Tuning advisor methodology for model predictive controllers

Winston Garcia-Gabin * **Michael Lundh** *

* *ABB, Corporate Research, SE72178, Västerås, Sweden*
e-mail:(winston.garcia-gabin,michael.lundh)@se.abb.com.

Abstract: In this paper is developed a methodology for monitoring the tuning of model predictive controllers (MPC). The importance of the proposed methodology is that it can help the tuning process during the design phase through simulations or monitoring how the selected tuning parameters affect the on-line PVs and MVs behavior. The index set is based on the cost function of the MPC and the concepts used are intuitive for the operators. The performance of the proposed method is illustrated through simulations with an MPC tuned in different conditions.

Keywords: Model predictive control, Performance monitoring.

1. INTRODUCTION

In the literature, several methodologies have been proposed for performance monitoring of control loops. Performance monitoring indexes for MPC have a special interest, because MPC is an advanced process control technique used frequently in the industry. Zhang and Henson (1999) proposed a model-based performance measure for multivariable MPC, based on the comparison between expected and actual process performance. The actual performance is determined from closed-loop data and the expected performance is computed using the process model and it is employed as a benchmark for comparison. The performance measure is based on a quadratic cost function that penalizes the tracking error over a moving horizon of past information. The actual and expected performances are defined as

$$J_{act}(k) = \sum_{j=1}^{N_p} e^T(k+j-N_p) Q_e e^(k+j-N_p) \qquad (1)$$

$$J_{exp}(k) = \sum_{j=1}^{N_p} e^{*T}(k+j-N_p) Q_e e^*(k+j-N_p) \qquad (2)$$

Where e are the errors between the measured process outputs and their set points over a moving horizon N_p of measured past information; Q_e is the weight matrix of the errors, and e^* are the errors between the prediction model outputs and their set points over a moving horizon N_p.

The performance index is computed as the ratio of the expected performance and actual performance indices:

$$I_{mpc}(k) = \frac{J_{exp}(k)}{J_{act}(k)} \qquad (3)$$

Zhang and Henson (1999) assumed that $I_{mpc}(k)$ is generated by an autoregressive moving average model. The estimated noise variance of the model is used to compute 95 % confidence limits on $\Delta I_m pc(k)$.

Patwardhan et al. (1998); Patwardhan (1999) proposed a historical index performance based on comparison of the achieved cost functions of the MPC in a good region with the current performance. This approach requires a priori knowledge of a case where the performance was good during a certain time period of time according to some expert assessment. The historical measure of the performance is given by

$$I(k) = \frac{J_{hist}}{J_{act}(k)} \qquad (4)$$

J_{hist} is the cost function value when the controller is known to deliver good performance in a particular conditions through some performance metric or knowledge such as operators experience (Patwardhan, 1999).The achieved cost function value $J_{ach}(k)$ is obtained at the sample k using the measured values of outputs and inputs. The index value is calculated N_p sampling instants later. The cost function is defined as

$$J(k) = \sum_{J=N_1}^{N_p} e^T(k+j) Q_e e(k+j) + \sum_{j=1}^{N_u} \Delta u^T(k+j-1) Q_u \Delta u(k+j-1) \qquad (5)$$

Where e are the errors between set points and the measured outputs; Δu are the control signal increments; Q_e is the weight matrix of the errors and Q_u is the weight matrix of the control increments; N_p is the prediction horizon and N_u is the control horizon.

An alternative approach proposed also by Patwardhan et al. (1998) is to evaluate the MPC performance using an index with the actual design cost function of the controller and then compare the achieved performance using measured data (Patwardhan, 1999; Shah et al., 2002). The index is defined below

$$I_{mpc}(k) = \frac{J_{design}(k)}{J_{actual}(k)} \tag{6}$$

The index uses the previous MPC cost function (1). J_{design} is the cost function value at the sample k using the future estimated control errors and the optimal control moves based on information at sample k. $J_{actual}(k)$ is the cost function value at the sample k calculated at $k + N_p$ using the measured values of outputs and inputs.

