A new method for deriving reduced models of one-dimensional distributed system

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Abstract

A new method for deriving reduced models of one-dimensional distributed systems is presented. The approach is both applicable to discrete (staged) and continuous systems. It is based on partitioning the system into steady-state subsystems, which are connected by dynamic elements. The steady-state subsystems can be solved off-line, and the required solutions can be substituted as functions of the states of the neighbouring dynamic elements into the equations of the latter. This way, the reduced models offer a considerable reduction in simulation time, making them attractive for real-time optimizing control applications (Linhart and Skogestad, 2008). An important feature of the reduced models is perfect steady-state agreement with the original models.

As an example for a discrete system, a binary distillation column with complex hydraulic and thermodynamic equations is presented. In this case, the model reduction method is a direct extension of the aggregated modeling method by Lévine and Rouchon (1991). However, it is shown that the notion of compartments, as used by Lévine and Rouchon, is not needed, allowing a greater freedom of choice for the parameters of the reduced model and the straightforward extension of the method to more complex models. The reduction procedure can be performed in two steps: first, the left-hand sides of the differential equations of some selected "aggregation stages" of the original model are modified by multiplying with large constants, and setting the left-hand sides of the remaining equations to zero. Secondly, the resulting algebraic equations are solved block-wise as functions depending on the neighbouring aggregation stages, and substituted into the equations of the aggregation trays. After the first step, a model with reduced dynamic order is obtained, which has the same number of equations as the full model. To be able to exploit the lower dynamic order for faster simulations, the second step has to be performed. Two ways of representing the numerically obtained solutions of the steady-state subsystems are demonstrated: tabulation and multi-linear interpolation of the function values, and polynomial approximation by linear regression. While polynomial approximation is easier to implement, tabulation and interpolation offers higher approximation accuracy. The resulting reduced model yields a very good approximation of the full model dynamics. The increase in simulation speed is proportional to the decrease in the number of dynamic trays in the model, which is typically around factor 10. An implementation with the DAE solver DASPK (Li and Petzold, 2000) is presented, giving details about the numerical performance of the individual parts of the full and the reduced model.

The method can be applied similarly to continuously distributed systems. The distributed system is partitioned into steady-state intervals connected by dynamic elements. For the derivation of the equations of the dynamic elemnts, the reduction procedure is applied to a finite-differences discretisation of the continuously distributed system. The reduced equations are then obtained by letting the number of discretisation points go to infinity. In analogy to the discrete case, the steady-state subsystems can be solved as boundary-value problems off-line and substituted into the equations of the dynamic elements.
Two examples for continuously distributed systems are presented: An adiabatic fixed-bed reactor with heat recycle exhibiting limit cycle oscillations (Liu and Jacobsen, 2004), and a linear heat exchanger model. For the fixed-bed reactor, it is shown that the reduced model is capable to reproduce the high-frequent limit cycle oscillations if the number of dynamic elements is large enough. The linear heat exchanger model has analytic steady-state solutions, which can be used to derive analytic reduced model equations. Compared to simple finite-differences discretisations, the reduced model shows a superior approximation quality. Responses of the outlet temperatures for disturbances of the inlet on the opposite side of the system, however, are due to the transport character of the system approximated well only with a high number of dynamic elements.
In the continuous case, the method can be applied as an alternative to established methods such as finite differences, finite volumes or collocation methods. In both discrete and continuous cases, a significant gain in computational performance can be expected, making the method attractive for real-time optimizing control applications.

References

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