# Extended Abstract
# Design of Measurement Noise Filters for PID Control

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## 1. INTRODUCTION

Control design requires a model of the process and its environment, as well as a collection of requirements such as robustness and performance. Robustness shows the sensitivity of the closed loop to process changes. Performance involves specifications with respect to load disturbance response as well as limitation of the control actions generated by measurement noise. Thus, the final design requires a compromise between the different requirements.

Most design methods focus on the attenuation of load disturbances and do not considere measurement noise. In this extended abstract the dicussion will focus on trade-offs between load disturbance attenuation, robustness and reduction of control actions due to measurement noise.

## 2. MODELING AND FILTER DESIGN

The process $P(s)$ is approximated with the standard FOTD system

$$P(s) = K_p \frac{1}{1+sT} e^{-sL}, \tag{1}$$

where $K_p$, $L$, and $T$ are the static gain, the apparent time delay, and the apparent time constant. The relative time delay $\tau = L/(L+T)$ is used to characterize process dynamics. The parameters $K_p$, $L$, and $T$ can be determined from a step response experiment.

The PI and PID controllers have the transfer functions

$$C_{PI}(s) = k_p + \frac{k_i}{s}, \quad C_{PID}(s) = k_p + \frac{k_i}{s} + k_d s, \tag{2}$$

where $k_p$, $k_i$, and $k_d$ are the controller parameters.

Measurement noise is reduced by a second order filter with the transfer function

$$G_f(s) = \frac{1}{1 + sT_f + s^2 T_f^2/2}, \tag{3}$$

where $T_f$ is the filter time constant. A second order filter is used to ensure roll-off in the PID controller.

The combinations of the controllers and the filter transfer functions are

$$C(s) = C_{PI}(s)G_f(s), \qquad C(s) = C_{PID}(s)G_f(s), \tag{4}$$

Using this representation ideal controllers can be designed for the augmented plant $P(s)G_f(s)$.

Control performance can be characterized by the integrated absolute error

$$IAE = \int_0^\infty |e(t)| dt, \tag{5}$$

where $e$ is the control error due to a unit step load disturbance. Here, it is assumed that the disturbance enters at the process input.

Robustness to process uncertainty can be captured by the maximum sensitivities $M_s$ and $M_t$.

It is important that the control actions generated by measurement noise are not too large. This can be observed in the transfer function from measurement noise to controller output of the closed loop system

$$G_{un}(s) = \frac{C(s)}{1 + G_l(s)} = C(s)S(s), \tag{6}$$

where $G_l(s) = P(s)C(s)$ is the loop transfer function, and $S(s)$ is the sensitivity function. In order to characterize the effects of measurement noise, the control bandwidth $\omega_{cb}$ is considered. This quantity represents the smallest frequency where the gain of $G_{un}$ is less than $\beta$, where $\beta$ is typically in the range 0.01–0.7. Considering that $S(s)$ in (6) approaches 1 for frequencies higher than the gain crossover frequency $\omega_{gc}$, the control bandwidth for PI and PID control can be approximated by

$$\omega_{cb}^{PI} \approx \frac{1}{T_f}\sqrt{\frac{2k_p}{\beta}} \qquad \omega_{cb}^{PID} \approx \frac{2k_d}{\beta T_f^2} \tag{7}$$

The largest gain $M_{un}$ of the transfer $G_{un}$ is another way to characterize the effect of measurement noise

$$M_{un} = \max_\omega |G_{un}(i\omega)|, \tag{8}$$

Adding a filter reduces the effects of measurement noise, but it also reduces robustness and deteriorate load disturbance responses. A compromise is to choose the filter so that the impact on robustness and performance is not too large. The design suggested here is formulated as a trade-off between performance (IAE), robustness ($M_s$, $M_t$) and filtering of measurement noise ($\omega_{cb}$, $M_{un}$), where the controller parameters and the filter time constant are calculated using an iterative procedure.