An additional aspect discussed in this paper is a tuning advisor for MPC. For fulfilling this objective an index set is developed in order to represent the prioritization of the controller variables and manipulate variables of the MPC. The methodology can be used for two different purposes. First, the indexes can measure the tuning behavior on-line in some particular situation for example set-point changes or disturbances rejection using real data. Second, the indexes can assist the selection of tuning parameters through simulations which is an off-line procedure. In the first case the operator has an indication about how the controller is behaving. On the other hand in the second purpose, the indexes quantify how the changes in the tuning parameters modify the tuning behavior.

2. INDEXES FOR TUNING MONITORING

The cost function used for defining the indexes must be the same used for the MPC. In this case it is defined as follows,

$$J(k) = \sum_{J=N_1}^{N_p} e^T(k+j)Q_e e(k+j) + \sum_{j=1}^{N_u} \Delta u^T(k+j-1)Q_u \Delta u(k+j-1) \tag{7}$$

$$Q_e = Diag[q_{e_{1,1}}, q_{e_{2,2}}, \cdots\ q_{e_{PV,PV}}] \tag{8}$$

$$Q_u = Diag[q_{u_{1,1}}, q_{u_{2,2}}, \cdots\ q_{u_{PV,PV}}] \tag{9}$$

where e are the errors between the measured values of the outputs and the set-points. Δu are the control signal increments applied to the process each sample time; N_p is the prediction horizon; N_1 is the initial prediction horizon and N_u is the control horizon. The weight matrixes Q_e and Q_u, which weight errors and Δu movements, are defined as diagonal matrixes in (2) and (2).

The cost function (2) can be rewritten as the contribution of two terms:

$$J(k) = J_e(k) + J_u(k) \tag{10}$$

The indexes use data applied to the process, this implies that for calculating the index at sample k, it is necessary to wait until the sample $k + N_p$, in this way, values of $J(k), J_e(k), J_u(k)$ are obtained every sample k. Then, indexes are calculated using the median of the in a time window (e.g. one day, one week, one moth) as it is shown below

$$Index\ \frac{J_e}{J} = M\left(\frac{J_e}{J}\right) \tag{11}$$

$$Index\ \frac{J_u}{J} = M\left(\frac{J_u}{J}\right) \tag{12}$$

where $M(\cdot)$ is the median operator applied to the time series. The indexes measure the relative contribution of each term $(J(k), J_e(k), J_u(k))$ in the total value of the cost function $(J(k))$. Additionally, the following indexes are defined for each PV and MV

$$Index\ \frac{J_{ePV}}{J_e} = M\left(\frac{\sum_{J=N_1}^{Np} e_{PV}(k+j)q_{e_{PV,PV}}e_{PV}(k+j)}{J_e}\right) \tag{13}$$

$$Index\ \frac{J_{uMV}}{J_u} = M\left(\frac{\sum_{J=1}^{Nu} \Delta u_{MV}(k+j)q_{u_{MV,MV}}\Delta u_{MV}(k+j)}{J_u}\right) \tag{14}$$

They indicate respectively the individual contribution of each PV and MV to the terms $J_e(k), J_u(k)$. An important characteristic is that the indexes have a value between zero and one.

3. INDEX EVALUATION

The indexes for monitoring the MPC tuning were evaluated as follows. First, it was designed a scenario. It consisted to apply a pulse train in the set-point of each PV of the process. Second, simulations of 400 samples long were run with different tuning parameters in the MPC. Finally the results are collected in tables and some figures are used to illustrate the results. The methodology is illustrated using a distillation column model with 22 states, which is a process with 3 inputs and 6 outputs. In this study, it is considered a square system with 3 outputs and 3 inputs in closed-loop configuration and the process model has white noise in all the process variables and manipulated variables.

