Robust Extremum Seeking Control with application to Gas Lifted Oil Wells

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Abstract: Classical extremum seeking control (ESC), when applied to systems with disturbances, can be subject to large deviations during transients caused by abrupt changes in the disturbances. In oil and gas production applications, such deviations can make ESC impractical. This paper presents a simple yet practical extension to the classical gradient-based extremum seeking control to make it robust to such disturbances, by removing the effect of the disturbance with a priori information of the disturbance model. Modelling and robustness of the disturbance models are discussed. The proposed method is demonstrated by a simulation-based study on gas lift optimization of a single well in oil and gas production.

Keywords: Extremum Seeking Control, Adaptive, Optimisation, Disturbance rejection

1. INTRODUCTION

Extremum seeking controller (ESC) is a class of data-driven adaptive control methods, where the steady state performance of the system is optimised in real time by applying constant perturbation to the system. The concept of extremum seeking control was first introduced in 1922, but gained steady interest only in the last decade after a rigorous proof of the classical ESC was provided in Krstic and Wang (2000). Various techniques have since then been developed to improve the performance of the extremum seeking control. The most popular ESC approach is the gradient-based approach due to its simplicity and guaranteed local convergence. The classical ESC identifies the extremum by estimating the gradient of input-output map by correlating the input perturbation signal with the measured performance function. The system is then driven towards the extremum by simply integrating the estimated gradient continuously, see Ariyur and Krstic (2003).

In many practical applications, however, the system is subject to disturbances which may change the performance function and the corresponding input-output map. A relatively fast and abrupt change of a disturbance and the corresponding effect on the performance function, can cause fast and large deviations in the estimated gradient and hence in the optimising parameter. Although the extremum seeking controller may eventually converge to the optimum after the disturbance has reached its new constant value as shown in Krstic (2000), the resulting transients may be far too large and long for practical applications. In some cases, this can even cause the ESC to converge to other stationary points that are no longer optimal, see Trollberg and Jacobsen (2013).

The data based disturbance feedforward method presented in Marinkov et al. (2014) addresses this issue by extending the classical ESC with additional blocks that detect abrupt changes in the performance function. The detected events are then used to stop the perturbation and wait for a predefined time to allow the disturbance or transients to damp out before starting the extremum search again. This method however may not be very practical for slow processes, where the waiting time maybe too long, or if the process is frequently subject to disturbances, where the extremum seeking scheme may spend a lot of time waiting for the transients from the disturbances to damp out.

Many applications in the oil and gas industry have slow system dynamics and may be subjected to disturbances often. The method proposed in Marinkov et al. (2014) may therefore not be very practical for such processes. In this paper we propose an alternative solution that addresses this problem for processes with slow dynamics. The method introduces robustness to disturbances by rejecting the effect of the disturbance from the performance function without stopping the perturbation or adaptation of the optimising parameter. Therefore the algorithm continuously seeks the extremum value without being affected by the disturbance nor waiting for the transients due to the disturbances to die out. The problem motivation and the proposed method are demonstrated through an application example of gas lift optimisation using extremum seeking control, as suggested in Peixoto et al. (2015).

The paper is organised as follows. Section 2 illustrates the issues with the classical gradient-based extremum seeking control using the gas lift optimisation example. Section 3 describes the problem formulation. Section 4 describes the proposed extremum seeking scheme with discussions on modelling the disturbance rejection block. Section 5 shows the results of the simulation example, before concluding the paper in section 6.

