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Optimal operation of simple refrigeration cycles Part II: Selection of controlled variables

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Abstract

The paper focuses on operation of simple refrigeration cycles and considers the selection of controlled variables for two different cycles. One is a conventional sub-critical ammonia refrigeration cycle and the other is a trans-critical CO_2 refrigeration cycle. There is no fundamental difference between the two cycles in terms of degrees of freedom and operation. However, in practical operation there are differences. For the ammonia cycle, there are several simple control structures that give self-optimizing control, that is, which achieve in practice close-to-optimal operation with a constant setpoint policy. For the CO_2 cycle on the other hand, a combination of measurements is necessary to achieve self-optimizing control.

Keywords: Operation; Self-optimizing control; Vapour compression cycle

1. Introduction

Refrigeration and heat pump cycles are used both in homes, cars and in industry. The load and complexity varies, from small simple cycles, like a refrigerator or air-conditioner, to large complex industrial cycles, like the ones used in liquefaction of natural gas.

The simple refrigeration process illustrated in Fig. 1 is studied in this paper. In Part I (Jensen & Skogestad, 2007b) we showed that the cycle has five steady-state degrees of freedom; the compressor power, the heat transfer in the condenser, the heat transfer in the evaporator, the choke valve opening and the "active charge". Different designs for affecting the active charge, including the location of the liquid storage, were discussed in Part I.

It was found in Part I that there are normally three optimally active constraints; maximum heat transfer in condenser, maximum heat transfer in evaporator and minimum (zero) super-heating. The cycle in Fig. 1 obtains the latter by having a liquid receiver before the compressor which gives saturated vapour entering the compressor. In addition, we assume that the load (e.g., cooling duty) is specified. There is then one remaining unconstrained steady-state degree of freedom, related to the outlet temperature of the condenser, which should be used to optimize the operation. The main theme of the paper is to select a "self-optimizing" controlled variable for this degree of freedom such that a constant setpoint policy (indirectly) achieves near-optimal operation.

We consider two systems:

- a conventional sub-critical ammonia cycle for cold storage ($T_C = -10\,^\circ C$);
- a trans-critical CO_2 cycle for cooling a home ($T_C = 20\,^\circ C$).

The CO_2 cycle is included since it always has an unconstrained degree of freedom that must be used for control. This is because there is no saturation condition on the high pressure side, which is usually said to introduce one extra degree of freedom to the cycle (Kim, Pettersen, & Bullard, 2004). However, as shown in Part I (Jensen & Skogestad, 2007b), this "extra" degree of freedom is also available in a conventional sub-critical cycle if we allow for sub-cooling in the condenser. The sub-cooling will to some extent decouple the outlet temperature and the saturation pressure in the condenser. More importantly, some sub-cooling is actually positive in terms of thermodynamic efficiency (Jensen & Skogestad, 2007b). The ammonia cycle is included to show that there are no fundamen-

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Nomenclature

A	heat transfer area
c	combined controlled variable
c_p	specific heat capacity
C_V	valve coefficient
d	disturbance variable
G	gain matrix
h	specific enthalpy
h_i	element in H
H	linear combination matrix
J	cost function
k	gain constant
L	loss
$\dot{m}$	mass flowrate
m	mass holdup
M_d	disturbance sensitivity
MW	mole weight
n	implementation error
P	pressure
PC	pressure controller
Q	heat transfer rate
R	universal gas constant
T	temperature
TC	temperature controller
u	unconstrained input variable
U	heat transfer coefficient
V	volume
W_s	shaft work
y	controlled variable
z	valve opening
η	isentropic efficiency
$\dot{m}$	mass flowrate

Superscript

s	setpoint

Subscripts

amb	ambient
con	condenser
C	cold source
gco	gas cooler
h	high pressure side
H	hot source
ihx	internal heat exchanger
l	liquid
l	low pressure side
opt	optimal
sat	saturated
sub	sub-cooling
sup	super-heating
vap	evaporator

tal differences between a sub-critical and a trans-critical cycle. There is some confusion in the literature on this.

Although there is a vast literature on the thermodynamic analysis of closed refrigeration cycles, there are few authors who discuss the operation and control of such cycles. Some discussions are found in text books such as Stoecker (1998), Langley (2002) and Dossat (2002), but these mainly deal with more practical aspects. Svensson (1994) and Larsen, Thybo, Stoustrup, and Rasmussen (2003) discuss operational aspects. A more comprehensive recent study is that of Kim et al. (2004) who consider the operation of trans-critical CO_2 cycles.

This paper considers steady-state operation and the objective is to find which controlled variables to fix. The compressor power is used as the objective function (cost $J = W_s$) for evaluating optimal operation.

2. Selection of controlled variable

We consider here the simple cycle in Fig. 1 where the liquid receiver on the low pressure side ensures that the vapour entering the compressor is saturated. Note that there is no liquid receiver after the condenser, and thus no assumption of having saturated liquid at the condenser outlet. Furthermore, it is assumed that the heat transfer in both the condenser and evaporator are maximized. Finally, a temperature controller on the stream to be cooled (here the building temperature T_C) is used to adjust the compressor power.

There then remains one unconstrained degree of freedom (choke valve position z) which should be used to optimize the operation for all disturbances and operating points. We could envisage an real-time dynamic optimization scheme where one continuously optimizes the operation (minimize compressor power) by adjusting z. However, such schemes may be quite complex and sensitive to uncertainty. These problems can be reduced by selecting a good control variable, and ideally one get a simple constant setpoint scheme, with no need for real-time optimization. What should be controlled (and fixed, at least on the short time scale)? Some candidates are:

- Valve position z (i.e., an open-loop policy where the valve is left in a constant position).
- High pressure (P_h).
- Low pressure (P_l).
- Temperature out of compressor (T_1).

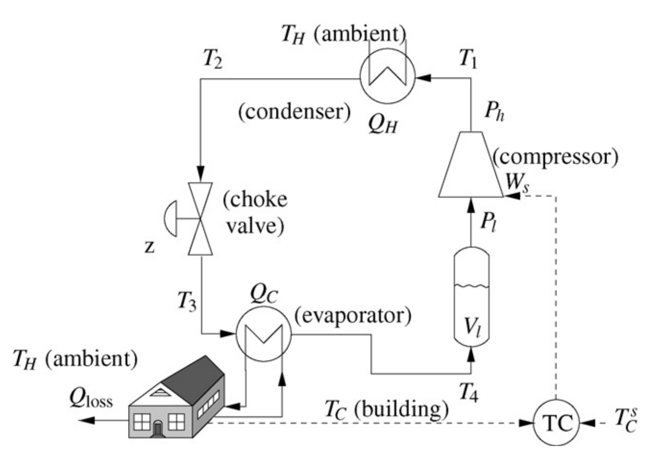

Fig. 1. Simple refrigeration cycle studied in this paper (shown for the ammonia case).