3.1 Control Weight

In this experiment the control weight of the MPC was changed from a value 10 times higher than the original tuning Q_u to a value of 0.1 times the original tuning Q_u. The elements in the diagonal of tuning Q_u were kept constant.

Figure 1 plots the PVs and MVs when pulse trains are applied as set-point for each PV. PVs and MVs a named 1,2,3 from top to bottom, thus PV_1 and MV_1 are blue lines, PV_2 and MV_2 are green lines and PV_1 and MV_1 are red lines. In Fig 1 the control weight is 10 times higher than the original control weight. Figure 1 shows that MVs have soft movements due to the high penalization in control actions, as a consequence PVs have big errors compared to the set-points.

The index J/J_e and J/J_u are collected in Table 1. The first row shows that Index $J_e/J = 0.9999$ and Index $J_u/J = 0.0001$ which means that control movements have small contributions on the total values of the cost function

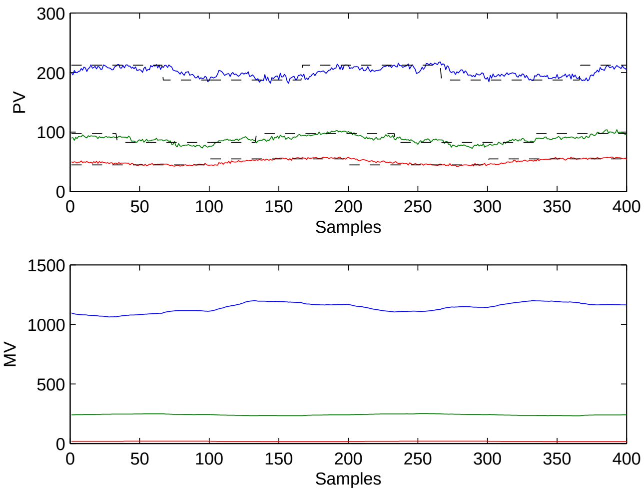

Fig. 1. Control weight 10 times higher than the original. PV_1 (blue), PV_2 (red) and PV_3 (green).

Tuning value	Index $\frac{J_e}{J}$	Index $\frac{J_u}{J}$
$Q_u * 10$	0.9999	0.0001
$Q_u * 5$	0.9982	0.0018
$Q_u * 2$	0.9631	0.0369
$Q_u * 1$	0.6163	0.3837
$Q_u * 0.5$	0.1392	0.8608
$Q_u * 0.1$	0.0009	0.9991

Table 1. Index J_e/J and Index J_u/J for changes in the control weight of the MPC.

and the errors are large. As can be seen in Table 1 when the value in the control weight is reduced the controller behaves in the opposite way. Thus when the control weight is reduce by a factor of 0.1 the controller is more aggressive, this situation is reflected in the indexes $J_e/J = 0.0009$ and $J_u/J = 0.9991$, which means that control movements have large contributions on the total values of the cost function and the errors are small.

The indexes related to PVs are collected in Table 2. The indexes show that the controller is tuning in order to give more importance to PV_3, then PV_2 and finally the less important is PV_1. Note that PV_1 contribute more on the J_e than PV_3. The MV indexes are presented in Table 3. They indicate that MV_1 has more activity than MV_2 and MV_3, almost all the control movements are focused on MV_1.