2. MOTIVATING EXAMPLE

In many oil production wells, when the reservoir pressure is not sufficient to lift the oil from the reservoir, artificial lift methods are employed. One commonly used method is

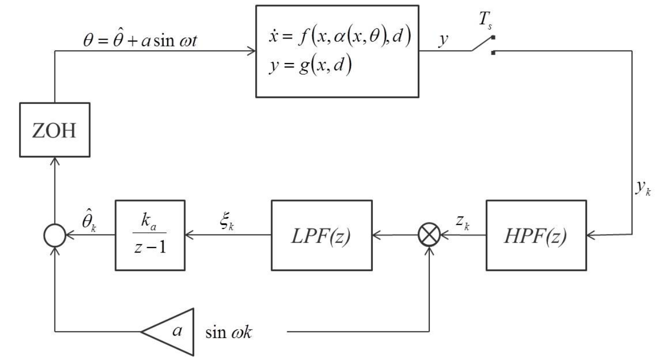

Fig. 1. Classic extremum seeking scheme

the gas lift method, where compressed gas is injected at the bottom of the well. As a result the fluid density decreases, thereby decreasing the hydrostatic pressure drop over the well. The pressure at the well bottom decreases and the inflow rate from the reservoir increases thus increasing the oil flow rate to the surface. However, injecting too much gas increases the frictional pressure drop which has an opposite effect on the flow rate. At some point the frictional drop becomes dominant over reduction of the hydrostatic pressure drop and causes the flow rate from the reservoir to decrease. Hence there exists an optimal gas lift injection rate that maximises the oil rate. The relation between the oil rate and the lift gas injection rate are called gas-lift performance curves, which have an optimum, see Golan and Whitson (1991) and Rashid et al. (2012).

The gas lift performance curves, however, change with changes in the wellhead pressure, injection gas pressure, reservoir productivity etc., which can considered as disturbances to the system. For example, the gas lift performance curves and their gradients for two different wellhead pressures are shown in Figure 3(red and blue lines). The goal of the controller is then to find the optimal gas lift injection rate (optimising input) that leads to maximum oil rate (performance function), see Peixoto et al. (2015).

In Figure 1, we show the block diagram of a classical gradient based extremum seeking scheme in discrete time with a sampling time T_s that is applied to such a process as suggested in Peixoto et al. (2015). To briefly explain the scheme, for the moment, we assume the disturbance is a constant. The scheme uses a sinusoidal dither signal $a \sin \omega k$ to perturb the optimising input(gas lift injection rate), which makes the performance function(oil rate) to vary according to the gradient of the gas-lift performance curve around the operating point. In essence, it is from the oscillating value of the performance function and that of the dither signal, that we are able to figure out how to move the value of the optimising input to maximise the performance function.

In discrete time setting, the gradient information is extracted in the stated scheme at each time step k via the following steps: remove the low frequency part of the output using a high-pass filter as shown in (1); correlate the outcome with the dither signal; apply a low-pass filter to the correlated signal as shown in (2). The estimated gradient is then used to update the optimising variable $\hat{\theta}_k$ (3). The filter cut-off time constants T_h and T_l, adaptation gain k_a and the dither amplitude a are tuning parameters

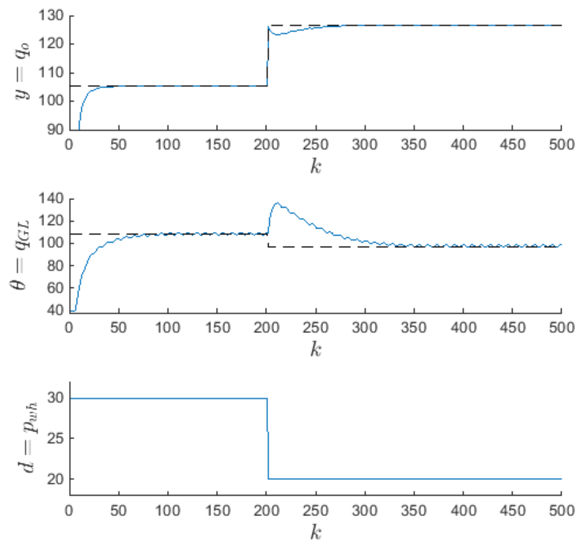

Fig. 2. Simulation results showing the effect of disturbance on the ESC

for the extremum seeking controller. For guidelines on parameter tuning, see Nesic (2009) and Tan et al. (2010).