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- Temperature before valve (T_2).
- Degree of sub-cooling in the condenser[1] ($\Delta T_{sub} = T_2 - T_{sat}(P_h)$).
- Temperature approach in hot source heat exchanger ($T_2 - T_H$).
- Temperature out of evaporator (T_4).
- Degree of super-heating in the evaporator[2] ($\Delta T_{sup} = T_4 - T_{sat}(P_l)$).
- Liquid level in the receiver (V_l) to adjust the active charge in the rest of the system.
- Liquid level in the condenser ($V_{l,con}$) or in the evaporator ($V_{l,vap}$).
- Pressure drop across the "extra" valve in Fig. 11.[2]

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation (Skogestad, 2000). Note that the selection of a good controlled variable is equally important in an "advanced" control scheme like MPC which also is based on keeping the controlled variables close to given setpoints.

The selection of controlled variables is a challenging task, especially if one considers in detail all possible measurements, so we will first use a simple screening process based on a linear model.

2.1. Linear analysis

To find promising controlled variables, the "maximum gain" rule (Halvorsen, Skogestad, Morud, & Alstad, 2003) will be used. For the scalar case considered in this paper the rule is:

Prefer controlled variables with a large scaled gain $|G'|$ from the input (degree of freedom) to the output (controlled variable).

2.1.1. Procedure scalar case

(1) Make a small perturbation in each disturbances d_i and re-optimize the operation to find the optimal disturbance sensitivity $\partial \Delta y_{opt}/\partial d_i$. Let Δd_i denote the expected magnitude of each disturbance and compute from this the overall optimal variation (here we choose the 2-norm):

$$\Delta y_{opt} = \sqrt{\sum_i \left(\frac{\partial \Delta y_{opt}}{\partial d_i} \cdot \Delta d_i\right)^2}$$

(2) Identify the expected implementation error n for each candidate controlled variable y (measurement).

(3) Make a perturbation in the independent variables u (in our case u is the choke valve position z) to find the (unscaled) gain,

$$G = \frac{\Delta y}{\Delta u}.$$

(4) Scale the gain with the optimal span (span $y \equiv \Delta y_{opt} + n$), to obtain for each candidate output variable y, the scaled gain:

$$|G'| = \frac{|G|}{\text{span } y}$$

The worst-case loss $L = J(u, d) - J_{opt}(u, d)$ (the difference between the cost with a constant setpoint and re-optimized operation) is then for the scalar case (Skogestad & Postlethwaite, 2005; p. 394):

$$L = \frac{|J_{uu}|}{2} \frac{1}{|G'|^2} \tag{1}$$

where $J_{uu} = \partial^2 J/\partial u^2$ is the Hessian of the cost function J. In our case $J = W_s$ (compressor work). Note that J_{uu} is the same for all candidate controlled variables y.

The most promising controlled variables should then be tested on the non-linear model using realistic disturbances to check for non-linear effects, including feasibility problems.

2.2. Combination of measurements

If the losses with a fixed single measurement are large, as for the CO_2 case study, then one may consider combinations of measurements as controlled variables. The simple null space method (Alstad & Skogestad, 2007) gives a linear combination with zero local loss for the considered disturbances,

$$c = h_1 \cdot y_1 + h_2 \cdot y_2 + \cdots \tag{2}$$

The minimum number of measurements y to be included in the combination is $n_y = n_u + n_d$. In our case $n_u = 1$ and if we want to consider combinations of $n_y = 2$ measurements then only $n_d = 1$ disturbance can be accounted exactly for. With the "exact local method" (Halvorsen et al., 2003) or the "extended null space method" (Alstad & Skogestad, 2007) it is possible to consider additional disturbances. The local loss is then not zero, and we will minimize the 2-norm of the effect of disturbances on the loss.

3. Ammonia case study

The cycle operates between air inside a building ($T_C = T_{room} = -10\,°C$) and ambient air ($T_H = T_{amb} = 20\,°C$). This could be used in a cold storage building as illustrated in Fig. 1. The heat loss from the building is

$$Q_{loss} = UA_{loss}(T_H - T_C) \tag{3}$$

The nominal heat loss is 15 kW. The temperature controller shown in Fig. 1 maintains $T_C = -10\,°C$ and will indirectly give $Q_C = Q_{loss}$ at steady-state.

3.1. Modelling

The structure of the model equations are given in Table 1 and the data are given in Table 2. The heat exchangers are modelled assuming "cross flow" with constant temperature on the air side

[1] Not relevant in the CO_2 cycle because of super-critical high pressure.
[2] Not relevant for our design (Fig. 1).

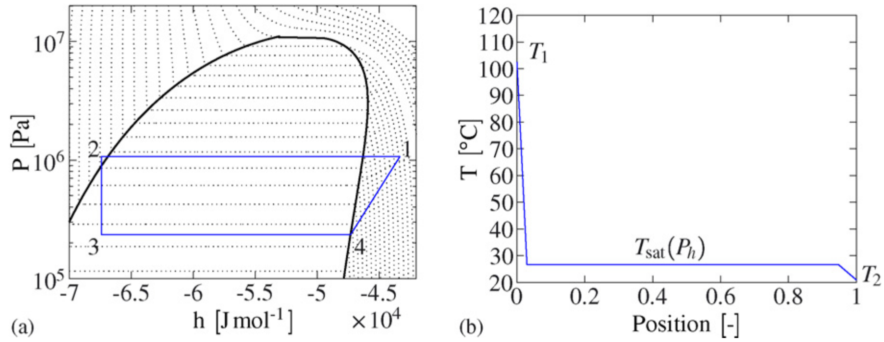

Fig. 2. Optimal operation for the ammonia case study. (a) Pressure enthalpy diagram. (b) Temperature profile in condenser.

($T_H = 20\,°C$ and $T_C = -10\,°C$). The isentropic efficiency for the compressor is assumed constant. The SRK equation of state is used for the thermodynamic calculations. The gPROMS model is available on the internet (Jensen & Skogestad, 2007a).

3.2. Optimal steady-state operation

At nominal conditions the compressor power was minimized with respect to the degree of freedom (z). The optimal results are given in Table 3, and the corresponding pressure enthalpy diagram and temperature profile in the condenser are shown in Fig. 2. Note that the optimal sub-cooling out of the condenser is 5.8 °C. This saves about 2.0% in compressor power (W_s) compared to the conventional design with saturation.

3.3. Selection of controlled variables

There is one unconstrained degree of freedom (choke valve opening z) which should be adjusted to give optimal sub-cooling in the condenser. We want to find a good controlled variable (see Section 2 for candidates) to fix such that we achieve close-to-optimal operation in spite of disturbances and implementation error ("self-optimizing control").