Figures 1 to 5 show the changes in the controller performance when control weight Q_u of the MPC was reduced and the indexes values are collected on Tables 1 to 3. It can be seen how the controller behavior is clearly represented through the index values between 0 to 1. Sluggish MPC tuning (Fig 1 and Fig 2) and aggressive MPC tuning (Fig 4 and Fig 5) are consistent with the index values on the Table 1. Table 1 shows how the indexes change from the values: Index $J_e/J = 0.9999$ and Index $J_u/J = 0.0001$ when the controller has a sluggish tuning to the index values $J_e/J = 0.0002$ and $J_u/J = 0.9998$ when the controller has an aggressive tuning. Note than the PV indexes and MV indexes have similar values because the parameters

Tuning value	Index $\frac{J_{e_{PV_1}}}{J_e}$	Index $\frac{J_{e_{PV_2}}}{J_e}$	Index $\frac{J_{e_{PV_3}}}{J_e}$
$Q_u * 10$	0.8571	0.1248	0.0181
$Q_u * 5$	0.8867	0.1022	0.0111
$Q_u * 2$	0.8610	0.1315	0.0076
$Q_u * 1$	0.8613	0.1327	0.0061
$Q_u * 0.5$	0.8536	0.1405	0.0059
$Q_u * 0.1$	0.8360	0.1588	0.0053

Table 2. $J_{e_{PV}}/J_e$ indexes for changes in the control weight of the MPC.

Tuning value	Index $\frac{J_{u_{MV_1}}}{J_u}$	Index $\frac{J_{u_{MV_2}}}{J_u}$	Index $\frac{J_{u_{MV_3}}}{J_u}$
$Q_u * 10$	0.9978	0.0022	0.0000
$Q_u * 5$	0.9985	0.0015	0.0000
$Q_u * 2$	0.9998	0.0022	0.0000
$Q_u * 1$	0.9999	0.0001	0.0000
$Q_u * 0.5$	0.9999	0.0001	0.0000
$Q_u * 0.1$	0.9993	0.0007	0.0000

Table 3. $J_{u_{MV}}/J_u$ indexes for changes in the control weight of the MPC.

in the diagonal of Q_u were kept constant. It can be seen how the indexes reflex the controller behavior. As it is expected, the MPC responses fast with an aggressive tuning and sluggish when the control movements has strong penalization.

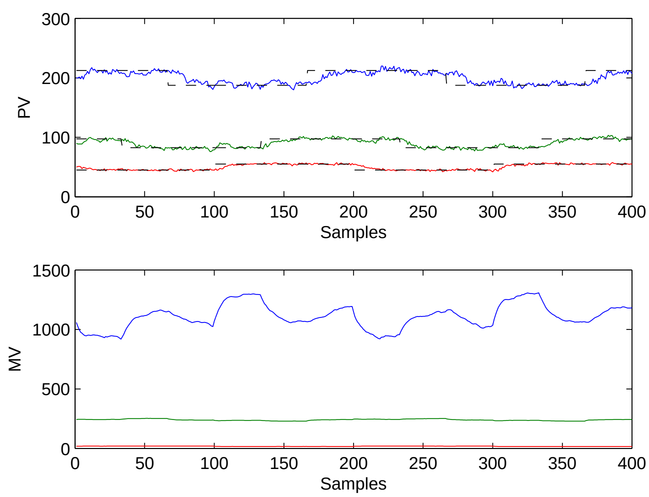

Fig. 2. Control weight 5 times higher than the original. PV_1 (blue), PV_2 (red) and PV_3 (green).

3.2 Tuning for prioritizing one PV

In this example three tuning sets are compared. The goal of the controller for each case is to minimize the errors of one PV to the detriment of the others PVs. For each case, the MPC was tuned manually changing the parameters of Q_e and Q_u in order to achieve the control objective. Figure 6 shows the PVs and MVs for cases where the controller was tuned for minimizing PV_1 (blue line), PV_2 (red line) and PV_3 (green line). The index values for each case are collected in Tables 4 to 6.

