Suppression of Heave-Induced Pressure Fluctuations in MPD ⋆

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Abstract: A model describing the flow and pressure fluctuations in the bore-hole due to drill-string movement has been presented. It consists of a pair of coupled nonlinear partial differential equations modelling the distributed pressure and flow in the well, and a superposition of multiple sine waves for the disturbance. Considering only top-side flow and pressure as measurements, it is shown that the model can be represented by a linear time invariant finite-dimensional system with output delay. This result is achieved by linearization and de-coupling using Riemann invariants. An infinite-dimensional observer is designed that estimates the disturbance, and the estimate is used in a controller that rejects the effect of the disturbance on the down-hole pressure. A model reduction technique based on the Laguerre series representation of the transfer function is used to derive a finite-dimensional, rational transfer function for the controller. The performance of the full-order and reduced-order controllers are compared in simulations, which show satisfactory attenuation of the heave disturbance for both controllers.

Keywords: Managed Pressure Drilling (MPD), Process control, Periodic disturbance rejection, Delayed systems, Infinite-dimensional systems

1. INTRODUCTION

In drilling operations, a fluid called mud is pumped down through the drill string and flows through the drill bit in the bottom of the well (see Figure 1). Then the mud flows up the well annulus carrying cuttings out of the well. To avoid fracturing, collapse of the well, or influx of fluids from the surrounding rock formations, it is crucial to control the pressure in the open part of the annulus within a certain operating window. In conventional drilling, this is done by mixing a mud of appropriate density and adjusting mud pump flow-rates. In managed pressure drilling (MPD), the annulus is sealed and the mud exits through a controlled choke, allowing for faster and more precise control of the annular pressure. In automatic MPD systems, the choke is controlled by an automatic control system which manages the annular mud pressure to be within specified upper and lower limits. Different aspects of modeling for MPD have been examined in the literature, see Landet et al. (2012); Kaasa et al. (2012); Petersen et al. (2008). Estimation and control design in MPD have been investigated by several researchers so far, see Kaasa et al. (2012); Gravdal et al. (2010); Stamnes et al. (2008); Breyholtz et al. (2010); Zhou et al. (2011); Zhou and Nygaard (2011). These works focus mainly on pressure control during regular drilling.

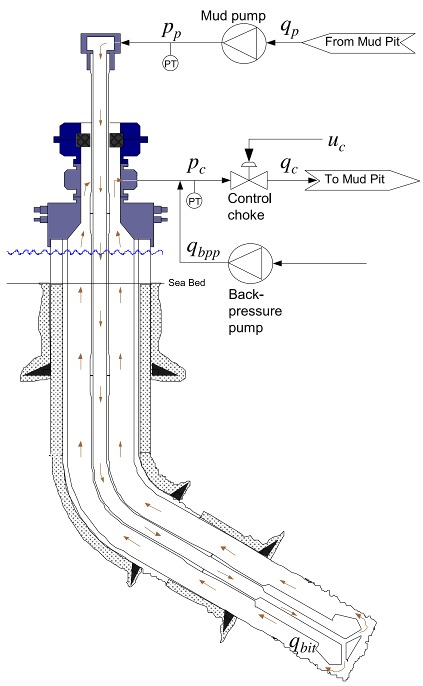

Fig. 1. Schematic of an MPD system, courtesy of G.-O. Kaasa, Statoil.

When designing MPD control systems, one should take into account various operational procedures and disturbances that affect the pressure inside the well. One such

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disturbance occurs when drilling from a floating rig. In this case, the rig moves with the waves, referred to as heave motion. As drilling proceeds, the drill string needs to be extended with new sections. Thus, every couple of hours or so, drilling is stopped to add a new segment of about 27 meters to the drill string. During drilling, a heave compensation mechanism isolates the drill string from the heave motion of the rig. However, during connections, the pump is stopped and the string is disconnected from the heave compensation mechanism and becomes rigidly connected to the rig. The drill string then moves vertically with the heave motion of the floating rig, and acts like a piston on the mud in the well. The heave motion is typically more than 3 meters in amplitude and has a period of 10-20 seconds, and the consequence is severe pressure fluctuations in the bottom of the well. Pressure fluctuations have been observed to be an order of magnitude higher than the standard limits for pressure regulation accuracy in MPD, which are about $\pm 2.5\ bar$. Downward movement of the drill string into the well gives pressure increase (surging), and upward movement gives pressure decrease (swabbing). Excessive surge and swab pressures can lead to mud loss resulting from high pressure fracturing the formation, or a kick-sequence (uncontrolled influx from the reservoir) that can potentially grow into a blowout, as a consequence of low pressure. A comparison and evaluation of some MPD methods for compensation of surge and swab pressure are presented in Rasmussen and Sangesland (2007). Two nonlinear control algorithms for handling heave disturbances in MPD operations are presented in Pavlov et al. (2010), and performance of both algorithms has been tested on a full-scale drilling rig. The tests indicated that for typical vertical drill-string movements, the problem of compensation of heave induced pressure oscillations remains open.

