Robust PID Design with Adjustable Control Signal Noise Reduction

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Background

- The PID controller is the most common controller in process industry today
  - Many poorly tuned
- Formula methods (like λ-tuning) and hand-tuning common
- D-part often disabled
  - One more parameter to choose
  - If chosen uncarefully it can lead to a noisy control signal and thus actuator wear

Goal and Purpose

What do we desire from a PI/PID design method?

- Robust control with good performance
  - Model errors and process changes should not lead to instability
- Applicable on a real process
  - Limits on control signal variation due to measurement noise
- PI or PID controller depending on which is preferable
  - Do not make it complicated if there is little to gain
- A simple method for deriving controllers
  - Use a new Matlab design tool
- A tool to examine if the controllers are reasonable
  - Compare with the best linear controllers (Youla)

Robust PI/PID Design - Software description

Matlab based software for design of robust PID controllers

$$C(s) = K(1 + \frac{1}{sT_i} + sT_d) \cdot \frac{1}{1 + sT_f + (sT_f)^2/2},$$

or PI controllers

$$C(s) = K(1 + \frac{1}{sT_i}) \cdot \frac{1}{1 + sT_f}.$$

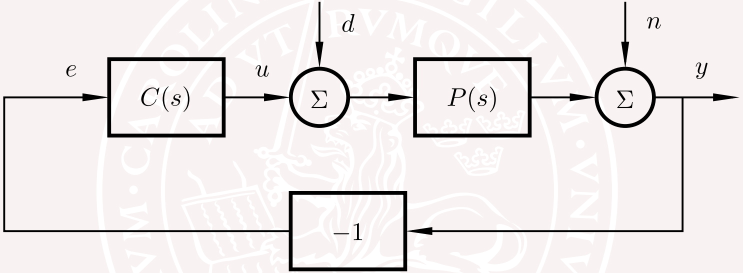
Where T_f is set in advance. Discrete controllers possible.

The software is

- Fast and robust
- Easy to use
- Free of charge: http://www.control.lth.se/user/olof.garpinger/

Robust PI/PID Design - Specified Control Structure

Consider the following system:



- d is a load disturbance
- n is measurement noise

$$T(s) = \frac{P(s)C(s)}{1+P(s)C(s)}, \quad S(s) = \frac{1}{1+P(s)C(s)}, \quad S_k(s) = \frac{C(s)}{1+P(s)C(s)}$$

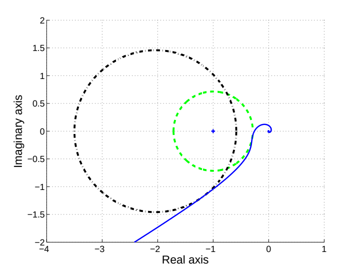
Robust PI/PID Design - Optimization Problem

The controllers are designed to minimize the IAE, during a load disturbance on the process input, with respect to robustness constraints.

$$\min_{K,T_i,T_d \in \mathcal{R}^+} \int_0^\infty |e(t)|dt = IAE_{load}$$

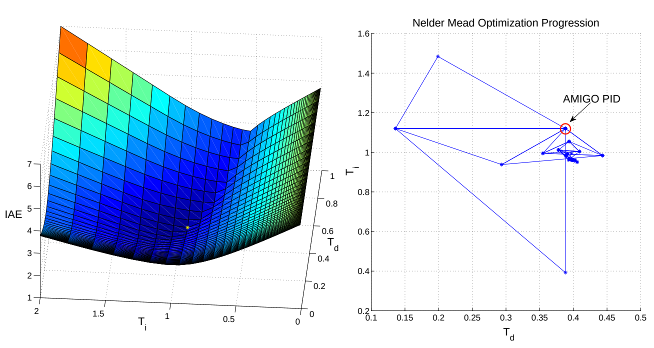
subject to $|S(i\omega)| \leq M_s, \quad |T(i\omega)| \leq M_p, \quad \forall \omega \in \mathcal{R}^+,$

$|S(i\omega^s)| = M_s$ and/or $|T(i\omega^p)| = M_p$



Robust PI/PID Design - Nelder Mead Optimization

Plots taken from the Matlab program:



- The Nelder Mead method has shown to work very well for the given optimization problem
- Similar for PI controllers

Introducing Four Parameter Design

$$C(s) = K\left(1 + \frac{1}{sT_i} + sT_d\right) \cdot \frac{1}{1 + sT_f + (sT_f)^2/2}$$

So far, it has been assumed that the lowpass filter time constant, T_f, was set in advance. But, is there a clever way to choose T_f, such that we get design of all four parameters?

The lowpass filter affects

- Performance
- Measurement noise throughput to the control signal

The idea is to choose T_f such that the variance constraint

$$\|S_k\|_2^2 = \left\| \frac{C}{1 + PC} \right\|_2^2 = \frac{\sigma_u^2}{\sigma_n^2} \leq V_k$$

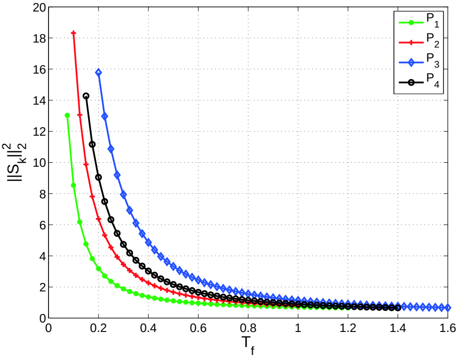
is fulfilled.