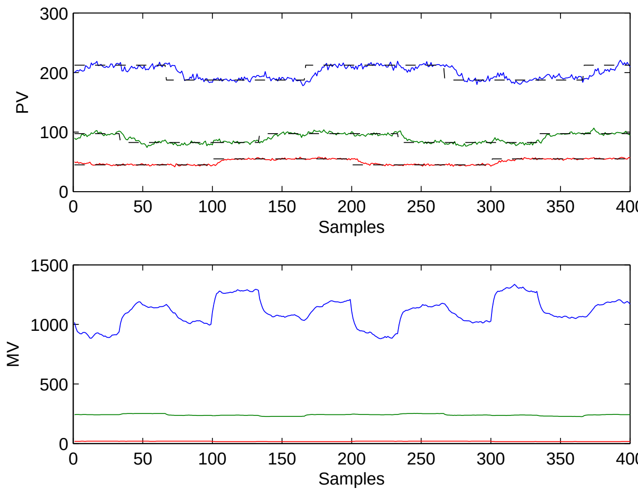

Fig. 3. MPC with the original control weight. PV_1 (blue), PV_2 (red) and PV_3 (green).

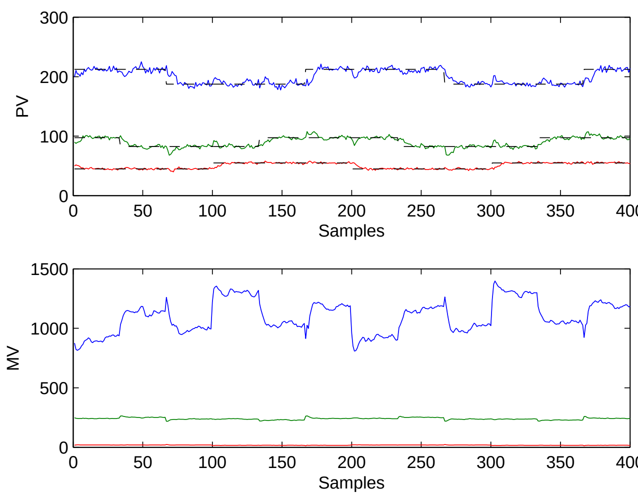

Fig. 4. Control weight 0.5 lower than the original. PV_1 (blue), PV_2 (red) and PV_3 (green).

Tuning for min	Index $\frac{J_e}{J}$	Index $\frac{J_u}{J}$
PV_1	0.1768	0.8232
PV_2	0.0273	0.9727
PV_3	0.1200	0.9988

Table 4. Index J_e/J and Index J_u/J for tuning sets with the objective of minimizing PVs

Table 4 collects index J_e/J and index J_u/J in all the cases the MPC has an aggressive tuning, which is reflected with small values of the J_e/J indexes and large values of J_u/J indexes. As can be seen in Table 5, PV indexes show values of zero when the controller was tuned for minimizing the correspondent PV. The indexes show clearly the prioritization of the PVs in the MPC tuning.

Table 6 collects the MV indexes. The indexes show that the control action is mainly driven by MV_1, in all the cases the index $J_{u_{MV_1}}/J_u$ are dominant.

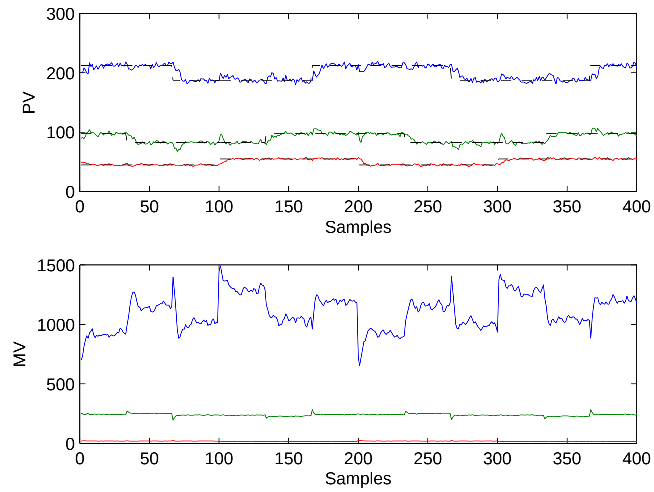

Fig. 5. Control weight 0.5 lower than the original. PV_1 (blue), PV_2 (red) and PV_3 (green).