$$z_k = \frac{T_h}{T_s + T_h}\left[z_{k-1} + y_k - y_{k-1}\right] \quad (1)$$

$$\xi_k = \left(1 - \frac{T_s}{T_s + T_l}\right)\xi_{k-1} + \frac{T_s}{T_s + T_l} z_k a \sin \omega k \quad (2)$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + T_s k_a \xi_k \quad (3)$$

So the classic extremum seeking scheme works fine for the cases in which the disturbance d is constant. However, when the disturbance changes abruptly and/or frequently with large magnitudes, the scheme may lead to quite undesirable outcomes. Briefly speaking, this is because a change in disturbance causes a change in the performance function, in addition to the change caused by the dither signal. Therefore, in this period it is no longer possible to extract reliably the information of the gradient w.r.t. the optimising input from the measured performance function. This results in wrong gradient estimation and hence driving the optimising input in a wrong direction during the transient period.

This point is further demonstrated using the simulation results from the gas-lift process mentioned above. The optimising parameter here is gas injection rate q_{GL}, the performance function (output) is oil production rate q_o and disturbance is wellhead pressure p_{wh}. Figure 3 shows the gas-lift performance curves for $d = d_1$ and $d = d_2$ with $d_1 > d_2$. When the disturbance changes from d_1 to d_2 abruptly, a steep rise in the oil rate occurs, i.e., the value of the performance function increases sharply. At the same time, if it happens that the dither signal $a \sin \omega t$ is positive, the extremum seeking scheme "thinks" that the small magnitude of increase in the optimising parameter could lead to large increase in the output. Therefore, the scheme increases the value of q_{GL} drastically making it deviate far away from where it should be. This transition is simulated in Figure 2. Note that the direction and the magnitude of the deviation in the optimising input q_{GL}

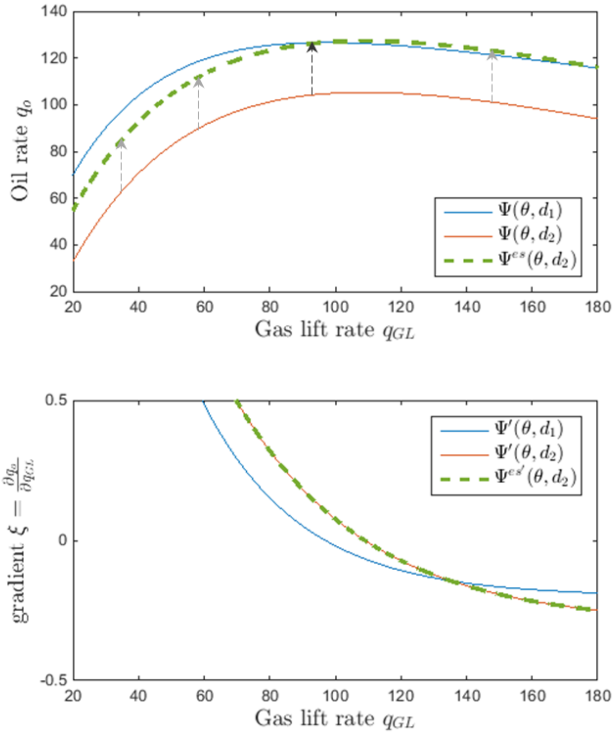

Fig. 3. input-output static map and the corresponding gradient for $d = d_1$ (blue curve), $d = d_2$ (red curve) and transition between d_1 to d_2 with the proposed method(green dotted curve)

depends on the phase of the sinusoidal perturbation at the time of the disturbance occurrence.

