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Output-feedback inclination control of directional drilling systems

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Abstract: Directional drilling techniques, based on rotary steerable systems, are used to generate complex curved boreholes. In practice, however, boreholes drilled with such systems often show instability-induced borehole spiraling, which negatively affects the borehole quality and increases drag losses while drilling. As a basis for controller synthesis, we present a directional drilling model in terms of delay differential equations characterizing the evolution of the borehole inclination. Next, the problem of curved well-bore generation is formulated as a tracking problem and an output-feedback control strategy is developed, solving this tracking problem while guaranteeing the prevention of borehole spiraling.

Keywords: Directional drilling, output-feedback control, tracking control, delay differential equations.

1. INTRODUCTION

Enhanced access to underground energy resources (such as oil and gas) requires drilling complex curved boreholes. So-called directional drilling rigs, which include downhole robotic systems known as rotary steerable systems (RSS), are used to drill such curved boreholes. This work focuses on a push-the-bit RSS, which steers the borehole propagation by exerting a force on the borehole using extendable pads, and pursues the development of novel strategies for inclination control.

Although RSSs are extensively used in drilling practice, it is known from experimental evidence that their usage can induce borehole oscillations, see e.g Prensky (2010); Marck et al. (2014). These oscillations in the borehole geometry are undesirable as they 1) endanger borehole stability, 2) induce increased drag while drilling (thereby reducing drilling efficiency), 3) reduce target accuracy, 4) make it more difficult to insert the borehole casing to prepare for production, and 5) reduce the rate-of-penetration (i.e. the speed of the drilling process). Current control techniques seem unable to prevent this so-called borehole spiraling. In this work, we aim to develop a *model-based* controller synthesis approach, which enables the drilling of complex borehole geometries while preventing borehole spiraling.

Many numerical directional drilling models exist, see Millheim et al. (1978); Amara (1985); Birades and Fenoul (1986); Rafie et al. (1986); Chen and Wu (2008), which do, however, not provide a closed-form dynamic model description for borehole propagation in directional drilling. In order to design a model-based controller for the directional drilling system, a closed-form dynamic model is needed to predict the bit trajectory given RSS actuation commands. Such a closed-form model, in terms of a delay differential equation (DDE) describing the borehole propagation, was first developed by Neubert and Heisig (1996). The next model development is due to Downton (2007), who formulated the borehole propagation equations for a class of directional drilling systems (either completely rigid or flexible with the addition of an equivalent spring) and analyzed the stability of the resulting (linear) DDE. The papers of Perneder (2013); Perneder and Detournay (2013b,a) and Downton and Ignova (2011) treat the BHA as an Euler-Bernoulli beam, similarly to Neubert and Heisig (1996), and consider a force actuation of a push-the-bit RSS. Although these two models describe the same physics, their formulation is different. The PD model in Perneder (2013); Perneder and Detournay (2013b,a, 2012); Detournay and Perneder (2011) is based on an angular description of the BHA and borehole tendencies and can thus naturally be used for describing boreholes undergoing large rotations, while the directional propagation of the borehole in the formulation of Downton (2007) and Downton and Ignova (2011) is described using the lateral displacement of the BHA with respect to an initial configuration, which needs to be regularly updated. Recently, it has been shown, using field data, that the PD model can predict the effect of borehole spiraling, see Marck et al. (2014).

Several works exist on the topic of the control of borehole propagation using an RSS. In Panchal et al. (2010, 2012b,a), controllers are developed based on empirical models of the borehole propagation process in which a direct link between the force applied by the RSS and the curvature of the borehole is assumed. This approach ignores (physically relevant) transient behavior of the borehole propagation, which is essential in preventing borehole spiraling. In Bayliss and Matheus (2009), a state-space model for borehole propagation is derived and on the basis of this model, a controller is designed. However, the essential delay nature of the borehole propagation dynamics is not captured in this model. In Sun et al. (2011), an $\mathcal{L}_1$ adaptive controller is designed, based on the directional drilling model of Downton (2007). In this approach, it assumed that the inclination of the borehole at the bit can be measured directly, which is generally not the case. The same restrictive assumption is made in most of the works above. This assumption is invalid in practice, since an inclination sensor can not be placed at (close to) the bit. In addition, even if available in practice, such an inclination sensor would measure the local inclination of the deformed BHA at the bit, which is not necessarily equal to the borehole inclination at the bit (due to bit tilt).