Table 1
Structure of model equations

Heat exchangers (condenser and evaporator)
$Q = U \cdot \int \Delta T\, dA = \dot{m} \cdot (h_{out} - h_{in})$
$P = P_{sat}(T_{Sat})$
$m = \frac{\rho}{V}$
Valve
$\dot{m} = z \cdot C_V \sqrt{\Delta P \cdot \rho}$; $h_{out} = h_{in}$
Compressor
$W_s = \dot{m}(h_{out} - h_{in}) = \frac{\dot{m} \cdot (h_s - h_{in})}{\eta}$

Table 2
Data for the ammonia case study

$T_H = 20\,°C$
$T_C = T_C^s = -10\,°C$
Condenser: $(UA)_C = 2500\,W\,K^{-1}$
Evaporator: $(UA)_E = 3000\,W\,K^{-1}$
Compressor: isentropic efficiency $\eta = 0.95$
Choke valve: $C_V = 0.0017\,m^2$
Building: $UA_{loss} = 500\,W\,K^{-1}$

Table 3
Optimal steady-state for ammonia case study

W_s (kW)	2.975
z	0.372
P_h (bar)	10.70
P_l (bar)	2.35
Q_H (kW)	17.96
$\dot{m}$ (kg s^{-1})	0.0127
$\Delta T_{sub} = T_2 - T_{sat}(P_h)$ (°C)	5.80
T_1 (°C)	102.6
T_2 (°C)	20.9
T_3 (°C)	−15.0
T_4 (°C)	−15.0

3.3.1. Linear analysis of alternative controlled variables

The following disturbance perturbations are used to calculate the optimal variation in the measurements y.[3]

- d_1: $\Delta T_H = \pm 10\,°C$;
- d_2 : $\Delta T_C^s = \pm 5\,°C$;
- d_3: $\Delta UA_{loss} = \pm 100\,W\,K^{-1}$.

The assumed implementation error (n) for each variable is given in Table 4 which also summarizes the linear analysis and gives the resulting scaled gains in order from low gain (poor) to high gain (promising).

Some notes about Table 4:

- P_l and T_4 have zero gains and cannot be controlled. The reason for the zero gains are that they both are indirectly determined by Q_{loss}.

$$Q_{loss} = Q_C + (UA)_C(T_4 - T_C) \quad \text{and} \quad P_l = P_{sat}(T_4) \tag{4}$$

- The degree of super-heating ΔT_{sup} can obviously not be controlled in our case because it is fixed at 0 °C (by design of the cycle).

[3] In order to remain in the linear region, the optimal variations were computed for a disturbance of magnitude 1/100 of this, and the resulting optimal variations were then multiplied by 100 to get $\Delta y_{opt}(d_i)$.

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Table 4
Linear "maximum gain" analysis of candidate controlled variables y for ammonia case study

Variable (y)	Nom.	G	$\|\Delta y_{opt}(d_i)\|$			$\|\Delta y_{opt}\|$	n	Span y	$\|G'\|$
			d_1 (T_H)	d_2 (T_C)	d_3 (UA_{loss})				
P_l (bar)	2.35	0.00	0.169	0.591	0.101	0.623	0.300	0.923	0.00
T_4 (°C)	−15.0	0.00	0.017	0.058	0.010	0.061	1.00	1.06	0.00
ΔT_{sup} (°C)	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
T_1 (°C)	102.6	−143.74	38	17.3	6.2	42.2	1.00	43.2	3.33
P_h (bar)	10.71	−17.39	4.12	0.41	0.460	4.17	1.00	5.17	3.37
z	0.372	1	0.0517	0.0429	0.0632	0.092	0.05	0.142	7.03
T_2 (°C)	20.9	287.95	10.4	0.20	0.300	10.4	1.00	11.4	25.3
V_l (m^3)	1.00	5.1455	9e−03	0.011	1.2e−03	0.0143	0.05	0.064	80.1
ΔT_{sub} (°C)	5.80	−340.78	2.13	1.08	1.08	2.62	1.50	4.12	82.8
$V_{l,con}$ (m^3)	0.67	−5.7	5.8e−03	2.4e−03	1.4e−03	0.0064	0.05	0.056	101.0
$T_2 - T_H$ (°C)	0.89	−287.95	0.375	0.174	0.333	0.531	1.50	2.03	141.8

- The loss is proportional to the inverse of squared scaled gain (see Eq. (1)). This implies, for example, that a constant condenser pressure (P_h), which has a scaled gain of 3.37, would result in a loss in compressor power $J = W_s$ that is $(82.8/3.37)^2 = 603$ times larger than a constant sub-cooling (ΔT_{sub}), which has a scaled gain of 82.8.
- The simple policies with a constant pressure (P_h) or constant valve position (z) are not promising with scaled gains of 3.37 and 7.03, respectively.
- A constant level in the liquid receiver (V_l) is a good choice with a scaled gain of 80.1. However, according to the linear analysis, the liquid level in the condenser ($V_{l,con}$) is even better with a scaled gain of 101.0.
- Controlling the degree of sub-cooling in the condenser ($\Delta T_{sub} = T_2 - T_{sat}(P_h)$) is also promising with a scaled gain of 82.8, but the most promising is the temperature approach at the condenser outlet ($T_2 - T_H$) with a scaled gain of 141.8.
- The ratio between the implementation error n and the optimal variation Δy_{opt} tells whether the implementation error or the effect of the disturbance is most important for a given control policy. For the most promising policies, we see from Table 4 that the contribution from the implementation error is most important.

3.3.2. Non-linear analysis

The non-linear model was subjected to the "full" disturbances to test more rigorously the effect of fixing alternative controlled variables. The main reason for considering the full disturbances is to check for non-linear effects, in particular possible infeasible operation, which cannot be detected from the linear analysis. Fig. 3 shows the compressor power W_s (left) and loss $L = W_s - W_{s,opt}$ (right) for disturbances in T_H (d_1), T_C (d_2) and UA_{loss} (d_3). $W_{s,opt}$ is obtained by re-optimizing the operation for the given disturbances. As predicted from the linear analysis, control of P_h or z should be avoided as it results in a large loss and even infeasibility (a line that ends corresponds to infeasible operation). Controlling the degree of sub-cooling ΔT_{sub} gives small losses for most disturbances, but gives infeasible operation when T_H is low. Controlling the liquid level, either in the receiver or in the condenser, gives small losses in all cases. Another good policy is to maintain a constant temperature approach out of the condenser ($T_2 - T_H$). This control policy was also the best in the linear analysis and has as far as we know not been suggested in the literature for ammonia cycles.

A common design for refrigeration cycles, also discussed in Part I, is to have no sub-cooling in the condenser. In practice, this might be realized with the design in Fig. 1 by adding a liquid receiver after the condenser and using the choke valve to control this liquid level, or using the design in Fig. 11 with the "extra" valve between the condenser and tank removed. The performance of this design ("no sub-cooling") is shown with the dashed line in Fig. 3. The loss (right graphs) for this design is always non-zero, as it even at the nominal point has a loss of 0.06 kW, and the loss increases with the cooling duty of the cycle. Nevertheless, we note that the loss with this design is low (less than about 0.2 kW or 3.5%) for all considered disturbances. This may be acceptable, although it is much higher than the best controlled variables (V_l, $V_{l,con}$ and $T_2 - T_H$) where the maximum losses are less than 0.005 kW.

