Subspace Identification of a Distillation Column

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Outline

- Background
  - State-space models for MIMO systems
  - Identification by PE methods
- Basic Idea of Subspace Identification
- Experimental design for MIMO systems
- Application to a pilot-scale distillation column
  - Identification experiments
  - N4SID identification
- Conclusions

Background

- State-space models
  - convenient for MIMO systems
  - problems with time delays

$$x(t+1) = Ax(t) + Bu(t) + Ke(t)$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

- Identification by PE methods
  - minimize $V_N(\theta) = \frac{1}{N}\sum_{t=0}^{N-1} \left\| \varepsilon(t,\theta) \right\|^2$ with respect to θ subject to

$$\hat{x}(t+1,\theta) = [A(\theta) - K(\theta)C]\hat{x}(t,\theta) + B(\theta)u(t) + K(\theta)y(t)$$
$$\varepsilon(t,\theta) = y(t) - C\hat{x}(t,\theta) - D(\theta)u(t)$$

  - nonlinear iterative optimization, usually ill-conditioned
  - local minima
  - choice of model structure is problematic

$\Rightarrow$ PE methods have inherent difficulties for MIMO systems (Katayama, 2005).

Basic Idea of Subspace Identification

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- Determine (A,B,C,D) directly from data through algebraic manipulations — i.e., no iterative optimization
  - If the state vector $\tilde{x}(t)$ can be estimated, (A,B,C,D) is obtained by

$$\begin{bmatrix} \hat{A} & \hat{B} \\ \hat{C} & \hat{D} \end{bmatrix} = \left(\sum_{t=0}^{N-1} \begin{bmatrix} \tilde{x}(t+1) \\ y(t) \end{bmatrix} \begin{bmatrix} \tilde{x}(t+1) \\ y(t) \end{bmatrix}^{\mathrm{T}} \right) \left(\sum_{t=0}^{N-1} \begin{bmatrix} \tilde{x}(t) \\ u(t) \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ u(t) \end{bmatrix}^{\mathrm{T}} \right)^{-1}$$

    There are ways of constructing $\tilde{x}(t)$ from input-output data (*direct N4SID*).

  - If the (extended) observability matrix Γ_r is known, (A,C) can be extracted. Since

$$y(t) = C(qI - A)^{-1} Bu(t) + Du(t) + \tilde{e}(t)$$

$$\Gamma_r = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-1} \end{bmatrix}$$

    (B,D) can also be determined.

    There are many ways of constructing Γ_r (or some similar matrix) from input-output data (*realization-based N4SID methods*).

Basic Idea of Subspace Identification

– One way is as follows (basically according to Ljung, 1999):

$$\mathbf{Y}_{0|-s_1} = \begin{bmatrix} y(0) & \cdots & y(N-1) \\ \vdots & \ddots & \vdots \\ y(-s_1) & \cdots & y(N-1-s_1) \end{bmatrix}, \quad \mathbf{U}_{0|-s_2} = \begin{bmatrix} u(0) & \cdots & u(N-1) \\ \vdots & \ddots & \vdots \\ u(-s_2) & \cdots & u(N-1-s_2) \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \mathbf{Y}_{0|-s_1} \\ \mathbf{U}_{0|-s_2} \end{bmatrix}, \quad G = \tfrac{1}{N}\mathbf{Y}_{1|r}\left[\mathbf{I} - \mathbf{U}_{1|r}^{\mathrm{T}}(\mathbf{U}_{1|r}\mathbf{U}_{1|r}^{\mathrm{T}})^{-1}\mathbf{U}_{1|r}\right]\Phi^{\mathrm{T}}$$

$$\hat{G} = W_1 G W_2 = USV^{\mathrm{T}} \approx U_1 S_1 V_1^{\mathrm{T}}, \quad \hat{\Gamma}_r = W_1^{-1} U_1 R$$

W_1, W_2 and R are weighting matrices given by the particular method.

– S_1 is a matrix of singular values obtained by omitting the insignificant singular values from S (note that data are corrupted by noise). In principle, this is a user choice.

