Optimal control of mass/energy distribution networks under uncertainties ⋆

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Abstract: This paper presents a general framework for the optimal control of nonlinear hydrodynamic systems under uncertainties. In this paper the tag *hydrodynamic* refers to systems with special decomposable state space structure, where the sub-spaces are constituted by the following triple: (1) mass/energy conservation law, (2) disturbance model and (3) auxiliary state components. For optimal control, *permutational invariance* is utilized executing stochastic approximate dynamic programming on a rolling horizon. The primary targets for the application are distribution systems e.g. water distribution networks (mass distribution) and district heating systems (energy distribution). However, the mathematical abstraction is suitable for power generation systems such as multi–reservoir and hydro–thermal grids.

Keywords: Optimal control, Distribution systems, Uncertain dynamic systems, Dynamic programming

1. INTRODUCTION

Mass/energy distribution systems play a key role in urbanization. No emerging city–life can be imagined without the transfer of clean water and heat to inhabitants. As supply and demand are spatially distributed, this operation requires a distribution network to convey the required resources from sources to consumers.

To satisfy consumer demand while minimizing operation costs under various operational constraints is an essential goal which has invoked a highly challenging research area. The problem of designing optimal management policy for such systems is governed by the combination of high dimensionality, nonlinearity (model, objectives) and strong uncertainties in the inputs.

A comprehensive literature review on the developed methods in the outlined area is given in the authors' former paper Selek et al. (2013) in the context of water systems. In Selek et al. (2013) a novel solution to the optimal control of stochastic nonlinear systems was proposed and applied to operational optimization of water distribution systems utilizing *permutational invariance*. This concept was first hinted in Bene and Selek (2012).

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The aim of the present paper is to give a brief overview about the achievements and generalize the presented results towards broader class of distribution systems.

The paper is organized as follows: Section 2 defines the systems of interest. The problem definition is given in Section 3. In Section 4, a solution is proposed for the outlined problems followed by an application to the optimal control of Sopron water distribution system presented under Section 5. Finally, Section 6 summarizes the results and draws conclusions.

2. SYSTEMS OF INTEREST

This paper focuses towards the operational optimization of mass/energy distribution networks which are defined as follows: mass/energy distribution network is a hydrodynamic system, comprised by a grid of interconnected pipes, where a work fluid is conveyed throughout the grid by active hydraulic elements (pumps, valves) in order to deliver mass (fluid) and/or energy from sources to consumers. Besides hydrodynamic grid, the system comprises three main building blocks: (1) source, (2) consumer and (3) storage.

(1) Source provides sufficient mass/energy which is conveyed throughout the distribution network to satisfy consumer demand
(2) Consumer is a sink, which utilizes the fluid and/or the energy content of the fluid representing the demand
(3) The role of storage is to accumulate mass/energy. Storage capacity is provided by the distribution network and/or separate units (tanks) which are located within the distribution network

A schematic representation of system of interest is depicted in Figure 1.

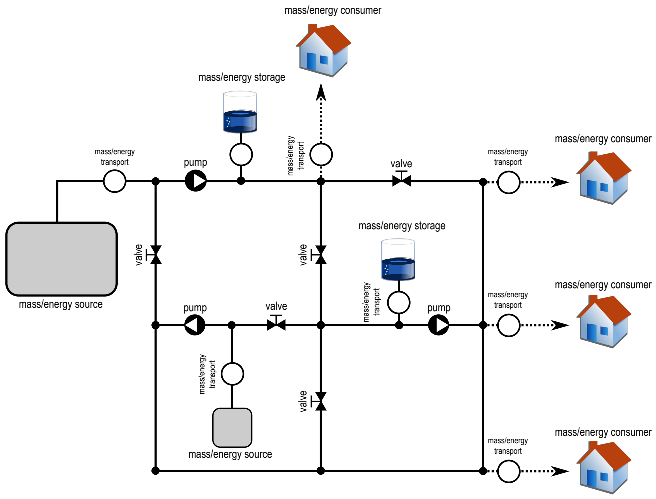

Fig. 1. Mass/energy distribution network

A broad range of systems can be treated under the umbrella of the outlined category. District Heating Systems and Water Distribution Networks are great examples.