The filter time constant is chosen as

$$T_f = \frac{\alpha}{\omega_{gc}}, \tag{9}$$

where $\omega_{gc}$ is the gain crossover frequency. Controllers with this filter constant will be designed for different values of $\alpha$, which is chosen as a trade-off between performance and robustness. For a given value of $\alpha$ the design procedure is

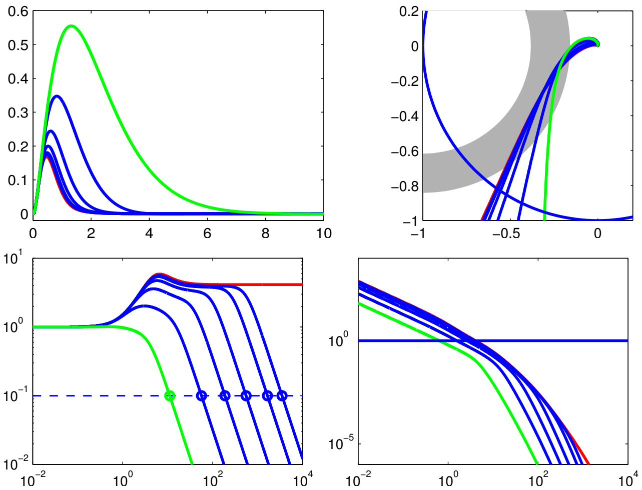


Figure 1. Performance and robustness for the process $P(s)$ with PI control. Load disturbance responses are shown on the top left and the Nyquist curve for $G_l$ on the top right. The bottom left plot shows the gain curve of $G_{un}$ and the bottom right the gain curve of $G_l$. The filter time constants are $T_f = \alpha/\omega_{gc}$ with $\alpha = 0$ (red), 0.01, 0.02, 0.05, 0.1, 0.15 and 0.2 (green).

- Optimize performance (IAE) for the process $P$ subject to robustness constraints $(M_s, M_t)$.
- Choose the filter time constant $T_f = \alpha/\omega_{gc}$.
- Repeat the procedure with $P$ replaced by $PG_f$ until convergence.

## 3. EXAMPLE

To illustrate the approach we consider the system

$$P(s) = \frac{1}{(s+1)(0.1s+1)(0.01s+1)(0.001s+1)} \tag{10}$$

The FOTD approximation of $P(s)$ gives $K_p = 1, T = 1.04$, $L = 0.08$, and $\tau = 0.07$, which shows the dominant lag dynamics of the process. Design of a PI controller using AMIGO [Åström and Hägglund (2005)] gives $k_p = 4.13$ and $k_i = 7.67$. These values are given in Table 1, which shows the influence of the filter time constant on the process and controller parameters as well as in the performance (IAE) and noise attenuation.

Figure 1 shows the effects on performance and robustness of the filter time constant for different $\alpha$ values. The top left plot shows the process output response to a unit step load disturbance. The top right shows the Nyquist plot of the loop transfer function and the region where the sensitivity is in the range $1.2 \leq M_s \leq 1.6$. The bottom left figure shows the magnitude $G_{un}$, the circles indicate the noise bandwidth $\omega_{cb}$ for $\beta = 0.1$. The lower right figure shows the gain curve of loop transfer function $G_l$.

The load disturbance response increases with increasing filtering, see Table 1. The maximum sensitivity remains essentially constant but the gain margin decreases with increased filtering. The noise attenuation decreases significantly with filtering, this is reflected by the decrements of $M_{un}$ and the ratio $\omega_{cb}/\omega_{gc}$. The gain crossover frequency decreases marginally with increased filtering. The process parameter $L$ and the controller parameters given by the iterative approach change significantly with $T_f$.

Table 1. Parameter dependence on the filter time constant for $P(s)$ using PI control

| $\alpha$ | $\tau$ | $L$ | $T$ | $k_p$ | $k_i$ | $T_f$ | $IAE$ | $\frac{\omega_{cb}}{\omega_{gc}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.07 | 0.08 | 1.04 | 4.13 | 7.67 | 0 | 0.13 | $\infty$ |
| 0.01 | 0.07 | 0.08 | 1.04 | 3.95 | 7.21 | 0.003 | 0.14 | 888.7 |
| 0.02 | 0.07 | 0.08 | 1.04 | 3.79 | 6.81 | 0.005 | 0.15 | 435.7 |
| 0.05 | 0.08 | 0.09 | 1.04 | 3.33 | 5.65 | 0.015 | 0.18 | 163.2 |
| 0.1 | 0.10 | 0.11 | 1.04 | 2.53 | 3.87 | 0.038 | 0.26 | 71.2 |
| 0.15 | 0.15 | 0.18 | 1.03 | 1.45 | 1.89 | 0.095 | 0.53 | 35.9 |
| 0.2 | 0.26 | 0.37 | 1.05 | 0.60 | 0.66 | 0.312 | 1.53 | 17.4 |

