Robustness Margins Separating Process Dynamics Uncertainties

Per-Ola Larsson, Tore Hägglund
Department of Automatic Control
Faculty of Engineering, Lund University, Sweden
{perola.larsson|tore.hagglund}@control.lth.se

Extended Abstract

This presentation describes a robustness measure that separates dead time and non-dead time uncertainties in process dynamics for SISO systems. The measure can easily be shown in a Nyquist diagram and gives good insight to the inherent problems of a time delay in a plant.

Background

Classic robustness measures such as phase, gain, and dead time margin do not guarantee stability when simultaneous changes in the process occur. To deal with simultaneous effects, one is often forced to use robust stability in the framework of robust control. This can be overly pessimistic in the sense that the process uncertainty might not have the shape of a disk, i.e., it is badly approximated by a norm bounded uncertainty. This is certainly the case for a dead time uncertainty, since this gives only a rotation of the Nyquist curve. There is thus a need for a robustness margin that especially take the characteristics of a dead time into account.

Sensitivity Functions Constraints

The derivation of the robustness margins are similar to the arguments in robust control, but the process uncertainties are now distinctly separated. Consider a process $P(s) = P_o(s)e^{-sL}$, where $P_o(s)$ is the delay free part and L is the time delay. We can now model the uncertainty in the delay free part as multiplicative or inverse multiplicative and the uncertainty in dead time as an interval $\Delta L \in [\Delta L_{\min},\ \Delta L_{\max}]$. The models including uncertainties are then

$$P_o(s)\left(1+\Delta_2(s)\right)e^{-s(L+\Delta L)}, \qquad \text{and} \qquad P_o(s)\left(1+\Delta_1(s)\right)^{-1}e^{-s(L+\Delta L)}.$$

By setting up the robust stability criteria, as with ordinary robust control, we get in return the constraints

$$\|T(s,\Delta L)\Delta_1(s)\|_\infty < 1, \qquad \text{and} \qquad \|S(s,\Delta L)\Delta_2(s)\|_\infty < 1$$

which corresponds to the two different modelling approaches and are very similar to ordinary robust control. The extended sensitivity functions $T(s,\Delta L)$ and $S(s,\Delta L)$ are defined as

$$T(s,\Delta L) = \frac{C(s)P(s)e^{-s\Delta L}}{1+C(s)P(s)e^{-s\Delta L}}, \qquad S(s,\Delta L) = \frac{1}{1+C(s)P(s)e^{-s\Delta L}}.$$

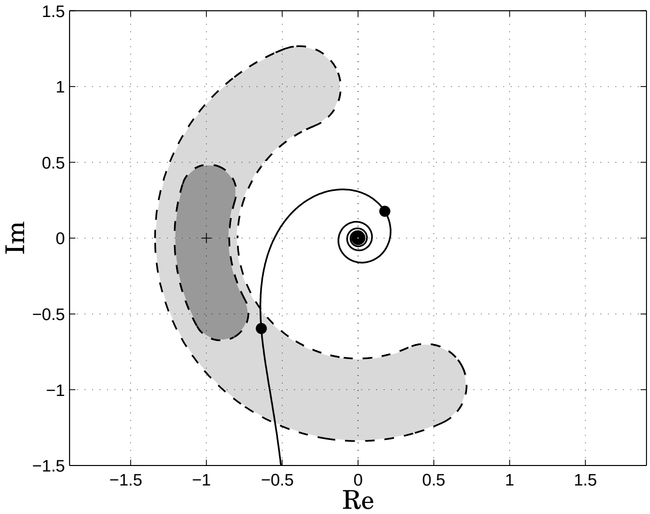

Figure 1: Nyquist curve and its prohibited areas at $\omega = 1$ (dark gray) and 4 rad/s (light gray) for the process and PI controller in the example. Dots indicate the two frequencies.

Interpretation in Nyquist Diagram

The two constraints are easily illustrated in a Nyquist diagram since they correspond to areas where the Nyquist curve is prohibited. The areas are defined by circles with centers in

$$\frac{1}{|\Delta_1(i\omega)|^2 - 1}\left(\cos(\omega\Delta L),\ \sin(\omega\Delta L)\right), \qquad \text{and} \qquad -1,$$

and radii

$$\frac{|\Delta_1(i\omega)|}{|1 - |\Delta_1(i\omega)|^2|}, \qquad \text{and} \qquad |\Delta_2(i\omega)|,$$

respectively, for the two different modelling choices. The areas are different from the ones in ordinary robust control. It has been seen that this measure can be less conservative than ordinary robust control especially when the largest process uncertainty is in the dead time.

Simple and fast algorithms to compute the margins have been constructed and implemented in Matlab.

Example — Robustness of PI-loop

Consider the first order process with dead time and PI controller

$$P(s) = \frac{1}{s+1}\, e^{-s}, \qquad C(s) = 1.02\left(1 + \frac{1}{1.45s}\right),$$

and assume we have 10% uncertainty in process time constant and gain while $\Delta L \in [-0.2, 0.5]$ The delay free part is modelled using multiplicative uncertainty. The prohibited areas for the two frequencies 1 and 4 rad/s are shown with dark and light grey, respectively, in Figure 1. It can easily be computed that the Nyquist curve is outside the specified area for all frequencies and hence the closed loop system is robustly stable to the specified uncertainties. The controller is the optimal controller for minimizing the integrated absolute error (IAE) with the above constraint on robustness and is not allowed by the ordinary robust control framework.