The relation between T_f and the variance of u

Using my software on

$$P_1(s) = \frac{1}{s+1}e^{-s},\ P_2(s) = \frac{1}{(s+1)^2}e^{-s},\ P_3(s) = \frac{1}{(s+1)^4},\ P_4(s) = \frac{(-0.5s+1)}{(s+1)^3}$$

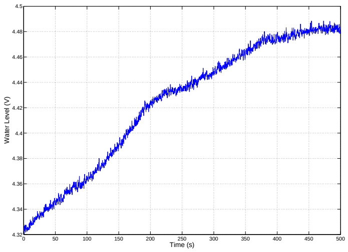
with different choices of T_f, reveals a clear relation between T_f and the control signal variance.

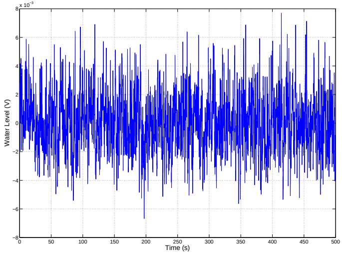


Choosing T_f - The Algorithm

A suggestion for design of PID controllers being performed as follows:

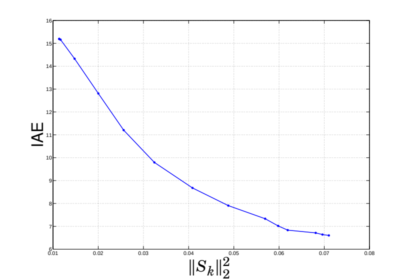
1. Collect noisy measurement data from the process, detrend it and estimate the variance σ_n^2.





Choosing T_f - The Algorithm

2. Choose a number of different T_f values. For each T_f
   - design a PID controller using the Matlab program.
   - simulate the closed loop system using the gathered noise data and estimate the variance, σ_u^2, of the control signal.
3. Plot IAE versus $\|S_k\|_2^2$.



4. Choose a PID controller, taking the trade-off between performance and control signal variance into account.

Controller Evaluation - Youla Controllers

Parameterizing a discrete controller, $C(z)$, as

$$C(z) = \frac{Q(z)}{1 + P(z)Q(z)}$$

with $Q(z)$ as a FIR filter

$$Q(z) = \sum_{l=0}^{N-1} q_l z^{-l},$$

it is possible to receive a good estimate of the best possible linear controller through convex optimization.

These so called Youla parameterized controllers have been used to evaluate the quality of the PI and PID controllers.

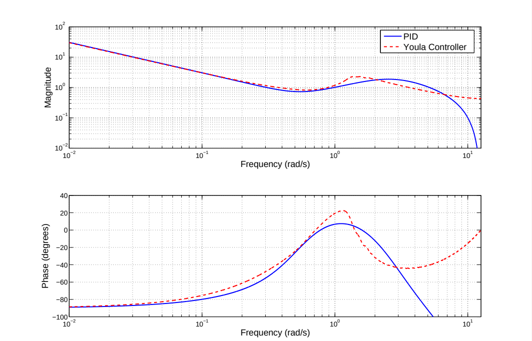
Design Example - Fourth Order Lag

$$P(s) = \frac{1}{(s+1)^4}, \;\; h = 0.25 \;\; \text{seconds}$$

$$V_k = 1, \;\; \text{White measurement noise} \;\; \sigma_n^2 = 1$$

$$Q(z) = 1 + q_1 z + \ldots + q_{149} z^{149}$$

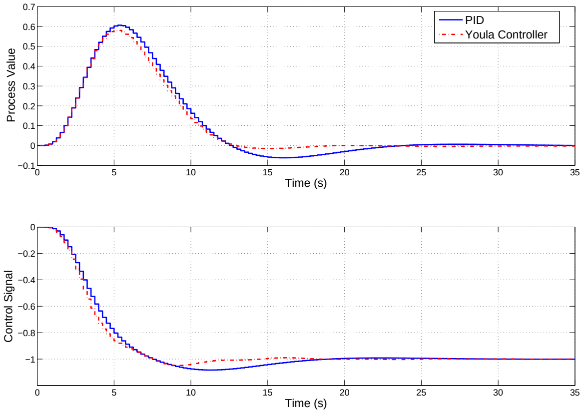
PID and Youla Designs:



Design Example - Fourth Order Lag

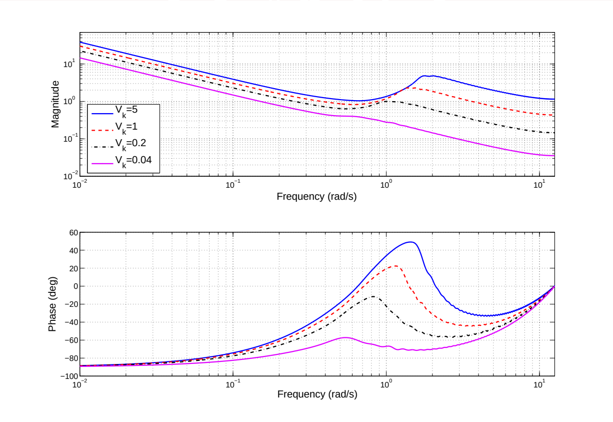
PID parameters: $K = 0.73$, $T_i = 2.38$, $T_d = 1.29$, $T_f = 0.52$

$IAE_{Youla} = 3.43$, $IAE_{PID} = 4.06$ (18% higher)



Design Example - Fourth Order Lag

Changing V_k for the Youla Design:



Prefered controller structure depends on desired trade-off between performance and control signal variance.
Thus, a PI controller may very well be prefered over a PID.

Summary and Future Work

The PI/PID design method

- is a simple way of deriving good controllers for a real plant
- takes the trade-off between noise throughput and performance into consideration
- can be evaluated using Youla parameterized controllers

Hopes for the future:

- Industrial tests
- For what types of processes are PI and PID controllers sufficient?