We do note that in the example the optimising parameter q_{GL} eventually converges to the new optimal value. This is because the disturbance stabilises at d_2 after the transition and the effect of the foregoing disturbance variation then dies out asymptotically due to the high-pass filtering. In other words, the classical extremum seeking scheme, by itself, is robust to slow-varying disturbance, see Krstic (2000). Although some may argue that the effect of the disturbance can be reduced by adjusting the tuning, this comes at the cost of affecting the convergence rate of the ES scheme. This would also require apriori knowledge of all expected disturbances inorder tune the ES scheme which may be overly conservative.

The transition issue due to disturbance variation described above is the main problem we will address in this work.

3. PROBLEM FORMULATION

Before introducing the proposed method, we first formulate the problem and set forth the assumptions that we consider in this paper. A process such as the gas-lift oil production may be represented by a general nonlinear dynamic system, which will be called the *plant model*:

$$\begin{aligned} \dot{x} &= f(x, u, d), \\ y &= g(x, d), \end{aligned} \tag{4}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control input, $d \in \mathbb{R}^l$ is the disturbance signal, and $y \in \mathbb{R}$ is the system output. For convenience of presentation, we simply assume enough smoothness of the functions $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^l \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}$.

In the gas-lift process, the input u includes the gas injection rate and some other controlled variables, the disturbance d may include wellhead pressure, injection gas pressure and reservoir productivity, and the output y is the produced oil rate.

Now suppose that we have a smooth state-feedback control law $u = \alpha(x, \theta)$, parameterized by θ; then the closed loop system becomes

$$\begin{aligned} \dot{x} &= f(x, \alpha(x, \theta), d), \\ y &= g(x, d). \end{aligned} \tag{5}$$

The parameter θ is called the *optimising parameter*; and a static map (defined below) relating the parameter θ to the system output y is the key to our goal of maximizing the output y by continuously updating the value of θ. In the gas-lift process, θ should be chosen as the gas injection rate.

The following assumption says that the system has an equilibrium point for each given pair of θ and d.

Assumption 1. There exists a smooth function $l : \mathbb{R} \to \mathbb{R}^n$ such that $f(x, \alpha(x, \theta), d) = 0$ if and only if $x = l(\theta, d)$.

When the system state is sitting at the equilibrium point $l(\theta, d)$, we have

$$y = g(l(\theta, d), d) := \Psi(\theta, d). \tag{6}$$

The static map Ψ is called the *output equilibrium map* and is the gas-lift performance curve in the gas-lift process. The following assumption says that for each given disturbance d, it is possible to manipulate the value of θ to achieve a maximum system output.

Assumption 2. There exists a smooth function $z(\cdot) : \mathbb{R}^l \to \mathbb{R}$ such that, for each d the output equilibrium map Ψ has a maximum at $\theta^* = z(d)$; and hence

$$\begin{aligned} \frac{\partial \Psi}{\partial \theta}(\theta^*, d) &= 0 \\ \frac{\partial^2 \Psi}{\partial \theta^2}(\theta^*, d) &< 0 \end{aligned} \tag{7}$$

A typical map Ψ for the gas-lift process is illustrated in Figure 3. To save space, we will use Ψ'_θ and Ψ''_θ to denote the first and second partial derivatives of the map Ψ w.r.t its first argument respectively.

An extremum seeking scheme here is meant to be a feedback mechanism of updating the parameter θ such that it will eventually reach θ^*, without the knowledge of θ^* and the static map Ψ.

4. PROPOSED METHOD

In section 2, we have seen that abruptly changing disturbance may lead to undesirable output when applying a classic extremum seeking scheme. To tackle this issue, we propose a simple yet effective extension to the classical extremum seeking scheme where the effect of the disturbance on the output y is approximated and mostly removed. For this purpose, in this work, we assume that the disturbance can be accurately measured or observed.

The proposed scheme is shown in Figure 4, in which one sees a block that maps the disturbance d to a quantity estimating the effect of the disturbance in the output y. This is called the *disturbance rejection model*. Unlike the classical extremum seeking scheme, in which the system output y is directly used as the feedback signal, the proposed scheme removes the estimated effect of the disturbance on the output and uses the resultant signal as the feedback.