Fig. 4 shows the sensitivity to implementation error for the four best controlled variables. Controlling a temperature difference at the condenser exit (either $T_2 - T_H$ or ΔT_{sub}) has a small sensitivity to implementation error. On the other hand, controlling either of the two liquid levels (V_l or $V_{l,con}$) might lead to infeasible operation for relatively small implementation errors. In both cases the infeasibility is caused by vapour at the condenser exit. In practice, this vapour "blow out" may be "feasible", but certainly not desirable.

A third important issue is the sensitivity to the total charge of the system which is relevant for the case where we control the liquid level in the receiver ($y = V_l$). There is probably some uncertainty in the initial charge of the system, and maybe more importantly there might be a small leak that will reduce the total charge over time. Optimally the total charge has no steady-state effect (it will only affect the liquid level in the receiver). However, controlling the liquid level in the receiver ($y = V_l$) will make the operation depend on the total charge, and we have lost one of the positive effects of having the liquid receiver. The other control structures will not be affected by varying total charge.

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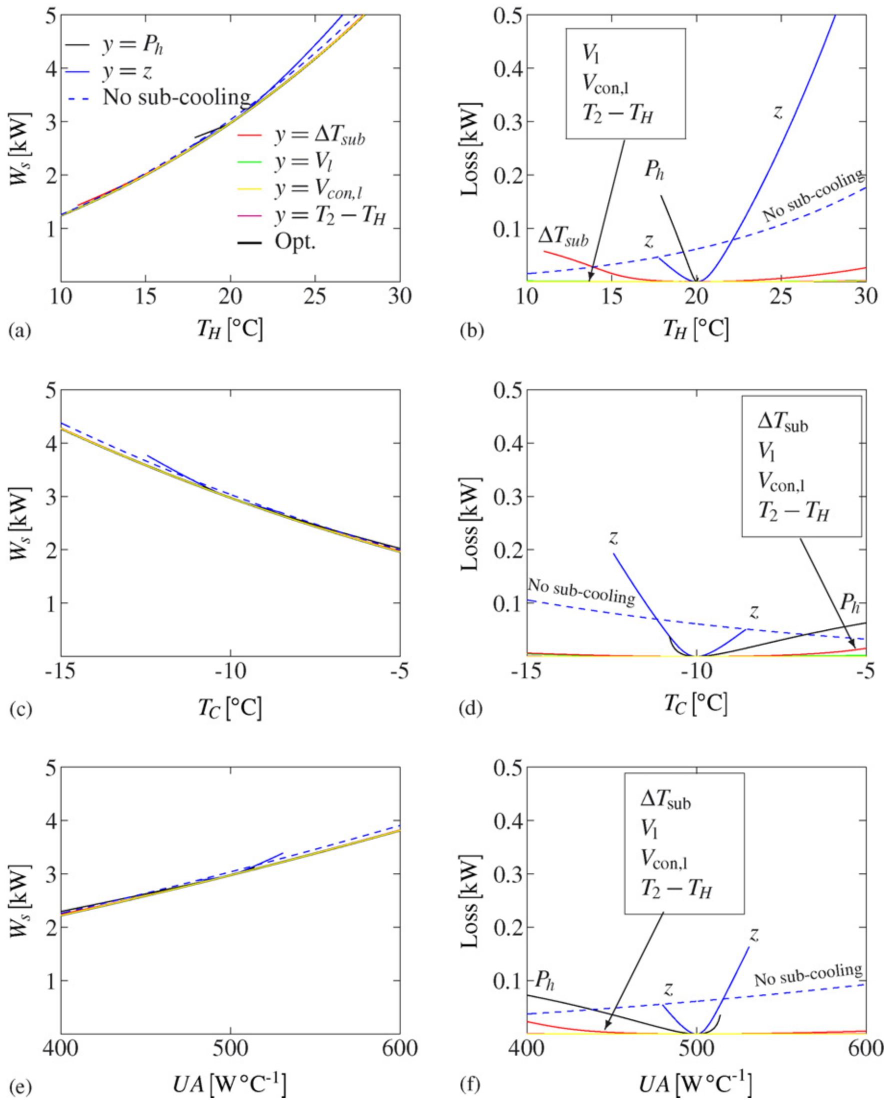

Fig. 3. Ammonia case: compressor power (left) and loss (right) for different disturbances and controlled variables. A line that ends corresponds to infeasible operation. (a and b) Disturbance in T_H (d_1). (c and d) Disturbance in T_C (d_2). (e and f) Disturbance in UA_{loss} (d_3).

3.3.3. Conclusion of ammonia case study

For the ammonia case, controlling the temperature approach at the condenser exit ($T_2 - T_H$) seems to be the best choice as the losses caused by implementation error (Fig. 4) and disturbances (Fig. 3) are very small. This control implementation is shown in Fig. 5 where we also have introduced an inner "stabilizing" loop for pressure. However, the *setpoint* for the pressure is used as a degree of freedom so this loop does not affect the results of this study, which are based on steady-state. Although not optimal even nominally, another acceptable policy is to use the conventional design with no sub-cooling (Fig. 11 with the "extra valve" removed and minimum superheating).

4. CO_2 case study

Neksaa (2002) shows that CO_2 cycles are attractive for several applications, both from an efficiency point of view and from an environmental perspective. Skaugen (2002) gives a detailed analysis of the parameters that affect the performance of a CO_2 cycle and discusses pressure control in these systems.

The simple cycle studied in this paper (see Fig. 6(a)) operates between air inside a room ($T_C = 20\,°C$) and ambient air ($T_H = 30\,°C$). This could be an air-conditioner for a home as illustrated in Fig. 6(a). The heat loss out of the building is given by Eq. (3), and the temperature controller shown in Fig. 6(a) indirectly gives $Q_C = Q_{loss}$. The nominal heat loss is 4.0 kW.