– ***Is this a problem for identification of ill-conditioned MIMO systems, where small singular values in the gain matrix are very relevant?***

Design of Identification Experiments

Preliminary analysis

- It is desirable to make the identification (equally) informative for all relevant "directions"
- Consider a singular value decomposition of the gain matrix, i.e.

$$y = Gu = U\Sigma V^{\mathrm{T}} u = \sum_{i=1}^{n} U_i \sigma_i V_i^{\mathrm{T}} u$$

  – the input $u = u^i = V_i \sigma_i^{-1}$ will produce the output $y = y^i = U_i$, $\left\| y^i \right\| = 1$

- To properly excite all directions i, $i = 1, \ldots, n$, we need to apply inputs u^i that vary (symmetrically) between

$$u_-^i = -\sigma_i^{-1} V_i \quad \text{and} \quad u_+^i = +\sigma_i^{-1} V_i$$

  – it is sufficient to know σ_i (a scalar) approximately; V_i may have to be more accurately estimated (but not difficult for distillation)

Some design options

- Excitation of one direction at a time
  - the input u is varied between u_-^1 and u_+^1 in one part of the experiment, between u_-^2 and u_+^2 in another part, etc.
- Excitation of all directions simultaneously
  - the input u is given by $u = \frac{1}{n}\sum_{i=1}^{n} u^i$, where the u^i:s are varied simultaneously in an uncorrelated way
- *Note 1:* The above principles apply irrespectively of what type of signal is used to move u^i between u_-^i and u_+^i (e.g., PRBS).
- *Note 2:* Perturbation of the inputs one at a time or simultaneously in uncorrelated ways are generally *not* optimal designs.

Application to Distillation

N4SID identification

- How sensitive is it to the experimental design?
- Is the choice of order a problem (in MATLAB's System Identification Toolbox)?
- How to handle time delays?

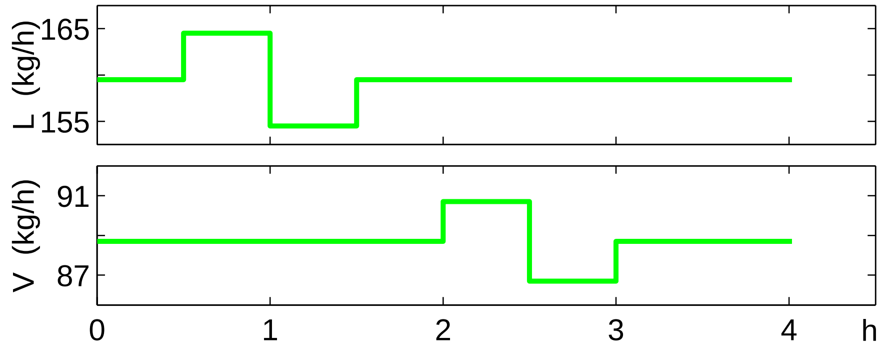
$$x(k+1) = Ax(k) + B\begin{bmatrix} u_1(k-\theta_1) \\ u_2(k-\theta_2) \end{bmatrix}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k+\theta_3) \end{bmatrix} = Cx(k)$$

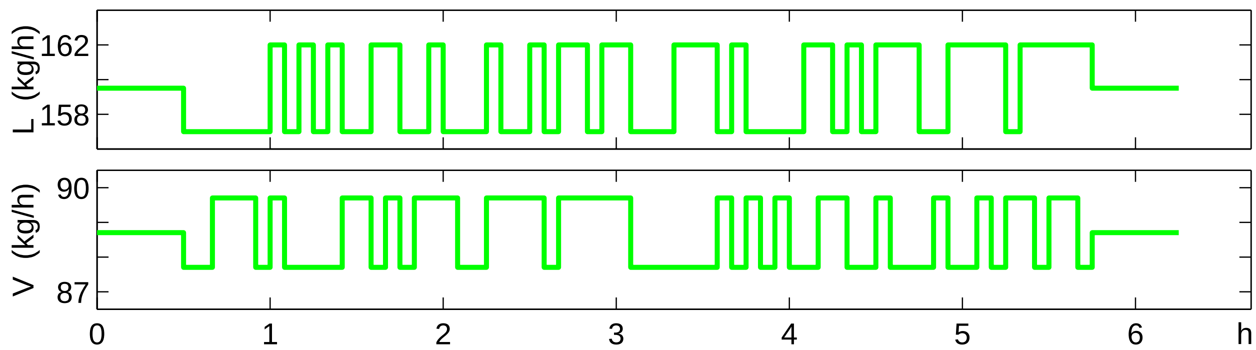
Pilot-scale distillation column at Åbo Akademi University