We now elaborate how the disturbance rejection works in discrete time. By doing this, we are not only able to avoid issues in dealing with discontinuous disturbance but also directly addressing the implementation in digital computers. The subscripts of y, d, θ below denote discrete time steps.

In this process, we assume that the dynamics of the plant system is sufficiently fast such that $x = l(\theta, d)$. Then we can write the output y_1 using Taylor series expansion as

$$\begin{aligned} y_1 &\approx \Psi(\theta_1, d_1) \\ &= \Psi(\theta_0, d_0) + \Psi'_\theta(\theta_0, d_0)\Delta\theta_0 + \\ &\qquad \Psi'_d(\theta_0, d_0)\Delta d_0 + o(\Delta\theta_0, \Delta d_0) \\ &= \Psi(\theta_0, d_0) + \Delta y_0^\theta + \Delta y_0^d + o(\Delta\theta_0, \Delta d_0) \\ &\approx y_0 + \Delta y_0^\theta + \Delta y_0^d + o(\Delta\theta_0, \Delta d_0). \end{aligned} \tag{8}$$

where we have defined

$$\Delta y_0^\theta = \Psi'_\theta(\theta_0, d_0)\Delta\theta_0 \qquad \Delta y_0^d = \Psi'_d(\theta_0, d_0)\Delta d_0,$$
$$\Delta\theta_0 = \theta_1 - \theta_0 \qquad \Delta d_0 = d_1 - d_0,$$

with Ψ'_d being the partial derivative of Ψ with respect to its second argument d.

It is clear from (8) that the term Δy_0^d accounts for the main part of the unwanted effect of disturbance in the output y. Hence naturally we may reject most of the effect of disturbance by subtracting Δy_0^d from y_1.

From (8) we have

$$y_1 - \Delta y_0^d \approx y_0 + \Delta y_0^\theta + o(\Delta\theta_0, \Delta d_0) \tag{9}$$

Let us proceed one time step further. Similar to the derivation in (8), one obtains

$$y_2 \approx y_1 + \Delta y_1^\theta + \Delta y_1^d + o(\Delta\theta_1, \Delta d_1). \tag{10}$$

It follows that

$$\begin{aligned} y_2 - (\Delta y_1^d + \Delta y_0^d) &\approx \\ &(y_1 - \Delta y_0^d) + \Delta y_1^\theta + o(\Delta\theta_1, \Delta d_1). \end{aligned} \tag{11}$$

Following this patten of derivation, we have that if a new signal y^{es} is defined as

$$\begin{aligned} y_0^{es} &= y_0, \\ y_k^{es} &= y_k - \sum_{i=0}^{k-1} \Delta y_i^d, \quad \forall k = 1, 2, ..., \end{aligned} \tag{12}$$

then y^{es} (approximately) follows the recursive update: for $k = 0, 1, 2, ...,$

$$y_{k+1}^{es} \approx y_k^{es} + \Delta y_k^\theta + o(\Delta\theta_k, \Delta d_k). \tag{13}$$

Note that if we ignore the higher-order term in (13) then the change of y^{es} (almost) only contains the response of the actual output y to the dither signal which manipulates the optimising parameter θ. It is for this reason that we can use the signal y^{es} to replace the output y in the classical extremum seeking scheme as the feedback signal. In other words, in view of (12), we remove the accumulated effect of the change of the disturbance $\sum_{i=0}^{k-1} \Delta y_i^d$ from y_k. The direct consequence is that the actual gradient w.r.t. θ can be recovered even with the presence of abrupt changes in the disturbance d. In fact, the statement holds if the higher-order term $o(\Delta\theta_k, \Delta d_k)$ is negligible or can be mostly "filtered out" by the series of blocks of high-pass filter, dither correlation, and low-pass filter in the extremum seeking scheme.