Currently there are no qualified solutions available for MPD from floating rigs considering the heave scenario in the north sea environment, and there is an urgent need for a solution. This is especially the case for depleted high-pressure and high-temperature reservoirs, which have very narrow drilling windows (Godhavn (2010)). Due to the propagation delays of the pressure, and the lack of downhole instrumentation, the design of a controller that counteracts the downhole pressure fluctuations by means of topside measurements, pumps and chokes is challenging.

In this paper, we model the annulus flow and design a controller that attenuates downhole pressure fluctuations based on estimating the disturbance from topside measurements. The estimator is infinite-dimensional, but a systematic model order reduction scheme shows that it can be truncated to a low order system corresponding to the number of harmonic disturbances. This basically amounts to the internal model principle.

The paper is organized as follows: In Section 2, we present a distributed-parameter-model based on mass and momentum balances that provides the governing equations for pressure and flow in the annulus. The controller is derived in Section 3, and controller order reduction is performed in Section 4. Section 5 provides simulation results and conclusions are offered in Section 6.

2. MODELING

2.1 *Annulus flow dynamics*

The governing equations for flow in an annulus can be derived from mass and momentum balances and written in the form

$$\frac{\partial P}{\partial t} + U\frac{\partial P}{\partial x} + (k+P)\frac{\partial U}{\partial x} = 0 \quad (1)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + \frac{c^2}{k+P}\frac{\partial P}{\partial x} + g\sin(\gamma) + f(x)U^2 = 0 \quad (2)$$

where $P(t,x)$ is pressure, $U(t,x)$ is fluid velocity, c is speed of sound in the fluid, g is acceleration of gravity, $\gamma(x)$ is the inclination angle of the pipe, and $f(x)$ is the friction factor. The constant k is related to the choice of equation of state. We use the following relation (Nieckele et al. (2001))

$$\rho(t,x) = \rho_{ref} + \frac{P(t,x) - p_{ref}}{c^2} \quad (3)$$

where $\rho(t,x)$ is fluid density, and ρ_{ref} and p_{ref} are reference values for the density and pressure, respectively. We then have that $k = c^2\rho_{ref} - p_{ref}$. Natural boundary conditions for (1)–(2) are flow in at the bottom ($x = 0$) and pressure at the outlet ($x = l$), that is

$$U(t,0) = U_0(t) \quad (4)$$

$$P(t,l) = P_l(t) \quad (5)$$

where l is the length of the well. For clarity of presentation, we will consider the well to be vertical, so that $\sin(\gamma(x)) = 1$. In the context of heave compensations, we are interested in the scenario of drillstring connection, when the flow through the drillstring is stopped (the pump has been shut off) and the flow in the annulus is only due to the motion of the drillstring acting like a piston. This results in zero flow-rate in the drillstring (there is a check valve at the drill bit preventing the back flow from the annulus into the drillstring) and relatively slow flow-rates in the annulus. Therefore we can develop a linear model that is valid around $\bar{U}(x) \equiv 0$. The corresponding steady pressure profile is given by (2) as

$$\frac{c^2}{k+\bar{P}}\frac{\partial \bar{P}}{\partial x} = -g \quad (6)$$

which we can integrate to obtain

$$\bar{P}(x) = (k+\bar{P}_l)e^{g(l-x)/c^2} - k\,. \quad (7)$$

A linearization of (1)–(2) around $(\bar{U},\bar{P})$ is obtained by inserting

$$U(t,x) = \bar{U} + u(t,x) = u(t,x) \quad (8)$$

$$P(t,x) = \bar{P}(x) + p(t,x) \quad (9)$$

into (1)–(2), and ignoring nonlinear terms. By using (6) and assuming p in comparison to $k+\bar{P}$ is negligible, the result is

