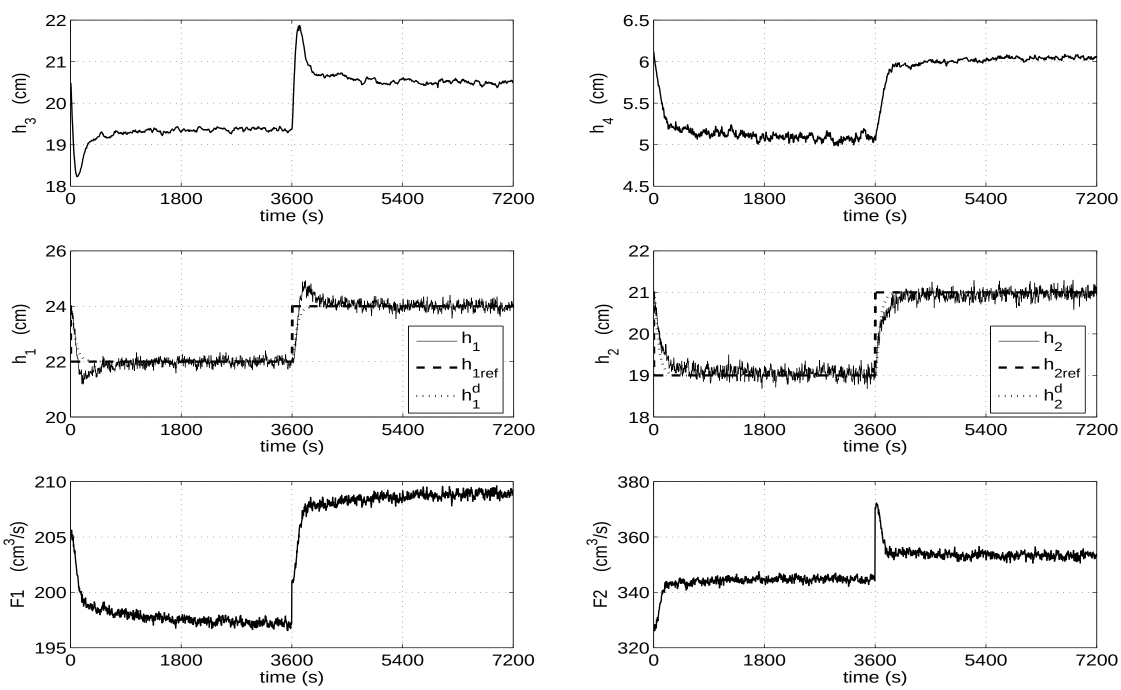

Fig. 4. Dynamic response of the pilot plant to +2 *cm*, simultaneous step changes in the reference to the two lower tanks. The responses are shown for the liquid level in all four tanks together with the desired response on the lower tanks. Furthermore the responses in the manipulated variable from the inventory controller are given. The implementation of the controller is based on the tuned set of controller parameters after 4 iterations.