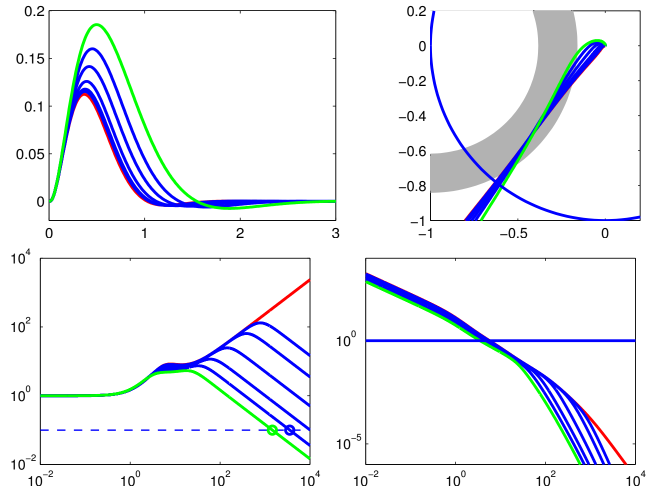


Figure 2. Performance and robustness for the process $P(s)$ with PID control. Load disturbance responses are shown on the top left and the Nyquist curve of $G_l$ on the top right. The bottom left plot shows the gain curve of $G_{un}$ and on the bottom right the gain curve of $G_l$. The filter time constants are calculated for $\alpha = 0$ (red), 0.01, 0.02, 0.05, 0.1, 0.15 and 0.2 (green).

Design of a PID controller with AMIGO gives $k_p = 6.44$, $k_i = 17.83$, and $k_d = 0.24$. Table 2 shows the dependence on the filter time constant of different parameters. Figure 2 shows the response to load disturbance, the Nyquist plot of the loop transfer function $G_l$, the gain curve of the noise transfer function $G_{un}$, as well as the gain curve of $G_l$.

Table 2 and Figure 2 show that for PID control, filtering has a significant effect on $M_{un}$, IAE and $\omega_{cb}/\omega_{gc}$. The gain crossover frequency $\omega_{gc}$ also decreases with increased filtering. Notice that the proportional and integral gains and $M_{un}$ are significantly higher for PID control.

Table 2. Parameter dependence on the filter time constant for $P(s)$ using PID control

| $\alpha$ | $\tau$ | $L$ | $T$ | $k_p$ | $k_i$ | $k_d$ | $T_f$ | $IAE$ | $\frac{\omega_{cb}}{\omega_{gc}} 10^3$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.07 | 0.08 | 1.04 | 6.4 | 17.8 | 0.24 | 0 | 0.059 | $\infty$ |
| 0.01 | 0.07 | 0.08 | 1.04 | 6.3 | 17.1 | 0.24 | 0.002 | 0.062 | 262.6 |
| 0.02 | 0.07 | 0.08 | 1.04 | 6.1 | 16.5 | 0.24 | 0.004 | 0.064 | 64.3 |
| 0.05 | 0.08 | 0.09 | 1.04 | 5.7 | 14.7 | 0.24 | 0.010 | 0.072 | 9.7 |
| 0.1 | 0.08 | 0.10 | 1.04 | 5.0 | 11.9 | 0.24 | 0.022 | 0.089 | 2.2 |
| 0.15 | 0.10 | 0.11 | 1.04 | 4.4 | 9.6 | 0.24 | 0.037 | 0.111 | 0.9 |
| 0.2 | 0.12 | 0.14 | 1.03 | 3.7 | 7.3 | 0.24 | 0.057 | 0.146 | 0.4 |

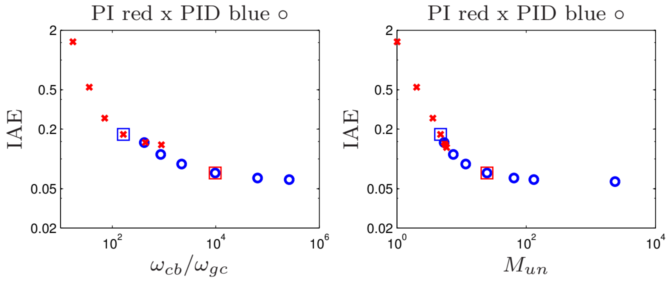


Figure 3. On the left performance IAE as a function of $\omega_{cb}/\omega_{gc}$ and on the right performance IAE as a function of $M_{un}$ for PI (x) and PID (∘) control of the process $P(s)$. The filter time constant is calculated for $\alpha = 0$, 0.01, 0.02, 0.05 (square), 0.1, 0.15 and 0.2.