3. PROBLEM STATEMENT

Let the evolution of the system of interest be described by a nonlinear discrete time model of the form

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k),\, \mathbf{u}(k),\, \mathbf{w}(k),\, k), \quad k = 0, \ldots, K-1 \quad (1)$$

where $\mathbf{x}(k) = (x_1(k), \ldots, x_n(k))^T \in X$ is the state vector. The control vector $\mathbf{u}(k) = (u_1(k), \ldots, u_m(k))^T \in U$ denotes the manipulated inputs of the system and $\mathbf{w}(k) = (w_1(k), \ldots, w_z(k))^T \in W(k)$ represents uncertainty (disturbance or noise). The available information on random variable $\mathbf{w}(k)$ is characterized by the uncertainty set W.

3.1 Model structure

The state vector $\mathbf{x}(k)$ represents all meaningful past and present information available at time k which can be used with advantage in selecting the appropriate control $\mathbf{u}(k)$. For systems of interest, the sate vector of the underlying model (1) can be decoupled as follows:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k) + \Delta t \mathbf{B}\mathbf{q}(k) + \mathbf{D}\, \mathbf{f}_{\mathrm{d}}(\check{\mathbf{x}}(k), \mathbf{w}(k), k) \quad (2a)$$
$$\check{\mathbf{x}}(k+1) = \mathbf{f}_{\mathrm{d}}(\check{\mathbf{x}}(k), \mathbf{w}(k), k) \quad (2b)$$
$$\tilde{\mathbf{x}}(k+1) = \mathbf{f}_{\mathrm{a}}(\tilde{\mathbf{x}}(k), \hat{\mathbf{x}}(k), \mathbf{u}(k), \mathbf{f}_{\mathrm{d}}(\check{\mathbf{x}}(k), \mathbf{w}(k), k)) \quad (2c)$$

subject to

$$\mathbf{q}(k) = \mathbf{f}_{\mathrm{q}}(\, \hat{\mathbf{x}}(k), \mathbf{u}(k), \mathbf{f}_{\mathrm{d}}(\check{\mathbf{x}}(k), \mathbf{w}(k), k)\,) \quad (2d)$$

where $\mathbf{q}(k)$ denotes the vector of reservoir mass/energy flow and Δt is the sampling time. The controlled state domain $\hat{\mathbf{x}}(k) \in \hat{X}$ represents conservation law, quantifying the amount of stored mass/energy in reservoirs. The uncontrolled component $\check{\mathbf{x}}(k) \in \check{X}$ includes a nonlinear disturbance model. The auxiliary state component $\tilde{\mathbf{x}}(k) \in \tilde{X}$ is required for the calculation of the step cost $c(\mathbf{x}(k), \mathbf{u}(k), \mathbf{w}(k), k)$ and has no effect on the system dynamics. Finally, equation (2d) is the hydrodynamic model of the distribution network, which describes the dependence of reservoir mass/energy flow on controllable and uncontrollable state components and control variable.

3.2 Control aim

The aim is to find an optimal control law (policy) π^* providing the control decision(s) based on the system's state. This policy consists of a sequence of functions $\boldsymbol{\pi}(., k) : X \to U$

$$\pi^* = \{\boldsymbol{\pi}(\mathbf{x}, 0), \ldots, \boldsymbol{\pi}(\mathbf{x}, K-1)\} \quad (3)$$

which maps the states into feasible controls $\mathbf{u}(k) = \boldsymbol{\pi}(\mathbf{x}(k), k)$ for all $\mathbf{x}(k) \in X$, and minimizes the associated cost

$$\lim_{K\to\infty} \frac{1}{K} \left(E \left\{ \sum_{k=0}^{K-1} c(\mathbf{x}(k),\, \boldsymbol{\pi}(\mathbf{x}(k), k),\, \mathbf{w}(k),\, k) \right\} \right). \quad (4)$$

The expectation is computed with respect to the joint distribution of the random variables $\mathbf{w}(k)$. The cost function is defined over an infinite horizon requiring the minimization of the average expected cost per stage, where $c(\mathbf{x}(k),\, \mathbf{u}(k),\, \mathbf{w}(k),\, k)$ is the state transition cost which accumulates over time. Equations (1)–(4) define an infinite horizon non–stationary stochastic optimal control problem.