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}\big((k+\bar{P})u\big) = 0 \quad (10)$$

$$\frac{\partial u}{\partial t} + \frac{c^2}{k+\bar{P}}\frac{\partial p}{\partial x} = 0\,. \quad (11)$$

We can further write (11) as

$$\frac{\partial}{\partial t}\Big(\frac{k+\bar{P}}{c}u\Big) + c\frac{\partial p}{\partial x} = 0 \quad (12)$$

and define

$$\tilde{u}(t,x) = \frac{k+\bar{P}}{c}u(t,x) \quad (13)$$

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to obtain

$$\frac{\partial p}{\partial t} + c\frac{\partial \tilde{u}}{\partial x} = 0 \tag{14}$$

$$\frac{\partial \tilde{u}}{\partial t} + c\frac{\partial p}{\partial x} = 0\,. \tag{15}$$

Let $p = \alpha - \beta$ and $\tilde{u} = \alpha + \beta$. α and β are the so-called Riemann invariants of the hyperbolic PDEs (14)–(15) and are obtained through diagonalization. In terms of the Riemann invariants, (14)–(15) become

$$\alpha_t + c\alpha_x = 0 \tag{16}$$

$$\beta_t - c\beta_x = 0\,. \tag{17}$$

We model the effect of friction factor and damping in the annulus by considering terms $m\alpha$ and $n\beta$ in (16)–(17) respectively

$$\alpha_t + c\alpha_x + m\alpha = 0 \tag{18}$$

$$\beta_t - c\beta_x + n\beta = 0\,. \tag{19}$$

The above are decoupled first order linear homogeneous partial differential equations with two independent variables. The general solution of (18) can be represented as the product

$$\alpha = \bar{\alpha}\alpha_0 \tag{20}$$

where $\bar{\alpha}$ is any nontrivial particular solution of this equation and α_0 is the general solution of corresponding truncated equation (with $m \equiv 0$).
Given two different (functionally independent) integrals,

$$u_1(t,x,\alpha) = C_1, \qquad u_2(t,x,\alpha) = C_2 \tag{21}$$

of the characteristic system

$$\frac{dt}{1} = \frac{dx}{c} = \frac{d\alpha}{-m\alpha} \tag{22}$$

the general solution of (18) can be expressed in terms of an arbitrary function of two variables as

$$\Phi(u_1,u_2) = \Phi(x - ct, \alpha e^{mt}) = 0 \tag{23}$$

Solving this equation for the second argument, we obtain the solution in explicit form

$$\alpha = e^{-mt}\phi(ct - x) \tag{24}$$

where ϕ is an arbitrary function. Similarly the solution of (19) in explicit form would be

$$\beta = e^{-nt}\psi(ct + x) \tag{25}$$

in which ψ is an arbitrary function.
Equations (24) and (25) can be solved to obtain for the boundary

$$\alpha(t,l) = e^{-ml/c}\alpha\Big(t - \frac{l}{c}, 0\Big), \tag{26}$$

$$\beta(t,0) = e^{-nl/c}\beta\Big(t - \frac{l}{c}, l\Big). \tag{27}$$

2.2 Disturbance due to drill-string movement

If we model the drill-string movement as a disturbance applied to flow in the bottom-hole, we have

$$\tilde{u}(t,0) = d(t) \tag{28}$$

which in terms of α and β is

$$\alpha(t,0) = -\beta(t,0) + d(t)\,. \tag{29}$$

Assuming the disturbance is a finite sum of single harmonics, Perez (2005), that is

$$d(t) = \sum_{j=1}^{N} a_j \sin(\omega_j t + \varphi_j) \tag{30}$$

it can be modeled as

$$\dot{x}_{1,j} = \omega_j x_{2,j} \tag{31}$$

$$\dot{x}_{2,j} = -\omega_j x_{1,j} \tag{32}$$

for $j = 1,2,3,...,N$, and therefore from (29), we obtain

$$\alpha(t,0) = -\beta(t,0) + \sum_{j=1}^{N} x_{1,j}\,. \tag{33}$$

The initial condition $x_{1,j}(0)$, $x_{2,j}(0)$ determines the phase, φ_j, and the amplitude, a_j, of the j^{th} component of the disturbance, which are unknown, while the frequency, ω_j, is assumed known.