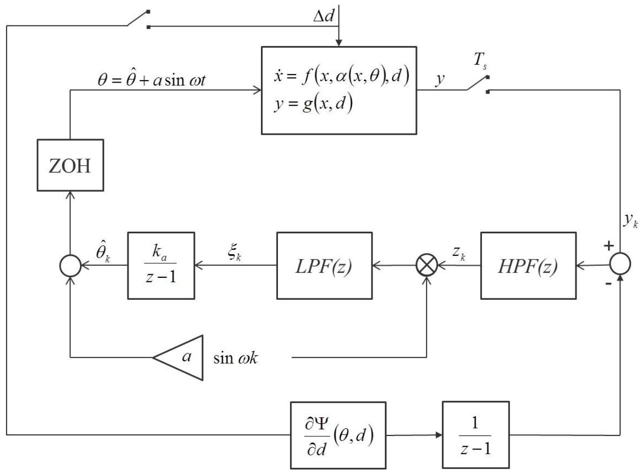

Fig. 4. Modified Extremum seeking scheme applied to system (4) with the disturbance rejection block

As mentioned, y^{es} is the output seen by the extremum seeking controller. Let us define the corresponding *output equilibrium map* seen by the extremum controller as

$$\Psi^{es}(\theta, d) := y^{es}(\theta, d) \tag{14}$$

In (12), by removing the effect of the change of the disturbance $\sum_{i=0}^{k-1} \Delta y_i^d$ from y_k, we essentially shift the output equilibrium map seen by the extremum seeking controller by the same quantity

$$\Psi^{es}(\theta_k, d_k) = \Psi(\theta_k, d_k) - \sum_{i=0}^{k-1} \Delta y_i^d, \quad \forall k = 1, 2, ..., \tag{15}$$

Taking the partial derivative of (15) w.r.t. θ,

$$\Psi_\theta^{es'}(\theta_k, d_k) = \Psi'_\theta(\theta_k, d_k), \quad \forall k = 1, 2, ..., \tag{16}$$

This shows that, by shifting the output equilibrium map, the optimal value $z(d_k)$ (see Assumption 2) remains unchanged. This is schematically represented in Figure 3, where the shifted map $\Psi^{es}(\theta, d_2)$ and the corresponding gradient are shown in green dotted curves.

According to the diagram of the extended extremum-seeking scheme (Figure 4), the disturbance rejection model is supposed to give out the value $\sum_{i=0}^{k-1} \Delta y_i^d$ at time k. This requires the knowledge about the partial derivative Ψ'_d, which we will address in Section 4.1.

Also note that the proposed extension of the extremum seeking scheme reduces to the classical one if the disturbance is constant as in this case we would have $y^{es} = y$.

4.1 On modelling the disturbance rejection block

As mentioned earlier, in principle, the disturbance rejection model shall output the accumulated values of Δy^d. However, this can be realized only when the partial derivative Ψ_d' is precisely known. It is clear that Ψ_d' describes how the (steady-state) output changes due to the change of the disturbance. Sometimes, this relationship can be modeled quite accurately by certain physical laws. When this cannot be easily achieved, we may identify the model using the measured disturbance and its response in the output y. Many system identification methods are available in literature for this purpose. Step response modelling is one of the simplest and easiest methods to identify such models and are common in most Model Predictive Control (MPC) applications, see Zhu (2001), Maciejowski (2002), Zhu et al. (1991) etc. Here, a step change is induced in the disturbance variable and the corresponding change in the performance function y is recorded. The empirical model is then identified using SISO, finite impulse response (FIR) or autoregressive (ARX) methods as described in Strand and Sagli (2003).