3.3 Periodicity

One of the main properties of the distribution systems is that, the uncertainty is subject to periodic events such as weather conditions and consumer behavior. Consequently, the disturbance pattern can be modeled as periodic with period one year. Likewise, the state transition function $\mathbf{f}(., k)$ and step cost and the step cost $c(., k)$ can be considered as periodic with a period one year. Taking into account the periodicity of the system, the optimal policy is a periodic sequence of control laws

$$\boldsymbol{\pi}(\mathbf{x}, k) = \boldsymbol{\pi}(\mathbf{x}, k + lT_{\mathrm{p}}), \quad l \in \mathbb{N}. \quad (5)$$

Assuming that the expected step cost is bounded

$$0 \le E\Big\{ c(\mathbf{x}(k),\, \mathbf{u}(k),\, \mathbf{w}(k),\, k) \Big\} \le c_{\max}, \quad c_{\max} < \infty \quad (6)$$

for all $(\mathbf{x}(k), \mathbf{u}(k), \mathbf{w}(k)) \in (X, U, W)$ and $k = 0, \ldots, K-1$, the average cost becomes well defined over an infinite number of stages, and it can be meaningfully minimized.

4. PROBLEM SOLUTION

This section gives a brief overview on the solution to the stated problem which was proposed in Selek et al. (2013).

To keep the review of the results as transparent as possible, it is assumed that the auxiliary sub–space $\tilde{X}$ is a null set.

The solution to the outlined problem was derived by observing permutational symmetries of the control sequences with respect to system's dynamics. Using this, the concept called *permutational invariance* was introduced which was first coined in Bene and Selek (2012) and later generalized in Selek et al. (2013).

In a nutshell, *permutational invariance* refers to the invariance of the controllable state domain of a dynamic system under the permutations of control sequences. To understand the key idea of this concept, let us consider a simple example where a single water tank is filled/drained. The system evolves according to the discrete time equation

$$x(k+1) = x(k) + u(k) - w(k) \tag{7}$$

where $x(k)$ denotes the stored water volume in the tank at time k, $u(k)$ and $w(k)$ are the water inflow and demand during the kth period. The water inflow is manipulated by a controller while demand is a random variable with a given (time variant) probability distribution. The system dynamics can be written in integrator form, as follows:

$$x(k+t+1) = x(k) + \sum_{\tau=0}^{t} u(k+\tau) - \sum_{\tau=0}^{t} w(k+\tau). \tag{8}$$

This representation highlights the fact that, the value of the actual state depends on the total delivered water (cumulative) rather than the schedule of individual water deliveries. In other words, for any admissible disturbance scenario $\{w(k), \ldots, w(k+t)\}$ the components of the inlet water sequence $\{u(k), \ldots, u(k+t)\}$ can be freely permuted without affecting the final state. This introduces permutational invariance.

4.1 Utilizing permutational invariance

In systems of interest, the permutational invariance is assured by the mass/energy conservation law (2a). However, due to the nonlinear characteristics of the hydrodynamic model (2d) , the system might not have the property of permutational invariance under the „primary" control variable which is usually composed by pump speeds, valve openings etc. To ensure permutational invariance through conservation law, a „dummy" control variable

$$\mathbf{u}_{\mathrm{q}}(k) = \Delta t \mathbf{q}(k) \tag{9}$$

is defined utilizing the output of the hydrodynamic model. It was pointed out in Selek et al. (2013) a pseudo dynamics can be created for the underlying system utilizing the dummy control variable. In this particular case, the pseudo dynamics is defined as follows,

$$\boldsymbol{\xi}(t+1) = \boldsymbol{\xi}(t) + \mathbf{u}_{\mathrm{q}}(k+t), \quad t = 0, 1, \ldots \tag{10}$$

subject to initial condition $\boldsymbol{\xi}(0) = (0, \ldots, 0)^T$. Using this, the reformulation of the conservation law (2a)

$$\hat{\mathbf{x}}(k+t+1) = \hat{\mathbf{x}}(k) + \mathbf{B}\boldsymbol{\xi}(t+1) + \ldots \tag{11}$$

highlights the fact that, the pseudo state $\boldsymbol{\xi}(k)$ carries necessary (but not sufficient) information for state transition.