Identification experiments

♦ Step changes of inputs one at a time (**SeqStep**)



♦ Simultaneous uncorrelated PRBS in inputs (**UncPRBS**)



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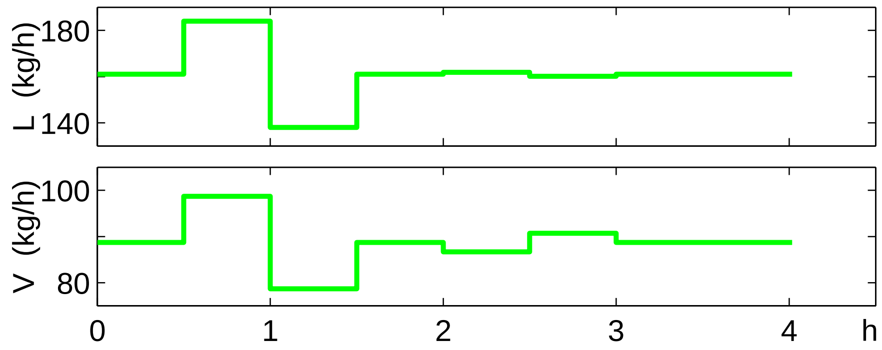
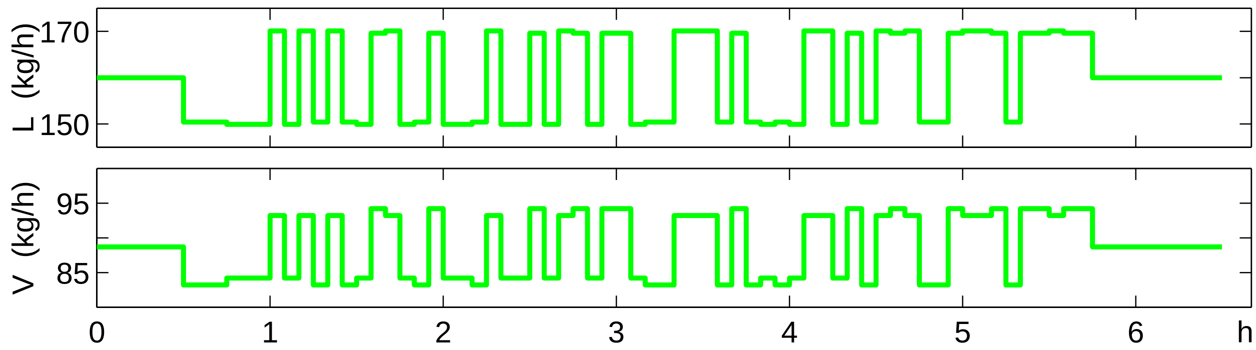
- Step changes in gain directions (**DirStep**)



- Simultaneous PRBS excitation of gain directions (**SimDirPRBS**)



N4SID Identification

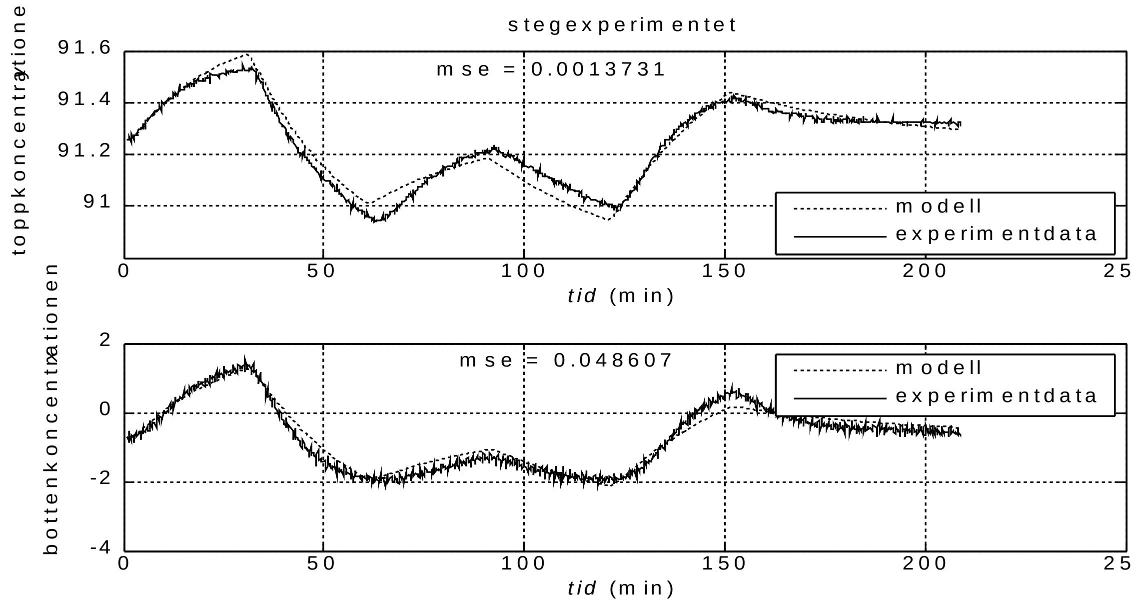
- Step changes of inputs one at a time (**SeqStep**)
  - Default order (figure)