3. CONTROL DESIGN

3.1 Known disturbance

We begin the control design by assuming that the disturbance is perfectly known by having access to d, and derive the desired feed-forward control for this case. In the next section, we will deal with the fact that d is unknown. The variations in bottomhole pressure is

$$p(t,0) = \alpha(t,0) - \beta(t,0)\,. \tag{34}$$

In order to perfectly cancel out the effect of the harmonic disturbance on $p(t,0)$, we need

$$\alpha(t,0) = \beta(t,0)\,. \tag{35}$$

From (35) and (29) we have that

$$\beta(t,0) = \frac{1}{2}d(t) \tag{36}$$

and from (27) we obtain

$$e^{-nl/c}\beta(t - \frac{l}{c}, l) = \frac{1}{2}d(t)\,. \tag{37}$$

Shifting time we get

$$\beta(t,l) = \frac{1}{2}e^{nl/c}d(t + \frac{l}{c}) = \frac{1}{2}e^{nl/c}\sum_{j=1}^{N} x_{1,j}(t + \frac{l}{c}) \tag{38}$$

$$= \frac{1}{2}e^{nl/c}\sum_{j=1}^{N}\cos(\frac{\omega_j l}{c})x_{1,j}(t) + \sin(\frac{\omega_j l}{c})x_{2,j}(t).$$

The desired feedforward topside control that attenuates downhole pressure oscillations is thus

$$\beta(t,l) = \frac{1}{2}e^{nl/c}BX(t) \tag{39}$$

where

$$B = \Big[\cos(\frac{\omega_1 l}{c})\ \sin(\frac{\omega_1 l}{c})\ \cdots\ \cos(\frac{\omega_N l}{c})\ \sin(\frac{\omega_N l}{c})\Big]$$

$$X(t) = [x_{1,1}(t)\ x_{2,1}(t)\ \cdots\ x_{1,N}(t)\ x_{2,N}(t)]^T$$

3.2 Unkown disturbance - observer design

The desired control signal (39) can not be implemented, since the disturbance $X(t)$ can not be measured in practice. At the topside, which is the only place we can collect measurements, we obtain from (26)–(27) by shifting time that

$$e^{ml/c}\alpha(t,l) = -e^{-nl/c}\beta(t - 2\frac{l}{c}, l) + \sum_{j=1}^{N} x_{1,j}(t - \frac{l}{c}) \tag{40}$$

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Let us now define our topside measurement as

$$Y(t) = e^{ml/c}\alpha(t,l) + e^{-nl/c}\beta(t-2\frac{l}{c},l) = \sum_{j=1}^{N} x_{1,j}(t-\frac{l}{c}) \tag{41}$$

Then, we have the system

$$\dot{X} = AX \tag{42}$$
$$Y(t) = CX(t-D) \tag{43}$$

where

$$A = \begin{bmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & A_N \end{bmatrix}, A_j = \begin{bmatrix} 0 & \omega_j \\ -\omega_j & 0 \end{bmatrix}$$

$$C = [1\ 0\ 1\ 0\ \cdots\ 1\ 0] \tag{44}$$
$$D = l/c \tag{45}$$

which is an LTI system with sensor delay. If (A, C) is observable, the results in Krstic and Smyshlyaev (2008) can be applied. In this case, we get the infinite-dimensional observer

$$\begin{aligned} \dot{\hat{X}} &= A\hat{X} + e^{AD}L(Y - \hat{z}(0)) \\ \hat{z}_t &= \hat{z}_x + Ce^{Ax}L(Y - \hat{z}(0)), x \in (0, D) \\ \hat{z}(D) &= C\hat{X} \end{aligned} \tag{46}$$

where

$$e^{Ax} = \begin{bmatrix} e^{A_1 x} & 0 & \cdots & 0 \\ 0 & e^{A_2 x} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & e^{A_N x} \end{bmatrix}$$

$$e^{A_j x} = \begin{bmatrix} \cos(\omega_j x) & \sin(\omega_j x) \\ -\sin(\omega_j x) & \cos(\omega_j x) \end{bmatrix} \tag{47}$$

and L must be chosen so that $A - LC$ is Hurwitz. According to Krstic and Smyshlyaev (2008), the observer error system is exponentially stable in the following norm

$$\Big(|X(t) - \hat{X}(t)|^2 + \int_0^D (z(t,x) - \hat{z}(t,x))^2 dx\Big)^{\frac{1}{2}} \tag{48}$$

where $z(t, x)$ represents the output equation (43) in the following manner

$$z_t = z_x, \quad x \in (0, D) \tag{49}$$
$$z(D) = CX \tag{50}$$
$$Y = z(0)\,. \tag{51}$$

It is straightforward to prove that (A, C) is observable if $\omega_1, \omega_2, ..., \omega_N$ are distinct.