The simplest way would be to carry out the model identification when the optimising parameter θ is fixed. This would, however, suspend the extremum seeking process. Alternatively, we may identify the disturbance rejection model online while the extremum seeking scheme is ongoing using closed loop identification methods as described in Zhu and van den Bosch (2000). For this purpose, we can either design the disturbance to have a special form such that its response in the output y can be distinguished from that of the dither signal, or utilize the fact that, for relatively large disturbance d, it would be the case that $\Delta y_d >> \Delta y_\theta$ (since the dither signal has very small magnitude) and the change of y is then roughly Δy_d. More details about this online identification/adaptation will be presented in future work.

4.2 Disturbance rejection for constrained extremum seeking

As described earlier, the method proposed in this paper shifts the static map Ψ when an abrupt change in the disturbance occurs (15). If not properly addressed, this can cause problems when handling constraints. Consider a constrained extremum seeking problem of the form,

$$\theta^* = \arg\max y \tag{17}$$

s.t.

$$y < y_{max} \tag{18}$$

where constraints are imposed on the output y. The constraints are handled by converting it to an unconstrained problem as shown in Tan et al. (2013)

$$\theta^* = \arg\max J = [y - max(0, y - y_{max})] \tag{19}$$

In such cases, to preserve the constraint fulfillment, the effect of the disturbance on the performance function must be subtracted from the modified performance cost J instead of the measured performance function y,

$$\Delta\hat{y}_\theta = \Delta J - \Delta\hat{y}_d \tag{20}$$

By enforcing the constraints before shifting the static map, we can ensure that the constraint handling will not be affected by the disturbance rejection scheme.

5. SIMULATION RESULTS

The proposed method was tested using an application example of gas lift optimisation. Modelling a gas lifted well is not the focus of this paper and many gas lifted well models are available in literature, see Peixoto et al. (2015) and Aamo et al. (2005). The partial derivative Ψ_d' was identified using step response models.

In the first simulation, the wellhead pressure decreases from 30bar to 20bar ($\Delta d_k = -10$bar) at sampling instant $k = 200$, when the dither signal $a \sin \omega t$ is positive. This disturbance changes the gas lift performance and causes an abrupt change in the oil rate ($\Delta y_k^d = 21$). The increase in the oil rate when correlated with the sinusoidal perturbation causes the estimated gradient ξ to increase sharply. This causes an undesirable overshoot in the optimal gas lift rate set by the extremum seeking controller. This is shown in Figure 5 in blue.

The same scenario was then tested with the method proposed in this paper, where the model from the wellhead pressure to the oil rate is assumed to have almost no model error. In this case, when the change in the disturbance $\Delta d_k = -10$bar occurs at $k = 200$, the estimated effect of the disturbance on the performance function Δy_k^d is subtracted from the measured oil rate y_k. This is shown in Figure 5, where the pre-conditioned cost y^{es} has no abrupt changes. Therefore, the output y_k^{es} given to the extremum seeking scheme is shifted to cancel out (almost) entirely the abrupt change in the measured oil rate. Thus the preconditioned cost y^{es} only contains the sinusoidal changes caused by the input perturbation, and the optimal gas lift rate set by the extremum seeking scheme converges to the optimal point without causing any undesired deviation. This is shown in Figure 5 in red.

To test the method where the model from the disturbance to the performance function is not very accurate, the same scenario was simulated with a model error $\epsilon = \pm 20\%$. In the case with $\epsilon = 20\%$, when the change in the disturbance $\Delta d_k = -10$bar occurs at $k = 200$, the disturbance rejection model overestimates the effect of the disturbance on the performance function. The performance function y_k^{es} given to the extremum seeking scheme is overcorrected. The relatively small abrupt increase in the performance cost seen by the extremum seeking controller , causes the optimal gas lift rate to increase slightly before converging to the optimal point. This is shown in Figure 5 in yellow.