    $\theta_1 = \theta_2 = 6,\ \theta_3 = 12$

    $n = 3,\ \ \bar{e}^2 = 6.79 \times 10^{-7}$

    $\sigma(A) = 1.00,\ 0.99,\ 0.81$

    $\sigma(K) = 0.784,\ 0.002$



  - Fix order = 3

    $\bar{e}^2 = 5.79 \times 10^{-7}$!!

    $\sigma(A) = 1.00,\ 0.96,\ 0.34$

    $\sigma(K) = 1.161,\ 0.028$!

  - Better order = 4 (?)

    $\bar{e}^2 = 4.65 \times 10^{-7}$

    $\sigma(A) = 1.05,\ 1.00,\ 0.99,\ 0.23$

    $\sigma(K) = 5.007,\ 0.036$

  - Better time delays: $\theta_1 = 12,\ \theta_2 = 15,\ \theta_3 = 9$ (?)

    $n = 4,\ \bar{e}^2 = 3.60 \times 10^{-7}$

    $\sigma(A) = 1.05,\ 1.00,\ 0.99,\ 0.28$ (consistent $\sigma(A)$!)

    $\sigma(K) = 0.467,\ 0.114$ (inconsistent $\sigma(K)$)

- Simultaneous uncorrelated PRBS (**UncPRBS**)

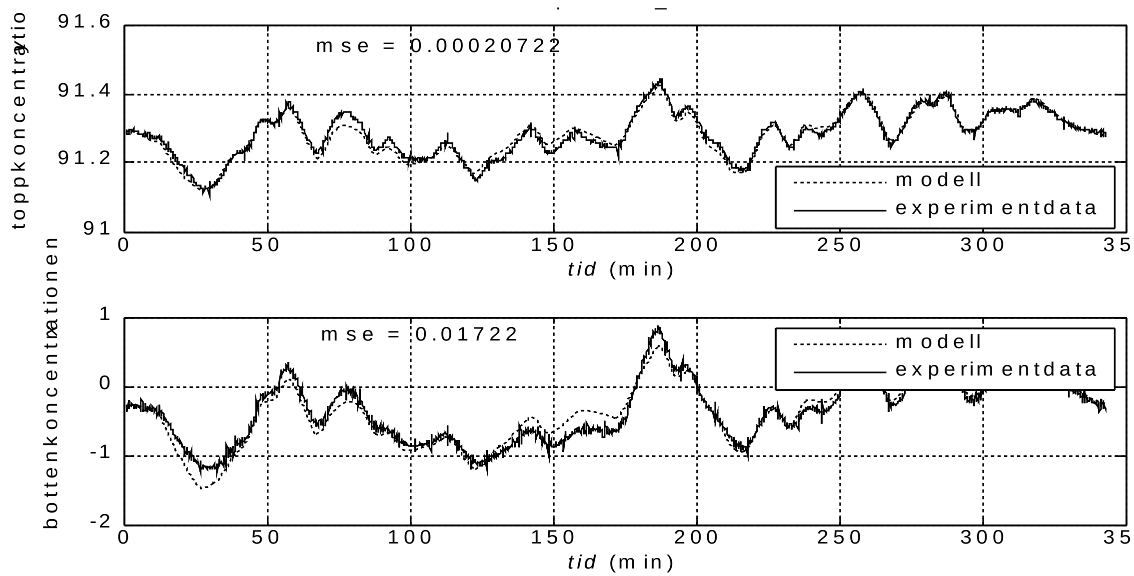
– Default order (figure)

$\theta_1 = \theta_2 = 6,\ \theta_3 = 12$

$n = 4,\ \bar{e}^2 = 7.78\times10^{-8}$

$\sigma(A) = 1.00,\ 0.99,\ 0.95,\ 0.26$

$\sigma(K) = 0.777,\ 0.001$



– Fix order = 4

$\bar{e}^2 = 8.61\times10^{-8}$!