3.3 Controller summary

The conventional way of controlling downhole pressure in an MPD system is to compute a set point for the choke pressure that corresponds to the desired downhole pressure and manipulate the choke opening to achieve this set point. Disregarding the heave-induced disturbance, this would be $P_{ref} = \bar{P}_l$. In the case with heave-induced disturbance, we can alter the set point by viewing $P(t, l)$ resulting from applying the desired $\beta(t, l)$ from (39) as the reference pressure signal. Of course, we have to replace $X(t)$ in (39) with its estimate $\hat{X}(t)$. The result, written in the original variables, is

$$P_{ref} = \frac{k + \bar{P}_l}{c} U(t,l) + \bar{P}_l - e^{nl/c} B\hat{X}(t)\,. \tag{52}$$

We assume that a choke controller is available that achieves $P_l(t) \approx P_{ref}(t)$.

4. CONTROLLER ORDER REDUCTION

The controller derived in (46) and (52) is infinite-dimensional. The implementation of this controller is computationally expensive, and therefore a finite-dimensional approximation of it is desired.

The model reduction technique used in this paper is based on the Laguerre representation of the transfer function. In Amghayrir et al. (2005), a model reduction technique that combines the Laguerre basis function and the gram matrix to reduce finite or infinite dimensional systems while minimizing a defined quadratic error is introduced. This method works based on the construction of a pencil of functions, using a one-order operator, and their projection on the basis of Laguerre functions. In order to apply the method, the transfer function for our controller is needed. It is derived next.

4.1 Derivation of the controller transfer function

Consider the control scheme as a system with input $Y(t)$ and output $\beta(t, l)$, given from (39) by replacing $X(t)$ with $\hat{X}(t)$. Taking the Laplace transform of (46) we have

$$s\hat{X} = A\hat{X} + e^{AD}L(Y - \hat{z}(0)) \tag{53}$$
$$s\hat{z} = \hat{z}_x + Ce^{Ax}L(Y - \hat{z}(0)),\ x \in (0, D) \tag{54}$$
$$\hat{z}(D) = C\hat{X} \tag{55}$$
$$\beta(t,l) = \frac{1}{2} e^{nl/c} B\hat{X}(t)\,. \tag{56}$$

From (53), we have

$$\hat{X} = (sI - A)^{-1} e^{AD} L(Y - \hat{z}(0)) \tag{57}$$

and we can solve (54) for $\hat{z}(0)$ by writing it as

$$\hat{z}_x = s\hat{z} - Ce^{Ax}L(Y - \hat{z}(0)) \tag{58}$$

and applying the variation of constants formula. We get

$$\hat{z}(D,s) = e^{sD}\hat{z}(0) - \int_0^D e^{s(D-y)} Ce^{Ay} L\big(Y - \hat{z}(0)\big) dy \tag{59}$$

Thus,

$$\hat{z}(0) = H^{-1}\Big(e^{-sD}C\hat{X} + (H-1)Y\Big) \tag{60}$$

where

$$H = 1 + C(A - sI)^{-1}(e^{(A-sI)D} - I)L\,. \tag{61}$$

Using (57) and (60) we have

$$\begin{aligned} \hat{X} = &\Big(I + J(H)^{-1}e^{-sD}C\Big)^{-1} J \\ &\Big(1 - (H^{-1}C(A - sI)^{-1}(e^{(A-sI)D} - I)L)\Big)Y \end{aligned} \tag{62}$$

where

$$J = (sI - A)^{-1} e^{AD} L\,. \tag{63}$$

Substituting (62) into (56), we finally obtain

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$$\beta(s,l) = \frac{1}{2}e^{nl/c}B\Big(I + J(H)^{-1}e^{-sD}C\Big)^{-1}J \Big(1-(H^{-1}C(A-sI)^{-1}(e^{(A-sI)D}-I)L)\Big)Y \quad (64)$$