Tuning for min	Index $\frac{J_{e_{PV_1}}}{J_e}$	Index $\frac{J_{e_{PV_2}}}{J_e}$	Index $\frac{J_{e_{PV_3}}}{J_e}$
PV_1	0.0000	0.5458	0.4542
PV_2	0.7132	0.0000	0.2868
PV_3	0.1200	0.9988	0.0000

Table 5. $J_{e_{PV}}/J_e$ indexes for tuning sets with the objective of minimizing PVs.

Tuning for min	Index $\frac{J_{u_{MV_1}}}{J_u}$	Index $\frac{J_{u_{MV_2}}}{J_u}$	Index $\frac{J_{u_{MV_3}}}{J_u}$
MV_1	0.7408	0.0006	0.2586
MV_2	0.8214	0.1200	0.0586
MV_3	0.9795	0.0035	0.0170

Table 6. $J_{u_{MV}}/J_u$ indexes for tuning sets with the objective of minimizing PVs.

3.3 Discussion

It was presented a tool for monitoring how the MPC is tuned. The goal of the tool is to assist the person who is tuning the MPC. Indexes J_e/J and J_u/J reflex how the balance between the errors and control movements is, and indicate how much aggressive the MPC is tuned. $J_{e_{PV}}/J_e$ indexes are defined for each PV. They show the prioritization of the errors between all PVs. Large values of this index indicate that this PV has a sluggish control (the loop takes too long to get to its set point after a disturbance or set point change) compared to the others PVs. Also, $J_{u_{MV}}/J_e$ indexes are defined for each MV. High values in one these indexes indicate more movements on the control signal compared to the others MVs. The indexes are normalized between zero and one, which facilitates index interpretation when the operator is tuning the MPC. The methodology is easier than visual inspection of the PVs and MVs.

An additional value of these indexes is that they can be used for both, the tuning process during the design phase through simulations and the monitoring procedure. In the last case, the indexes verify that the MPC in closed-loop

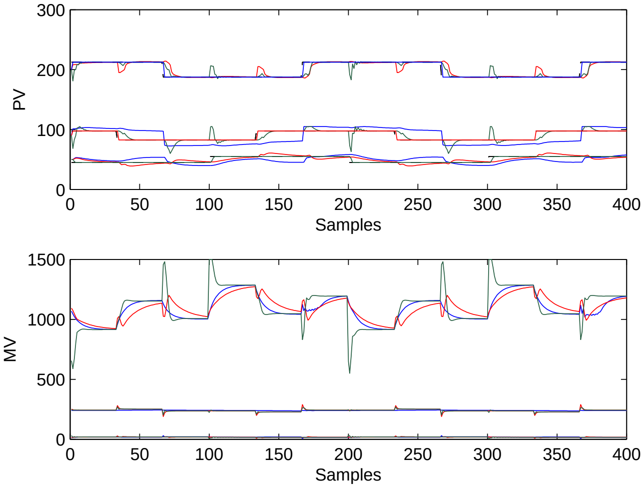

Fig. 6. Tuning for minimizing: PV_1 (blue), PV_2 (red) and PV_3 (green).

satisfies the PVs and MVs prioritization with the selected tuning parameters.

4. CONCLUSION

A tuning advisor methodology for MPC was proposed. The tuning monitoring indexes indicate how the MPC is tuned in a range between zero to one. Using these indexes the operator has numerical values that show the prioritization of PVs, also show which MV has more control movement relative to the rest of MVs. The indexes show also whether the controller has a sluggish or aggressive tuning through comparing the contributions of the errors and control movements to the total value of the MPC cost function. The results show how the index are useful for reflex how the MPC is tuned, avoiding the difficult task of visual comparison of the time series of MVs and PVs, particulary in the case of multivariable MPCs.

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