The main contribution of this work is the development of a synthesis strategy for output feedback inclination controllers for directional drilling systems. More detailed contributions are as follows. Firstly, this synthesis method is based on a closed-form (PD) model description of the borehole propagation, which captures the essential, physically relevant, behavior of a directional drilling system. Secondly, the resulting controllers can be used to generate complex borehole geometries. Unlike existing control methods, the goal of the controller synthesis method is to design a controller which reduces borehole spiraling and prevents oscillations in the transient closed-loop response (both of which are detrimental to borehole quality). Thirdly, we assume that only local inclination measurements of the deformed BHA are available at discrete locations other than at the bit. For this reason, an observer-based output feedback control strategy is developed. Lastly, the influence of (quasi-) constant disturbances, such as the influence of gravitational effects, on the accuracy of borehole propagation, is reduced by dedicated designs of both the controller and observer.

2. DIRECTIONAL DRILLING MODEL

In this work, we only consider the directional propagation of the borehole in a vertical plane. The directional drilling model used here builds upon the work in Perneder (2013); Perneder and Detournay (2013b,a). It consists of three components, as illustrated in Figure 1. Firstly, the forces and moments acting on the bit are calculated by modeling the deformation of the BHA inside the borehole. Since the BHA is constrained in the borehole by the stabilizers in contact with the borehole wall, see Figure 2, the existing borehole geometry affects the forces and moments on the bit in a spatially delayed manner. Secondly, the bit-rock interface laws govern how these forces and moments acting on the bit are related to the penetration of the bit into the rock. Finally, the bit motion is related to the propagation of the borehole geometry through kinematic relationships.

2.1 Borehole evolution equations

This model leads to the formulation of an evolution equation for the borehole inclination Θ, defined in Figure 2, in terms of a single delay differential equation:

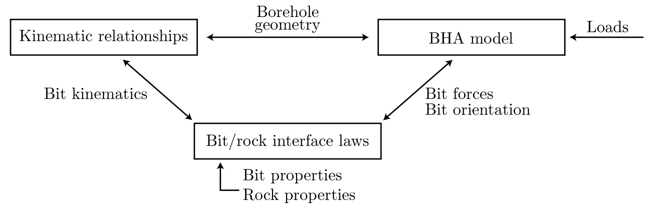

Fig. 1. Three components of the model and their interaction (Perneder and Detournay (2013b)).

$$\begin{aligned}\chi\Pi\Theta' &= \mathcal{M}_b\left(\langle\Theta\rangle_1 - \Theta\right) + \frac{\chi}{\eta}\mathcal{F}_b\left(\Theta - \Theta_1\right)\\ &+ \sum_{i=1}^{n-1}\left(\frac{\mathcal{F}_b\mathcal{M}_i - \mathcal{F}_i\mathcal{M}_b - \mathcal{M}_i\eta\Pi}{\eta\Pi}\right)\left(\langle\Theta\rangle_i - \langle\Theta\rangle_{i+1}\right)\\ &- \frac{\chi}{\eta}\sum_{i=1}^{n-1}\mathcal{F}_i\left(\frac{\Theta_{i-1} - \Theta_i}{\varkappa_i} - \frac{\Theta_i - \Theta_{i+1}}{\varkappa_{i+1}}\right)\\ &+ \frac{\mathcal{F}_b\mathcal{M}_w - \mathcal{F}_w\mathcal{M}_b - \mathcal{M}_w\eta\Pi}{\eta\Pi}\mathcal{W}\\ &+ \frac{\mathcal{F}_b\mathcal{M}_r - \mathcal{F}_r\mathcal{M}_b - \mathcal{M}_r\eta\Pi}{\eta\Pi}\Gamma - \frac{\chi}{\eta}\mathcal{F}_r\Gamma'.\end{aligned} \tag{1}$$