4.2 Optimization

Due to it's weak coupling, the pseudo dynamics (10) provides an ideal medium for solving the underlying problem by dynamic programming. To eliminate dimensionality issues, the essential idea is to use an aggregation function over the space of pseudo states and construct a simpler more traceable problem. The resulted problem has a reduced dimension which gives the possibility to solve it by dynamic programming. A one dimensional aggregated variable is constructed by summing the components of the pseudo state vector, that is

$$y(t) = \sum_{i=1}^{N_{\mathrm{q}}} \xi_i(t). \tag{12}$$

The use of pseudo states and aggregation eases the optimization, but causes information loss since the optimal policy is calculated over the space of aggregated variable. To compensate this effect, this policy must be updated at every time instant k in order to be able to take into account all meaningful (past and present) information available which can be used with advantage in selecting the appropriate control.

To achieve this, the receding horizon principle is applied, that is, an optimal policy

$$\pi_Y^* = \{\boldsymbol{\pi}_Y(y, 0), \ldots, \boldsymbol{\pi}_Y(y, N_{\mathrm{t}} - 1)\} \tag{13}$$

is calculated over the space of aggregated variables $y(t) \in Y$ $(\boldsymbol{\pi}_Y(., t) : Y \to U)$ on a finite time horizon $[k, k+N_{\mathrm{t}}-1]$ by solving the Bellman equation

$$V(y(t), t) = \min_{u(k+t)} \underset{w(k+t)}{E} \Big\{ c(\mathbf{x}(k), y(t), \mathbf{u}(k+t), \mathbf{w}(k+t), k+t) + V(y(t+1), t+1) \Big\}$$

subject to

$$y(t+1) = y(t) + \sum_{i=1}^{N_{\mathrm{q}}} f_{\mathrm{q},i}(\hat{\mathbf{x}}(k+t), \mathbf{u}(k+t), \mathbf{f}_{\mathrm{d}}(\check{\mathbf{x}}(k+t), \mathbf{w}(k+t), k+t)) \tag{14}$$

$$\mathbf{g}(\hat{\mathbf{x}}(k+t), \mathbf{u}(k+t)) \leq 0. \tag{15}$$

Equation (15) represents the constraint system (reservoir bounds, control constraints, etc.) and $y(0) = 0$. Once the policy is obtained, only the first decision is implemented for the system as a sub–optimal control law at time k,

$$\boldsymbol{\pi}(\mathbf{x}(k), k) := \boldsymbol{\pi}_Y(y(0), 0). \tag{16}$$

The decision making then repeatedly continues by shifting the optimization horizon $[k+1, k+N_{\mathrm{t}}]$.

5. APPLICATION

The outlined approach was implemented at the regional water distribution network of Sopron, Hungary. The networks serves the city of Sopron and its surroundings, with a total population of about 120,000. The topology of the water distribution system is shown in Figure 2. The network includes 11 pumping stations, 8 reservoirs and 5 main consumer demands allocated to the corresponding service reservoirs. Each pumping station in the model represents a group of individual pumps running in parallel indicated as "Pump" units in Figure 2.