We consider a cycle with an internal heat exchanger (see Fig. 6(a)). This heat exchanger gives further cooling before the choke valve by super-heating the saturated vapour from the evaporator outlet. This has the advantage of reducing the expansion loss through the valve, although super-heat increases the compressor power. For the CO_2 cycle it has been found that the internal heat exchanger improves efficiency for some operating points (Domanski, Didion, & Doyle, 1994). For the CO_2 cycle,

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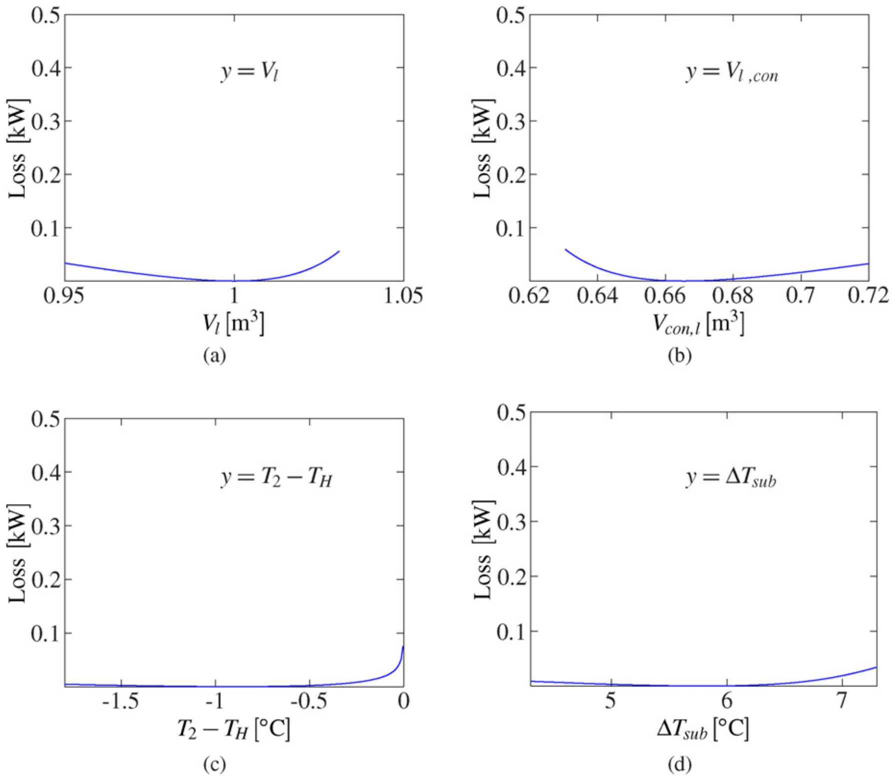

Fig. 4. Ammonia case: loss as function of implementation error. A line that ends corresponds to infeasible operation.

we find that the internal heat exchanger gives a nominal reduction of 9.9% in W_s. For the ammonia cycle, the effect of internal heat exchange to give super-heating is always negative in terms of efficiency.

4.1. Modelling

Table 1 shows the structure of the model equations and the data are given in Table 5. Constant air temperature is assumed in the evaporator (T_C). The gas cooler and internal heat exchanger are modelled as counter-current heat exchangers with 6 control volumes each. The Span–Wagner equation of state (1996) is used for the thermodynamic calculations. The MATLAB model is available on the internet (Jensen & Skogestad, 2007a).

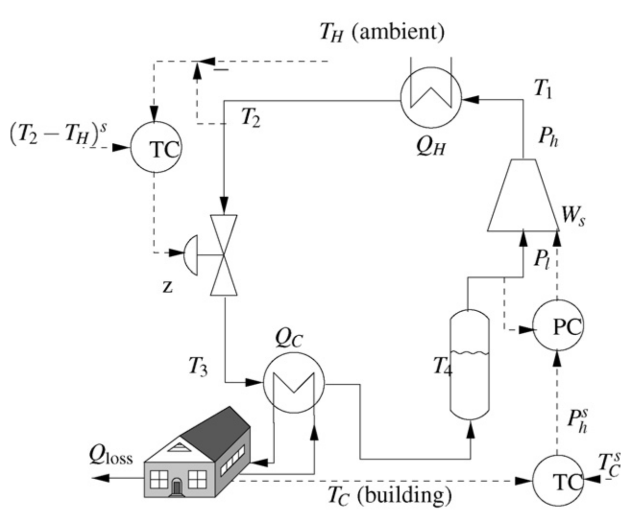

Fig. 5. Proposed control structure for the ammonia cycle.

4.2. Optimal operation

Some key parameters for optimal operation of the CO_2 cycle are summarized in Table 6 and the pressure enthalpy diagram is given in Fig. 6(b). Fig. 7 shows the optimal temperature profiles in the gas cooler and in the internal heat exchanger.

Note that when the ambient air goes below approximately $T_H = 25\,^{\circ}C$ the optimal pressure in the gas cooler is sub-critical. We will only consider trans-critical operation, so we assume that the air-conditioner is not used below 25 °C.

4.3. Selection of controlled variable

We want to find what the valve should control. In addition to the variables listed in Section 2, we also consider internal temperature measurements in the gas cooler and internal heat

Table 5
Conditions for the CO_2 case study

Evaporator: $(UA)_{vap} = 798\ W\,^{\circ}C^{-1}$
Gas cooler: $(UA)_{gco} = 195\ W\,^{\circ}C^{-1}$
Internal heat exchanger: $(UA)_{ihx} = 153\ W\,^{\circ}C^{-1}$
Compressor: isentropic efficiency $\eta = 0.75$
Ambient: $T_H = 30\,^{\circ}C$
Air flow gas cooler: $\dot{m}c_p = 250\ J\,^{\circ}C^{-1}\,s^{-1}$
Room : $T_C = T_C^s = 20\,^{\circ}C$
Room: $UA_{loss} = 400\ W\,^{\circ}C^{-1}$
Choke valve: $C_V = 1.21 \times 10^{-6}\ m^2$

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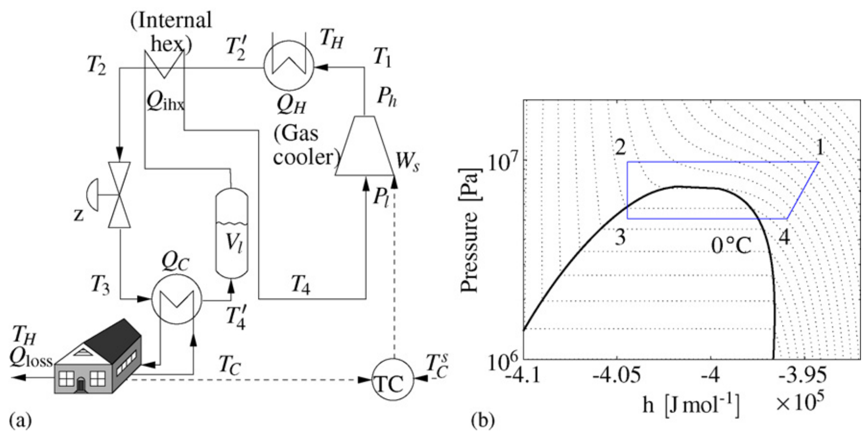

Fig. 6. The CO_2 cycle operates trans-critical and is designed with an internal heat exchanger. (a) The CO_2 cycle and (b) pressure enthalpy diagram.

Table 6
Optimal operation for CO_2 case

W_s (kW)	958
z	0.34
P_h (bar)	97.61
P_l (bar)	50.83
Q_H (kW)	−4958
Q_{ihx} (W)	889
$\dot{m}$ (kg s^{-1})	0.025
T_1 (°C)	89.6
T_2 (°C)	25.5
T_3 (°C)	15.0
T_4 (°C)	31.2

exchanger. Note that the "no sub-cooling" policy is not possible for the CO_2 cycle because it operates trans-critical.