After a few simplifying steps we have

$$\beta(s,l) = G(s)Y(s) \quad (65)$$

with

$$G(s) = \frac{1}{2}e^{nl/c}B\Big((1+C(A-sI)^{-1}(e^{(A-sI)D}-I)L) e^{-AD}(sI-A)+Le^{-sD}C\Big)^{-1}L. \quad (66)$$

4.2 Laguerre-based model order reduction

The procedure developed in Amghayrir et al. (2005) is used for computing rational, finite-dimensional approximations of the transfer function (66) with parameters given in Table 1 and heave disturbance frequency vector as follows

$$\omega_i = \begin{bmatrix} 0.21 & 0.31 & 0.52 & 0.63 & 0.9 \end{bmatrix} \quad [rad/s] \quad (67)$$

The bode plots of the original infinite-dimensional transfer function and its approximations are shown in Figure 2. Obviously, the 10th-order approximation matches the original infinite-dimensional transfer function extremely well.

Table 1. Parameter Values

Parameter	Value	Parameter	Value
l	5000 [m]	c	1.2271×10^3 [m/s]
ρ_{ref}	870 $[kg/m^3]$	p_{ref}	1.013 [bar]
m	0.1	n	0.1

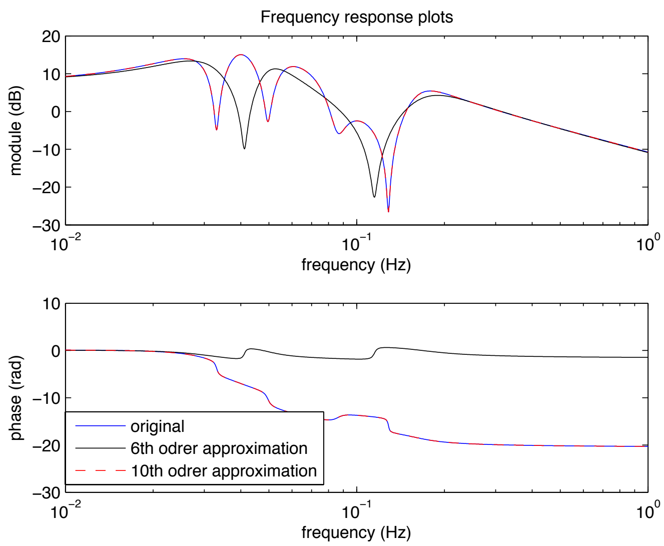

Fig. 2. Comparison of the Bode plots of original model and reduced order approximations.

5. SIMULATIONS

In this part we present the results of simulations. The linear PDE model is used in simulations. The performance of controller with infinite-dimensional observer is compared to the reduced-order controller. Figure 3 shows comparison of bottom-hole pressure for the cases of control with infinite-dimensional observer and reduced-order controller, and the corresponding top-side pressures are illustrated in

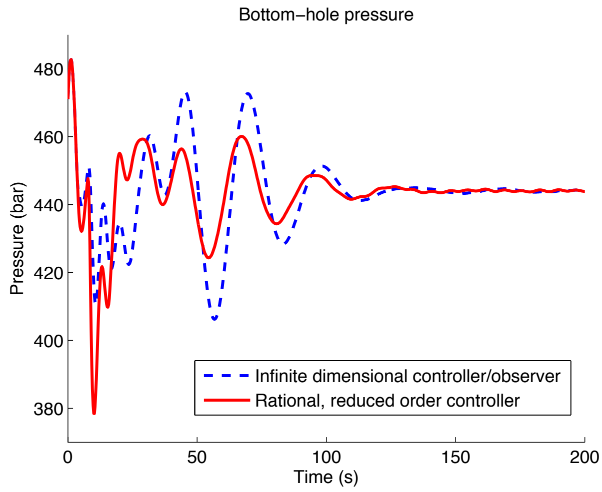

Fig. 3. Comparison of the bottom-hole pressures.