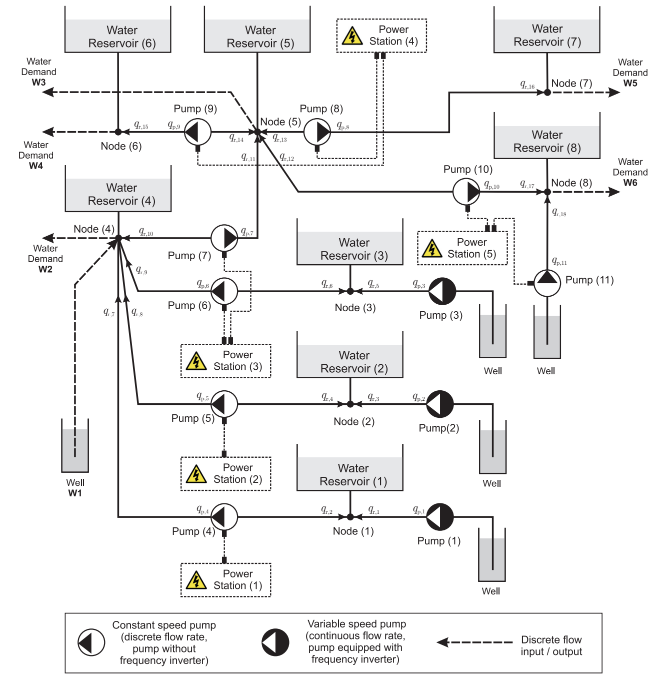

Fig. 2. The topology of the regional water distribution network of Sopron, Hungary (Schematic).

5.1 Optimization model

The hydraulic characteristic of the presented network allows the use of mass–balance modeling. For this particular case, a mass–balance description provides a computationally traceable model for real time control with sufficient accuracy. Mass balance models rely on the following assumptions Jowitt and Germanopoulos (1992); Goryashko and Nemirovski (2011):

- The water distribution network is a well designed network, where internal network pressure remains within acceptable bounds for allowable service reservoir storage fluctuations;
- The head lift for each pumping station is large compared to the network nodal head changes induced by pump/valve switchings elsewhere in the system;
- The flows a given pumping station will deliver depend on zonal consumer demands, and not on the changes in the network head/flow pattern caused by pump/valve switchings elsewhere in the system.

The outlined assumptions were well confirmed by a full–hydraulic simulator of the network. This allows the pumping stations to be represented by a set of flow rates and corresponding energy consumptions. Hence, there is a unique control action for each discharge of the pumping station, including operation rules (pump speeds, pump switchings) for the individual pumps within the group.

Using the vector of reservoir inflows $\mathbf{q}_{\mathrm{r}}(k) = (q_{\mathrm{r},1}(k), \ldots, q_{\mathrm{r},18}(k))^T$ (indicated in Figure 2) the mass conservation law can be written as follows:

$$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k) + \Delta t \mathbf{B} \mathbf{q}_{\mathrm{r}}(k) + \mathbf{D}\mathbf{w}(k) \tag{17}$$

where $\hat{\mathbf{x}}(k) = (\hat{x}_1(k), \ldots, \hat{x}_8(k))^T$ and $\mathbf{w}(k) = (w_1(k), \ldots, w_6(k))^T$. The hydraulic model of the network becomes

$$\mathbf{q}_{\mathrm{r}}(k) = \mathbf{F}\mathbf{q}_{\mathrm{p}}(k) \tag{18}$$

where $\mathbf{q}_{\mathrm{p}}(k) = (q_{\mathrm{p},1}(k), \ldots, q_{\mathrm{p},11}(k))^T$ denotes the pump discharges. The water demand $\mathbf{w}(k)$ was implemented as a random truncated Gaussian noise,

$$w_i(k) \sim \mathcal{N}(\mu_i(k), \sigma_i(k)) \text{ and } w_i^{\min}(k) \le w_i(k) \le w_i^{\max}(k)$$

where $i = 1, \ldots, 6$. The parameters of the distributions $\mu_i(k)$, $\sigma_i(k)$, $w_i^{\min}(k)$ and $w_i^{\max}(k)$ were obtained using empirical values (mean, standard deviation, min and max demand) computed from historical records of the water demand. The parameters were considered as periodic with a period of one year.

Finally, the model of the system is formulated as follows

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \Delta t \overbrace{\mathbf{B}\mathbf{F}}^{\widetilde{\mathbf{B}}} \mathbf{q}_{\mathrm{p}}(k) + \mathbf{D}\mathbf{w}(k). \tag{19}$$

Since demand is represented as state noise the state space X is totally controllable (i.e. $X \equiv \hat{X}$).The dummy control variable is defined by pump discharges $\mathbf{u}(k) = \Delta t \mathbf{q}_{\mathrm{p}}(k)$. The state of the system is observed on hourly basis $\Delta t = 1\,\mathrm{h}$ which is a good compromise between computational complexity of the model and flexibility of the operation.