$\sigma(A) = 1.18, 1.00,\ 0.99,\ 0.08$

$\sigma(K) = 0.550,\ 0.003$

– Better order = 5 (?)

$\bar{e}^2 = 5.22\times10^{-8}$

$\sigma(A) = 1.04, 1.00, 0.99, 0.81, 0.63$

$\sigma(K) = 0.770,\ 0.003$

– Better time delays: $\theta_1 = 12,\ \theta_2 = 15,\ \theta_3 = 9$ (??)

$n = 4,\ \bar{e}^2 = 7.49\times10^{-8}$

$\sigma(A) = 1.00,\ 0.99,\ 0.97,\ 0.23$

$\sigma(K) = 0.803,\ 0.000001$!!!

- Simultaneous PRBS in gain directions (**SimDirPRBS**)

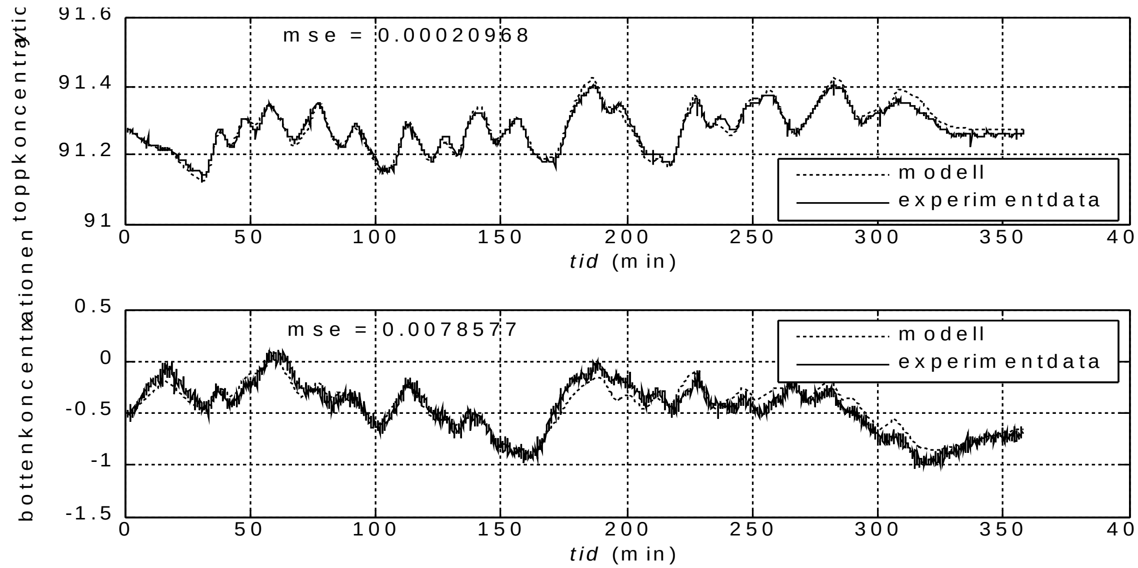
– Default order (figure)

$\theta_1 = \theta_2 = 6,\ \theta_3 = 12$

$n = 3,\ \bar{e}^2 = 6.73 \times 10^{-8}$

$\sigma(A) = 1.00, 0.99, 0.92$

$\sigma(K) = 1.387, 0.013$



– Fix order = 3

$\bar{e}^2 = 8.44 \times 10^{-8}$!!

$\sigma(A) = 1.00, 0.99, 0.43$

$\sigma(K) = 1.220, 0.023$

– Better order = 4 (??)

$\bar{e}^2 = 9.92 \times 10^{-8}$!!