where $(\cdot)'$ indicates a derivative with respect to the (dimensionless) length of the borehole $\xi := L/\lambda_1$ with L the length of the borehole and λ_1 the distance between the bit and the first stabilizer, see Figure 2. Moreover, Π denotes the active weight-on-bit (which is assumed to be constant) and η, χ are respectively the lateral and angular steering resistance of the bit. The number of stabilizers is given by n. In (1), the inclination at the 'delayed' location of the i-th stabilizer is given as $\Theta_i := \Theta(\xi_i)$, for $i \in \{1, 2, \ldots, n\}$, with $\xi_i := \xi - \sum_{j=1}^{i}\varkappa_j$ and $\varkappa_j := \lambda_j/\lambda_1$ the dimensionless length of the j-th BHA segment between two adjacent stabilizers. The average inclination of the i-th BHA segment $\langle\Theta\rangle_i$ is given as:

$$\langle\Theta\rangle_i := \frac{1}{\varkappa_i}\int_{\xi_{i-1}}^{\xi_i}\Theta(\sigma)d\sigma, \tag{2}$$

which induces terms with distributed delays in (1). The factors $\mathcal{F}$ and $\mathcal{M}$ in (1) (with appropriate indices) only depend on the specific configuration of the BHA, see

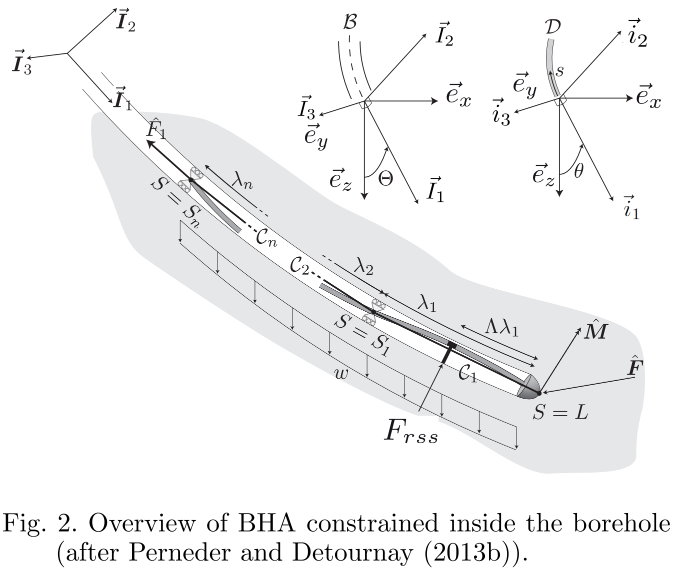

Fig. 2. Overview of BHA constrained inside the borehole (after Perneder and Detournay (2013b)).

Perneder (2013). The term involving $\mathcal{W} := \Upsilon \sin\langle\Theta\rangle_1$ in (1), with Υ a scaled measure of the BHA weight, is related to the influence of gravity on the BHA. We consider this term to be a quasi-constant perturbation, since the average inclinations only change slowly with the distance drilled. Finally, $\Gamma := \frac{F_{\text{rss}}}{F_*}$ is the (scaled) RSS force, i.e. the control input, where F_{rss} is the RSS actuation force and $F_* := \frac{3EI}{\lambda_1^2}$ with EI denoting the BHA's bending stiffness.

2.2 State-space model formulation

In this section, the DDE (1) with distributed delays is transformed into a first-order state-space formulation with point-wise delays by considering the average inclinations $\langle\Theta\rangle_i$ as states:

$$x'(\xi) = A_0 x(\xi) + \sum_{i=1}^{n} A_i x(\xi_i) + B_0\Gamma + B_1\Gamma' + B_2\mathcal{W}, \quad (3)$$

where the state vector is given as $x := [\Theta, \langle\Theta\rangle_1, .., \langle\Theta\rangle_n]^T$ and the system matrices are in accordance with (1) and (2). In this respect, we note that the expressions for the derivatives $\langle\Theta\rangle_i'$ in (3) are obtained by differentiation of (2) with respect to ξ. As a consequence, n additional poles at zero have been added to (3). It can be shown that these poles are inconsequential for the stability of the original DDE in (1) and hence can be ignored in the stability analysis of (3), see Kremers (2013).