The goal is to find an optimal water pump operation policy which minimizes the cost of the electric energy required by pumping while satisfying the water demand subject to reservoir constraints. The cost of energy has the following form:

$$\lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} \Bigg(\sum_{i=1}^{3} Q_i(u_i(k))) + \mathbf{a}_1^T \mathbf{u}_{4:11}(k) + \mathbf{a}_2^T \mathbf{u}_{4:11}^2(k) + \mathbf{a}_3^T \mathbf{u}_{4:11}^3(k) \Bigg) e_{\mathrm{c}}(k), \tag{20}$$

which must be minimized, subject to reservoir constraints $\mathbf{x}^{\min} \le \mathbf{x}(k) \le \mathbf{x}^{\max}$ and control constraints

$$\mathbf{u}(k) = \Delta t \begin{pmatrix} q_{\mathrm{p},1}(k) \in [0,\ 270] \\ q_{\mathrm{p},2}(k) \in [0,\ 250] \\ q_{\mathrm{p},3}(k) \in [0,\ 460] \\ q_{\mathrm{p},4}(k) \in \{0, 150, 360\} \\ q_{\mathrm{p},5}(k) \in \{0, 110\} \\ q_{\mathrm{p},6}(k) \in \{0, 270, 500\} \\ q_{\mathrm{p},7}(k) \in \{0, 550\} \\ q_{\mathrm{p},8}(k) \in \{0, 66, 116\} \\ q_{\mathrm{p},9}(k) \in \{0, 66, 118, 148\} \\ q_{\mathrm{p},10}(k) \in \{0, 90, 114\} \\ q_{\mathrm{p},11}(k) \in \{0, 72, 130\} \end{pmatrix}. \quad (21)$$

where $\mathbf{u}^p_{4:11}(k) = (u^p_4(k), \ldots, u^p_{11}(k))^T$, $(p = 2, 3)$ and $Q_i(.)$ are non-polynomial nonlinear functions.

The energy tariff $e_c(k)$ varies during the day, involving peak (price of electric energy is high) and off peak periods (price of electric energy is low). The tariff has the following pattern: 1 (Unit) $\{[0\mathrm{h} - 7\mathrm{h}), [13\mathrm{h} - 17\mathrm{h}), [20\mathrm{h} - 24\mathrm{h})\}$ and 1.25 (Unit) $\{[7\mathrm{h} - 13\mathrm{h}), [17\mathrm{h} - 20\mathrm{h})\}$. Unit denotes the price of the electric energy in terms of a given currency (e.g. EUR/kWh, USD/kWh etc.).

The complete problem definition (including water demand data, cost function coefficients etc.) can be downloaded from Selek (2011).

5.2 Results

The presented method was implemented under MATLAB R2011b and executed on a computer equipped with Intel Core i7 CPU (2.93 GHz) using parallelization (the computation tasks were distributed to 8 cores). In order to ensure the reproducibility of the presented results, the implemented algorithm is available to download from Selek (2011).

Some experiments are presented to illustrate the performance of the control system. The obtained results are presented for 10 days of operation using a randomly chosen feasible state as initial condition. Figure 3 shows the control strategy and the evolution of reservoirs. The water demand is satisfied without significant constraint violations. As expected, reservoir filling is performed when the electrical tariff is low (off peak period is uncolored in figures). This confirms intuitively the cost efficiency of the control. The corresponding average cost per stage is 319.19 units.

6. SUMMARY AND CONCLUSIONS

A general framework to the optimal control of nonlinear hydrodynamic systems under uncertainties was presented. It was pointed out that, if the state space of the system model is decomposable to the following triple: (1) conservation law, (2) disturbance model and (3) auxiliary state component, then permutational symmetries can be utilized to construct a one dimensional equivalent problem for the original (high dimensional) system in the subspace of the 1st and 2nd state components.

Consequently, the optimal control solution of a high dimensional stochastic dynamic system is obtained by the application of Stochastic Dynamic Programming (SDP) on the associated one dimensional problem. The presented method in this study resolves the curse of dimensionality on the outlined sub–space of the state domain and it takes nonlinearity, as well as uncertainty fully into account.

The presented approach has been applied to real time management of the regional water distribution network of the city of Sopron in Hungary. Although the presented case study is preliminary, it clearly highlights the good potential of the proposed approach and suggest further studies.