As discussed in more detail below, there are no obvious single measurements to control for this application. One exception is the holdup m on the high pressure side of the cycle. However, measuring the holdup of a super-critical fluid is not easy (one might use some kind of scale, but this will be to expensive in most applications). Thus, we will consider measurement combinations. First, we will try to combine two measurements, and if this is not acceptable for all disturbances, we may try more measurements. Any two measurements can be combined, and we choose here to combine P_h and T_2. The reason is that P_h is normally controlled anyway for dynamic reasons, and T_2 is simple to measure and is promising from the linear analysis. Also, temperature corrected setpoint for pressure has been proposed before (Kim et al., 2004). We use the "exact local method" (Alstad & Skogestad, 2007) and minimize the 2-norm of $M_d = HFW_d$, where $F = \partial y_{opt}/\partial d_i$ is the optimal sensitivity of $y' = [P_h\ T_2]$ with respect to disturbances $d' = [T_H\ T_C\ (UA)_{loss}]$. The magnitude of the disturbances are given in W_d. We find that the linear combination $c = h_1 \cdot P_h + h_2 \cdot T_2$ with $k = h_2/h_1 = -8.53\ \text{bar}\ {}^\circ\text{C}^{-1}$ minimizes the 2-norm of the three disturbances on the loss. This can be implemented in practice by controlling the combined pressure and temperature

$$P_{h,combine} = P_h + k \cdot (T_2 - T_{2,opt}) \tag{5}$$

where $T_{2,opt} = 25.5\ {}^\circ\text{C}$ and $k = -8.53\ \text{bar}\ {}^\circ\text{C}^{-1}$. An alternative is to use a more physically-based combination. For an ideal gas we have $m = (PV \cdot \text{MW})/RT$, and since the gas cooler holdup m_{gco} seems to be a good variable to control, we will include P/T in the gas cooler as a candidate controlled variable.

4.3.1. Linear method

We first use the linear "maximum gain" method to find promising controlled variables. The following disturbances[3] are considered:

- d_1: $\Delta T_H = \pm 10\ {}^\circ\text{C}$;
- d_2: $\Delta T_C = \pm 5\ {}^\circ\text{C}$;
- d_3: ΔUA_{loss} from -100 to $+40\ \text{W}\ {}^\circ\text{C}^{-1}$.

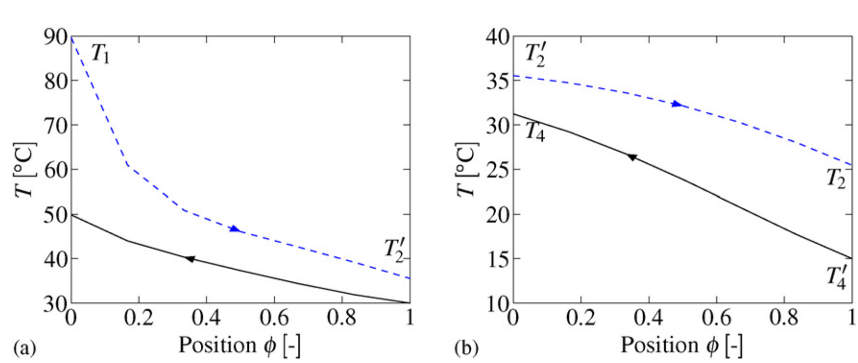

Fig. 7. CO_2 case: temperature profile in gas cooler and internal heat exchanger. (a) Gas cooler and (b) internal heat exchanger.

Table 7
Linear "maximum gain" analysis of controlled variables for CO_2 case

Variable (y)	Nom.	G	$\|\Delta y_{opt}(d_i)\|$			$\|\Delta y_{opt}\|$	n	Span y	$\|G'\|$
			d_1 (T_H)	d_2 (T_C)	d_3 (UA_{loss})				
P_h/T'_2(bar °C^{-1})	0.32	−0.291	0.140	−0.047	0.093	0.174	0.0033	0.177	0.25
P_h (bar)	97.61	−78.85	48.3	−15.5	31.0	59.4	1.0	60.4	1.31
T'_2 (°C)	35.5	36.7	16.27	−2.93	7.64	18.21	1	19.2	1.91
$T'_2 - T_H$ (°C)	3.62	24	4.10	−1.92	5.00	6.75	1.5	8.25	2.91
z	0.34	1	0.15	−0.04	0.18	0.24	0.05	0.29	3.45
V_l (m^3)	0.07	0.03	−0.02	0.005	−0.03	0.006	0.001	0.007	4.77
T_2 (°C)	25.5	60.14	8.37	0.90	3.18	9.00	1	10.0	6.02
$P_{h,combine}$ (bar)	97.61	−592.0	−23.1	−23.1	3.91	33.0	9.53	42.5	13.9
m_{gco} (kg)	4.83	−11.18	0.151	−0.136	0.119	0.235	0.44	0.675	16.55