Figure 3, together with Table 1 and Table 2, show the trade-offs between load disturbance attenuation and measurement noise injection for PI and PID control. The load disturbance response deteriorates with filtering, the range is larger for PI control than for PID control, hence it seems advisable to use smaller values of $\alpha$ for PI control. The figure also shows that filtering has a significant effect on the magnitude of the unwanted control actions created by measurement noise. Both $M_{un}$ and $\omega_{cb}/\omega_{gc}$ decrease rapidly with filtering. According to these results, reasonable values of $\alpha$ are in the range of 0.01 to 0.05.

## 4. DESIGN RULES

The iterative design is based on the FOTD model and the dynamics of the filter is accounted for by changing the apparent delay $L$ and the apparent time constant $T$.

Figures 3 shows that filtering has a significant effect on the trade-off between performance and noise attenuation. The trade-off is governed by the design parameter $\alpha$. A small value of $\alpha$ emphasizes performance and larger values emphasize noise rejection. The choice is problem dependent, but a $\alpha = 0.05$ is a reasonable nominal value.

For the example the filter time constant is related to the gain crossover frequency, however for design rules it is useful to relate the filter time constant to the controller parameters. The example as well as others [Romero and Hägglund and Åström (2013)] not included here for space reasons, show that the filter time constant depends on the process. Figure 4 which has been obtained using FOTD models illustrates this dependency, it shows the ratios $T_f/T_i^0$ and $T_f/T_d^0$ as a function of the relative time delay $\tau$ for different values of the design parameter $\alpha$. The parameters $T_i^0$ and $T_d^0$ are the integral time and the derivative time computed for the controller without filtering.

Simple parameter fits in Figure 4 give the following approximate rules for PI and PID control

$$T_f = 6\alpha\tau T_i^0 \ \text{(PI)} \qquad T_f = 4.5\alpha T_d^0 \ \text{(PID)} \qquad (11)$$

The rules hold for $\alpha < 0.1$. The rule for PI control is valid for all $\tau$ but the rule for PID control only holds for lag-dominated and balanced systems. Derivative action is however of little value for delay-dominated systems. A reasonable standard value is $\alpha = 0.05$.

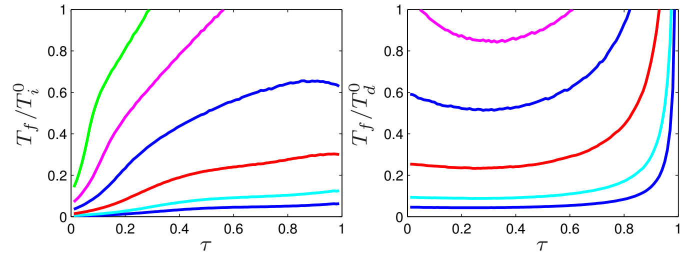


Figure 4. Filter time constant as a function of $\tau$ for PI (left) and PID (right) control. The filter time constant is given by (9) with $\alpha = 0.01$ (bottom blue), 0.02, 0.05, 0.1, 0.15 and 0.2 (green).

## 5. SUMMARY

A drawback of feedback is that measurement noise is fed into the system, but the undesired control actions generated by the noise can be reduced using filtering. Filtering introduces additional dynamics which have to be considered in the control design. Insight into the choice of filtering has been obtained by investigating design of PI and PID controllers as a trade-off between performance and robustness.

The design problem has been solved iteratively. Process dynamics has been approximated by FOTD models and controller parameters have been determined using the AMIGO rule which give sensitivities less than 1.4. The filter has been chosen as a second order Butterworth filter which is characterized by one parameter, the filter time constant $T_f$. The iterative process starts with the nominal process dynamics $P$. The crossover frequency $\omega_{gc}$ has been determined and the filter time constant has been chosen as $\alpha/\omega_{gc}$. A new process model has then been determined by fitting an FOTD model to $PG_f$ and the process has been repeated until convergence.

The results have shown that the control actions generated by measurement noise can be reduced significantly by filtering with only a moderate decrease of performance while maintaining robustness.

Simple design rules for choosing the filter time constant have also been developed (11).

The analysis has been made based on a particular design method AMIGO and the matching method of fitting FOTD models. It would be interesting to investigate if the design rules are similar if other methods for PID design are used.

## REFERENCES

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Romero Segovia, V. and Hägglund, T. and Åström, K.J. (2005). Noise Filtering in PI and PID Control. In *2013 American Control Conference*.