In the case with $\epsilon = -20\%$, the disturbance rejection model underestimates the effect of the disturbance, which causes the optimal input to undershoot slightly before converging to the optimal point. This is shown in Figure 5 in purple. Although, due to model error, the effect of the disturbance is not entirely nullified, most of the abrupt change in the performance function is compensated and the undesired oscillations in the gas lift rate set by the extremum seeking controller have significantly reduced.

In an other scenario, shown in Figure 6, the same disturbance ($\Delta d_k = -10$bar) occurs at $k = 150$, when the dither signal $a \sin \omega t$ is negative. In this scenario, the increase in the oil rate when correlated with the input perturbation causes the estimated gradient ξ to decrease rapidly contrary to the previous case. This causes an

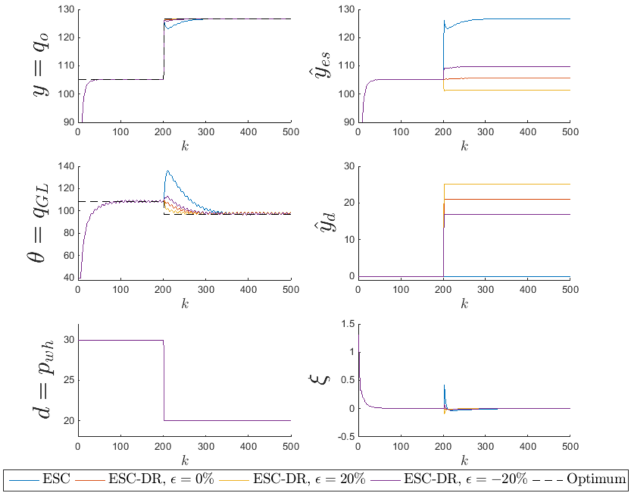

Fig. 5. simulation results 1. The classical extremum seeking scheme is shown in blue, the proposed method is shown in red, the proposed method with 20% model error in yellow and -20% model error in purple.

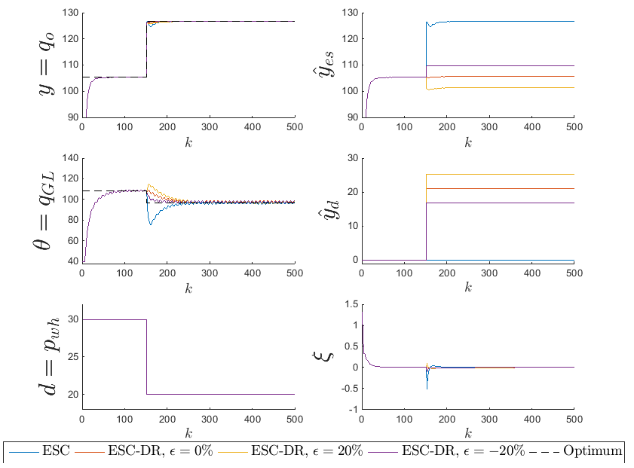

Fig. 6. Simulation results 2 where the same disturbance causes an undesired oscillation in the opposite direction

undesirable undershoot in the optimal gas lift rate set by the extremum seeking controller. The proposed method was then tested when the disturbance rejection model is near accurate ($\epsilon \approx 0\%$), overestimates ($\epsilon = 20\%$) and underestimates ($\epsilon = -20\%$). As seen from Figure 6, in this case the proposed method also allows one to avoid the large transient deviations observed in classic ESC.

6. CONCLUSION

In this paper, we presented a simple extension to the classical extremum seeking scheme to improve its robustness to abrupt changes in the disturbance. The preliminary results of the proposed method tested on the gas-lift system example show the improvement in the performance of the extremum seeking controller. A brief overview of model identification for the disturbance rejection block was discussed. The effect of the model error and possible directions to improve the robustness were also discussed.

Although we understand that the proposed method might be restrictive due to the requirement of the disturbance model to be known, the methods described in section 4.1 are commonly used in many MPC applications. However, work on developing a method that removes this restriction is ongoing and is for future work.

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