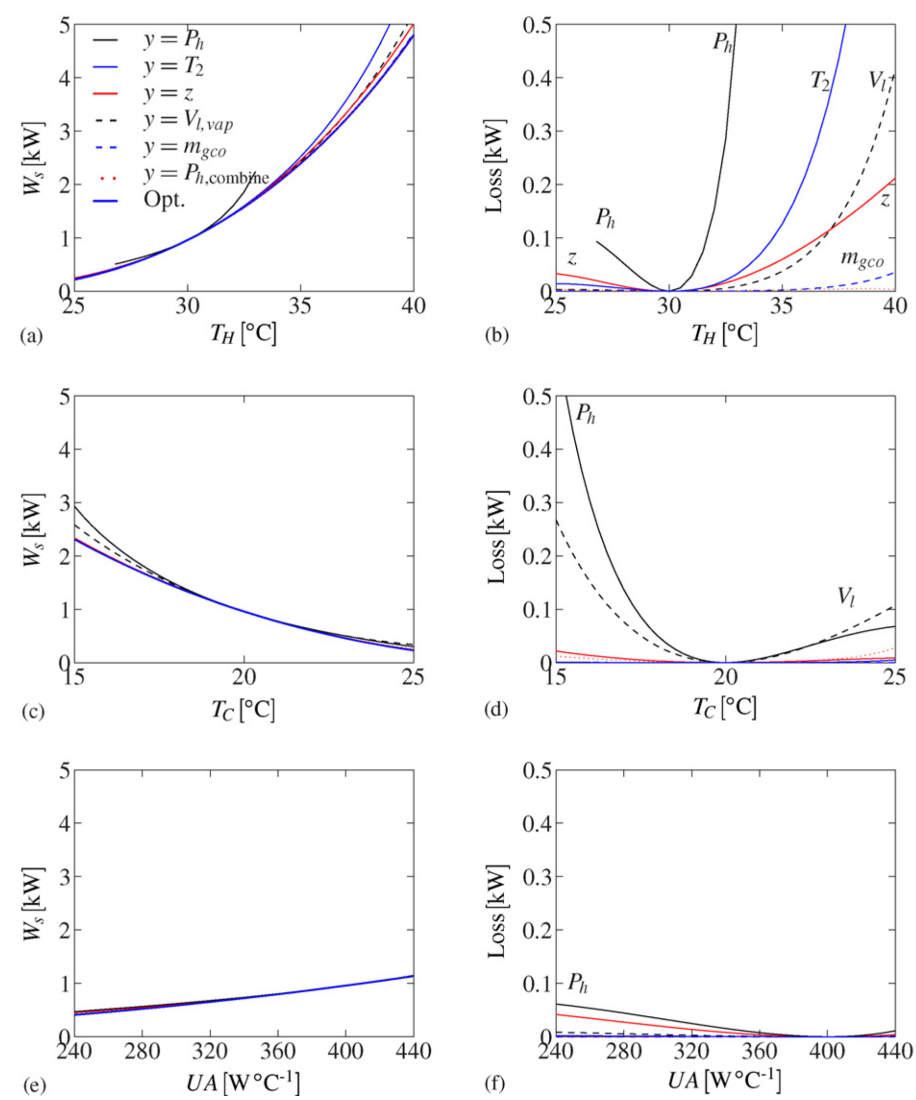

Fig. 8. CO_2 case: compressor power (left) and loss (right) for different disturbances and controlled variables. A line that ends corresponds to infeasible operation. (a and b) Disturbance in T_H (d_1). (c and d) Disturbance in T_C (d_2). (e and f) Disturbance in UA_{loss} (d_3).

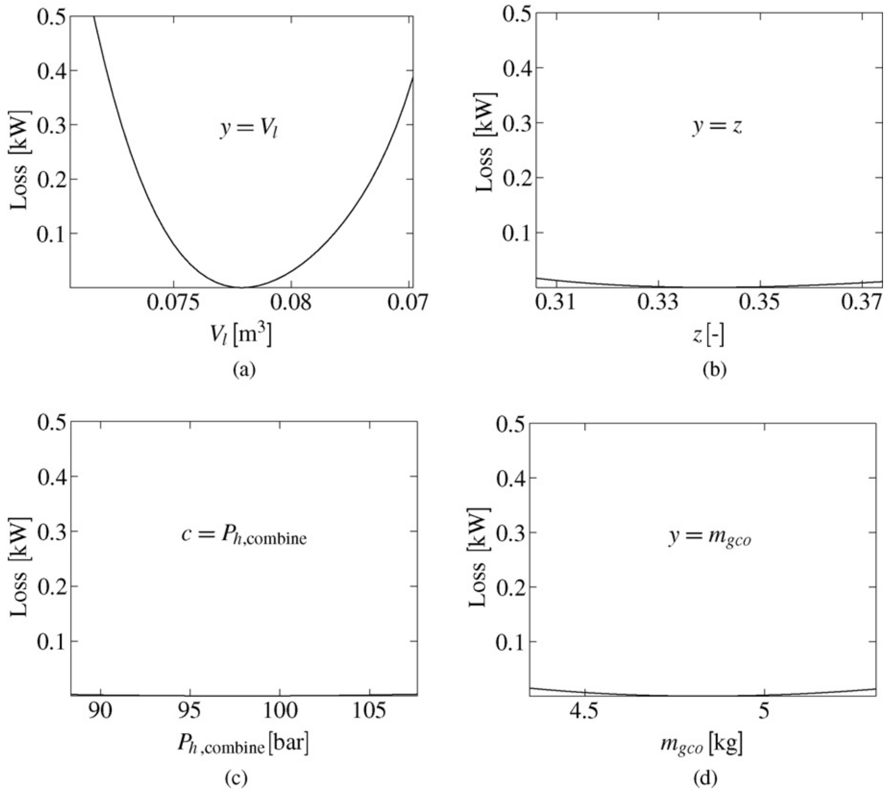

Fig. 9. CO_2 case: loss as function of implementation error.

The linear results are summarized in Table 7. Some controlled variables (P_l, T_4', ΔT_sub and ΔT_sup) are not considered because they, as discussed earlier, cannot be fixed or are not relevant for this cycle. The ratio P_h/T_2' in the gas cooler is not favourable with a small scaled gain. This is probably, because the fluid in the gas cooler is far from ideal gas so P_h/T_2' is not a good estimate of the holdup m_gco. From Table 7 the most promising controlled variables are the holdup in the gas cooler (m_gco) and the linear combination ($P_\mathrm{h,combine}$). Fixing the valve opening z_s (no control) or the liquid level in the receiver (V_l) are also quite good.

4.3.2. Non-linear analysis

Fig. 8 shows the compressor power (left) and loss (right) for some selected controlled variables. We see that the two most important disturbances are the temperatures T_H and T_C which gives larger losses than disturbance in the heat loss out of the building. Controlling the pressure P_h gives infeasible operation for small disturbances in the ambient air temperature (T_H). The non-linear results confirm the linear gain analysis with small losses for $P_\mathrm{h,combine}$ and m_gco.

Another important issue is the sensitivity to implementation error. From Fig. 9 we see that the sensitivity to implementation error is very large for $y = V_\mathrm{l}$. The three best controlled variables are constant valve opening (z), constant holdup in the gas cooler (m_gco) and the linear combination ($P_\mathrm{h,combine}$).

4.3.3. Conclusion of CO_2 case study

For this CO_2 refrigeration cycle we find that fixing the holdup in the gas cooler m_gco gives close to optimal operation. However, since the fluid is super-critical, holdup is not easily measured. Thus, in practice, the best *single* measurement is a constant valve opening z ("no control"). A better alternative is to use *combinations* of measurements. We obtained the combination $P_\mathrm{h,combine} = P_\mathrm{h} + k\cdot(T_2 - T_{2,\mathrm{opt}})$ using the "exact local method". This implementation is shown in Fig. 10. The disturbance loss compared with single measurements is significantly reduced and the sensitivity to implementation error is very small.

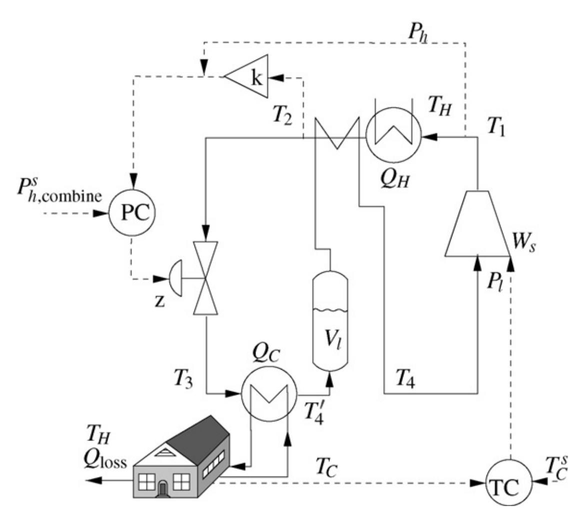

Fig. 10. Proposed control structure for the CO_2 cycle.