$\sigma(A) = 1.41, 1.00, 0.99, 0.19$

$\sigma(K) = 1.318, 0.019$

– Better time delays: $\theta_1 = 12, \theta_2 = 15, \theta_3 = 9$

$n = 4,\ \bar{e}^2 = 5.10 \times 10^{-8}$ ($n = 4$ is default choice!)

$\sigma(A) = 1.01, 0.99, 0.98, 0.30$

$\sigma(K) = 1.221, 0.014$ (very consistent $\sigma(K)$)

— N4SID identification

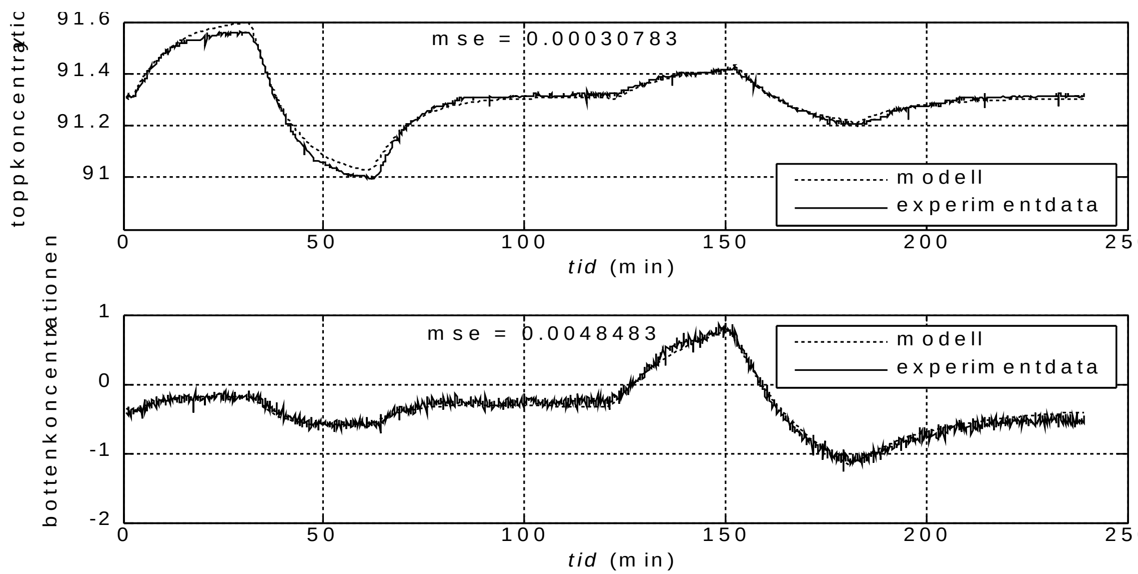
- Step changes in gain directions (**DirStep**)
  - Default order (figure)

    $\theta_1 = \theta_2 = 6,\ \theta_3 = 12$

    $n = 4,\ \ \bar{e}^2 = 1.33\times 10^{-7}$

    $\sigma(A) = 1.09,\ 0.99,\ 0.98,\ 0.24$

    $\sigma(K) = 1.459,\ 0.013$



  - Fix order = 4

    $\bar{e}^2 = 1.43\times 10^{-7}$!!

    $\sigma(A) = 1.44, 0.99, 0.99, 0.03$!

    $\sigma(K) = 1.532,\ 0.014$

  - Better order = 3 (???)

    $\bar{e}^2 = 1.21\times 10^{-7}$!!

    $\sigma(A) = 0.99,\ 0.99,\ 0.40$

    $\sigma(K) = 1.558,\ 0.015$

  - Better time delays: $\theta_1 = 12, \theta_2 = 15, \theta_3 = 9$ (?)

    $n = 4,\ \bar{e}^2 = 1.04\times 10^{-7}$

    $\sigma(A) = 1.00,\ 0.99,\ 0.98,\ 0.06$!!

    $\sigma(K) = 1.453,\ 0.013$ (very consistent $\sigma(K)$)

Conclusions

- Some observations about the N4SID algorithm:
  - The “loss function” (~variance of the disturbance model) does not always decrease with increasing model order.
  - The default choice of model order does not always seem “right”.
  - Fixing the model order to the default order changes the loss function and can dramatically change estimated parameters (maybe different weight matrices are used?).
- Some other observations:
  - Consistent gain estimates require experiments that properly excite the “gain directions”.
  - A good choice of time delays is a demanding task.