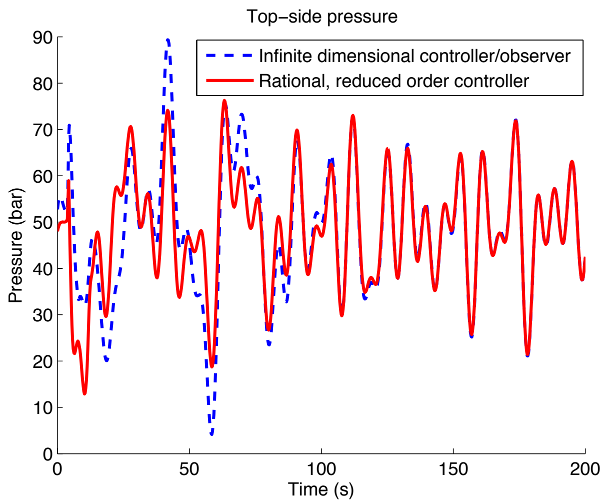

Fig. 4. Comparison of the top-side pressures.

Figure 4. Clearly, both cases show very good disturbance rejection capabilities after some initial oscillations.

The heave-induced pressure disturbance and its estimate are illustrated in Figure 5. The pressure variations around hydrostatic pressure in the whole well for the case of rational reduced order controller is shown in Figure 6.

6. CONCLUSIONS

In this paper a dynamical equation describing the flow and pressure in the annulus is derived. The coupled nonlinear partial differential equation is linearized, and decoupled using Riemann invariants. The disturbance due to drill-string movement is modeled as a superposition of multiple sinusoidal waves applied to flow in the bottom-hole. The state-space realization of heave disturbance is shown to be a delayed LTI system considering the flow and pressure measurements at the top-side.
An infinite dimensional observer is designed to estimate both the disturbance state and the measurement state. Moreover a controller is considered to reject the disturbance completely. Next, the irrational infinite-dimensional transfer function between the measurement and control input is obtained. A model reduction technique based on

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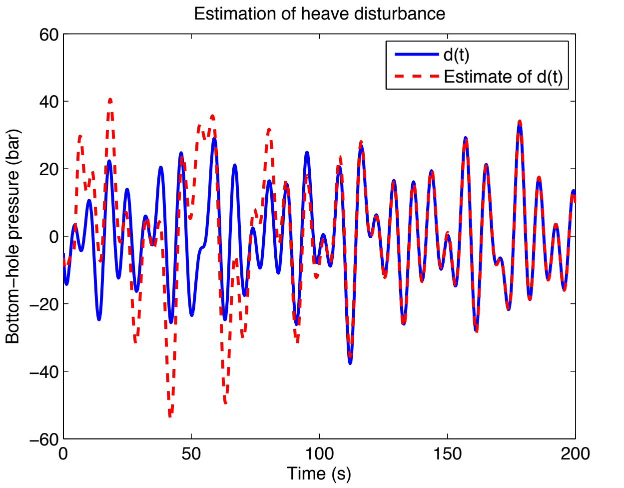

Fig. 5. The heave-induced pressure disturbance and its estimation.

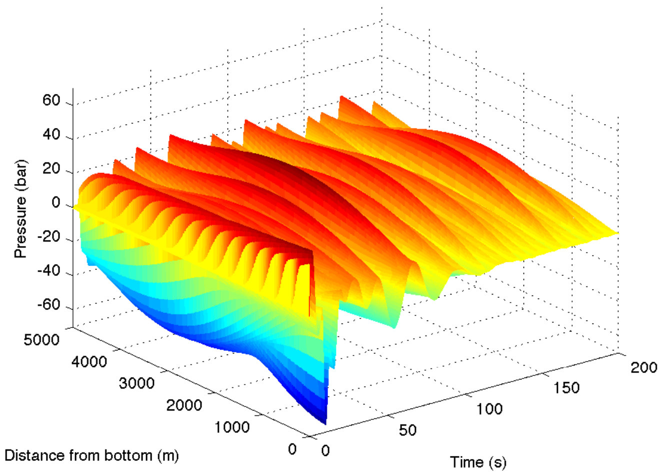

Fig. 6. Pressure variations around hydrostatic pressure for the case of rational reduced order controller.

Laguerre description of the transfer function is used to derive a simplified, rational, finite-dimensional controller for the system. Finally the simulation results are presented, which shows satisfactory attenuation of the heave disturbance.
Future work includes investigating how to deal with transient response of the control system, dealing with friction in a more rigorous manner, and robustness with respect to modeling errors.

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