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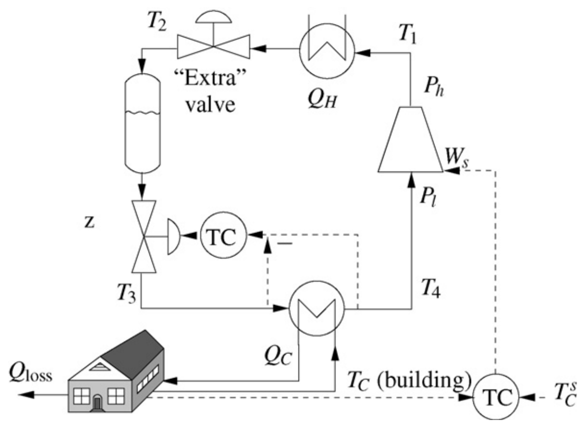

Fig. 11. Alternative refrigeration cycle with liquid receiver on high pressure side and control of super-heating.

5. Discussion

5.1. Super-heating

An important practical requirement is that the material entering the compressor must be vapour (either saturated or super-heated). Saturation can be achieved by having a liquid receiver before the compressor as shown in Fig. 1. However, in many designs the receiver is located at the high pressure side and super-heating may be controlled with the choke valve (e.g., thermostatic expansion valve, TEV) as shown in Fig. 11. A minimum degree of super-heating is required to handle disturbances and measurement errors. Since super-heating is not thermodynamically efficient (except for some cases with internal heat exchange), this minimal degree of super-heating becomes an active constraint. With the configuration in Fig. 11, the "extra" valve is the unconstrained degree of freedom (u) that should be adjusted to achieve optimal operation. Otherwise the results from the study hold, both for the ammonia and CO_2 cycle.

5.2. Heat transfer coefficients

We have assumed constant heat transfer coefficients in the heat exchangers. Normally, the heat transfer coefficient will depend on several variables such as phase fraction, velocity of the fluid and heat transfer rate. However, a sensitivity analysis (not included) shows that changing the heat transfer coefficients does not affect the conclusions in this paper. For the CO_2 cycle, we did some simulations using a constant air temperature in the gas cooler, which may represent a cross flow heat exchanger and is an indirect way of changing the effective *UA* value. We found that the losses for a constant liquid level control policy ($y = V_l$) was slightly smaller, but the analysis presented here is still valid and the conclusion that a combination of measurements is necessary to give acceptable performance, remains the same.

5.3. Pressure control

This paper has only considered steady-state operation. For dynamic reasons, in order to "stabilize" the operation, a degree of freedom is often used to control one pressure (P_l or P_h). However, the *setpoint* for the pressure may be used as a degree of freedom at steady-state, so this will not change the results of this study. An example of a practical implementation using cascade control is shown in Fig. 5 where the temperature difference at the condenser outlet is controlled which was found to be the best policy for the ammonia case study. The load in the cycle is controlled by adjusting the setpoint to the pressure controller that stabilize the low pressure (P_l).

6. Conclusion

For a simple cycle, there is one unconstrained degree of freedom that should be used to optimize the operation. For the sub-critical ammonia refrigeration cycle a good policy is to have no sub-cooling. Further savings at about 2% are obtained with some sub-cooling where a good control strategy is to fix the temperature approach at the condenser exit ($T_2 - T_H$) (see Fig. 5). One may argue that 2% savings is very little for all the effort, but larger savings are expected for cases with smaller heat exchanger areas (Jensen & Skogestad, 2007b), and allowing for sub-cooling shows that there is no fundamental difference with the CO_2 case.

For the trans-critical CO_2 cycle, the only single "self-optimizing" measurement seems to be the holdup in the super-critical gas cooler (m_{gco}). However, since this holdup is difficult to measure a combination of measurements is needed. We propose to fix a linear combination of pressure and temperature, $P_{h,combine} = P_h + k \cdot (T_2 - T_{2,opt})$ (see Fig. 10). This is a "self-optimizing" control structure with small losses for expected disturbances and implementation errors.

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References

Alstad, V., & Skogestad, S. (2007). The null space method for selecting optimal measurement combinations as controlled variables. *Industrial and Engineering Chemistry Research*.

Domanski, P. A., Didion, D. A., & Doyle, J. P. (1994). Evaluation of suction-line/liquidline heat exchange in the refrigeration cycle. *International Journal of Refrigeration*, *17*, 487–493.

Dossat, R. J. (2002). *Principles of refrigeration*. Prentice Hall.

Halvorsen, I. J., Skogestad, S., Morud, J. C., & Alstad, V. (2003). Optimal selection of controlled variables. *Industrial and Engineering Chemistry Research*, *42*, 3273–3284.

Jensen, J. B., & Skogestad, S. (2007a). gPROMS and MATLAB model code for ammonia and CO_2 cycles. See additional material for paper at homepage of S. Skogestad.

Jensen, J. B., & Skogestad, S., (2007b). Optimal operation of simple refrigeration cycles. *Computers and Chemical Engineering*, doi:10.1016/j.compchemeng.2006.12.003.

Kim, M. H., Pettersen, J., & Bullard, C. W. (2004). Fundamental process and system design issues in CO_2 vapor compression systems. *Progress in Energy and Combustion Science*, *30*, 119–174.

Langley, B. C. (2002). *Heat pump technology*. Prentice Hall.

Larsen, L. S., Thybo, C., Stoustrup, J., & Rasmussen, H. (2003). Control methods utilizing energy optimizing schemes in refrigeration systems. In *European control conference (ECC)*

Neksaa, P. (2002). CO_2 heat pump systems. *International Journal of Refrigeration*, *25*, 421–427.

Skaugen, G. (2002). *Investigation of transcritical CO_2 vapour compression systems by simulation and laboratory experiments*. Ph.D. thesis. Norwegian University of Science and Technology.

Skogestad, S. (2000). Plantwide control: The search for the self-optimizing control structure. *Journal of Process Control*, *10*(5), 487–507.

Skogestad, S., & Postlethwaite, I. (2005). *Multivariable feedback control* (2nd ed.). John Wiley & Sons.

Span, R., & Wagner, W. (1996). A new equation of state for carbon dioxide covering the fluid region from the triple-point temperature to 1100 K at pressures up to 800 MPa. *Journal of Physical and Chemical Reference Data*, *25*(6), 1509–1596.

Stoecker, W. F. (1998). *Industrial refrigeration handbook*. McGraw-Hill.

Svensson, M. C. (1994). *Studies on on-line optimizing control, with application to a heat pump*. PhD thesis. Norwegian University of Science and Technology, Trondheim.

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