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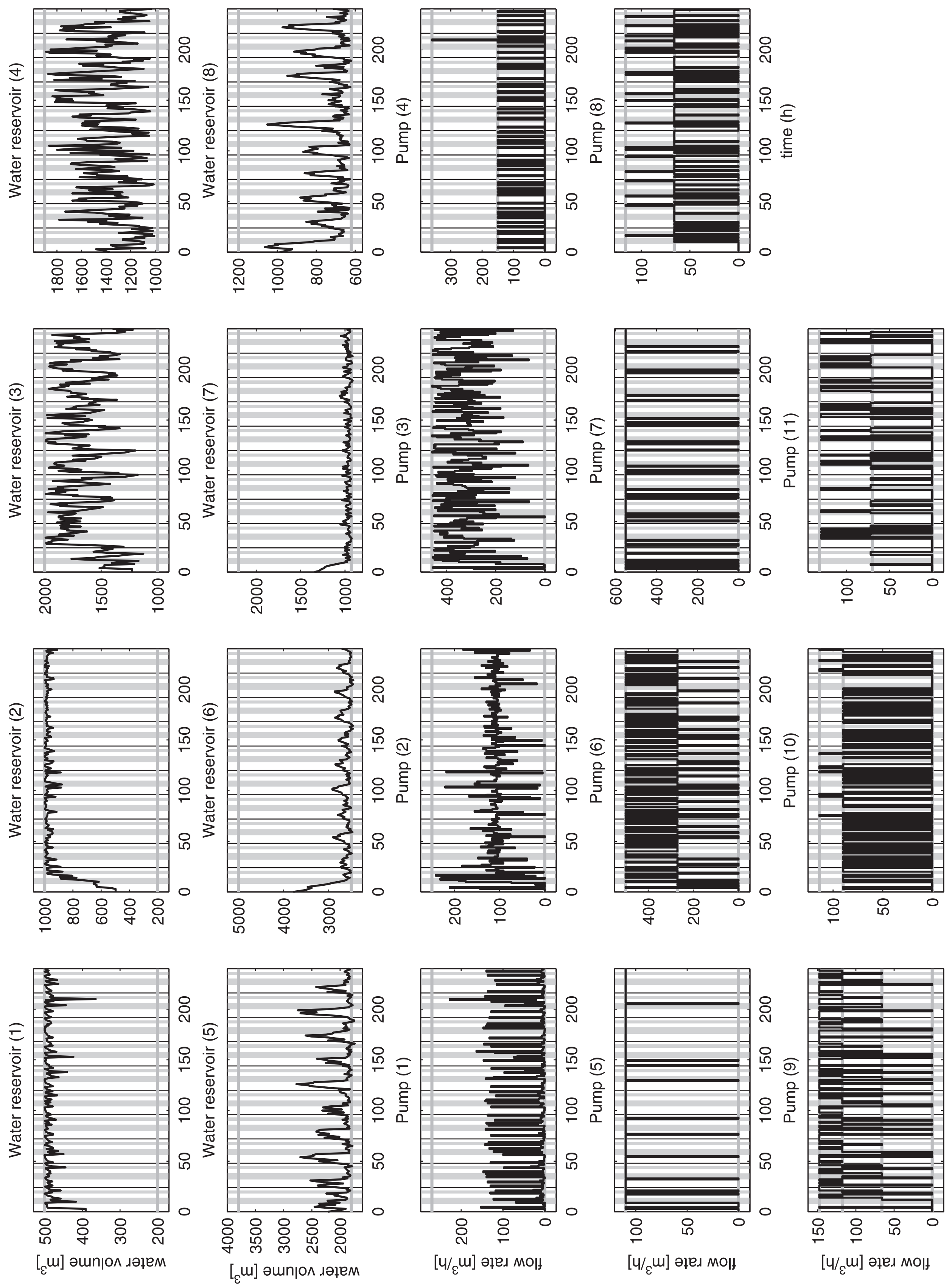

Fig. 3. 10 day sub–optimal pump control policy. Peak charging period is gray shaded while off–peak periods are uncolored. Reservoir upper and lower bounds are indicated by solid gray lines.