In practice, the states of the DDE model in (3) can not be measured. Only sensors measurements of the local BHA inclination are available (not of the borehole inclination). Such measured outputs, denoted by y_m, can generally be expressed as a function of the state vector x and the input force Γ:

$$y_m = Cx + D\Gamma, \quad (4)$$

where the matrices C, D depend on the configuration of the BHA and the location of the inclination sensors. The influence of the gravity term $\mathcal{W}$ on the measured output y_m is generally very small compared to the influence of $\mathcal{W}$ in (3), and is hence neglected in (4).

3. CONTROL PROBLEM FORMULATION

The main goal of directional drilling is the generation of a borehole with some particular geometry. In terms of the model in (3), this objective can be formulated as a tracking problem. More specifically, we aim to track some inclination reference trajectory $\Theta_r(\xi)$, for $\xi \in [-\varkappa_{tot}, \infty]$, with $\varkappa_{tot} := \sum_{i=1}^{n} \varkappa_i$. Note that defining $\Theta_r(\xi)$ immediately results in the state reference trajectory $x_r(\xi)$ being fully defined, since the average inclination state references $\langle\Theta_r\rangle_i,\ i = 1, 2, \ldots, n$, can be obtained by integration of Θ_r, see (2). Hence, the problem can be formulated as a state tracking problem. We will assume that $\Theta_r(\xi)$ is continuously differentiable, which is reasonable at the scale at which the problem is treated as it avoids curvature discontinuities. Given the directional drilling model (3), (4), an output-feedback controller needs to be designed such that the control input $\Gamma(\xi)$ renders $x_r(\xi)$ the globally asymptotically stable solution of the closed-loop system.

Besides the above formulation of the control goal as a state tracking problem, certain additional objectives stem from the fact that the spiraling behavior in the borehole, which is often observed in practice, needs to be reduced/eliminated. Such borehole spiraling can either be caused by poles in the right-half complex plane (i.e. instability), which is avoided if the state tracking problem is solved, or by weakly damped poles (i.e. undesired transient behavior). For this reason, we focus on appropriate placement of the poles of the tracking error dynamics (with the tracking error defined as $e := x - x_r$), in order to reduce/eliminate borehole spiraling. Another control objective is related to the fact that there exist several sources of force disturbances. We focus on the effect of the gravity-induced forces here. Although strictly speaking, the gravity term in (1) acts as a non-linear term in the DDE, it can be seen as a slowly varying quasi-constant disturbance force, see (3), since the average inclination $\langle\Theta\rangle_1$, on which the gravitational term in the directional drilling model depends, only changes slowly with the distance drilled ξ. We aim to reduce the influence of this disturbance on the steady-state inclination error $e_\Theta := \Theta - \Theta_r$.

4. CONTROLLER SYNTHESIS APPROACH

4.1 Controller structure

For the sake of transparency, we focus on the directional drilling model for the case of a two-stabilizer BHA, i.e. (3) with $n = 2$. This model contains terms in both the RSS force Γ and its derivative Γ'. In support of controller design, we introduce a control input u defined by $Bu(\xi) = B_0\Gamma(\xi) + B_1\Gamma'(\xi)$, with $B = [1, 0, 0]^T$, which is well-defined by the grace of the specific structure of B and that of B_0 and B_1 (namely, $B_0 = [b_0, 0, 0]^T$ and $B_1 = [b_1, 0, 0]^T$). Substituting this expression for u in (3) (for $n = 2$) results in the following DDE model:

$$x'(\xi) = A_0 x(\xi) + A_1 x(\xi_1) + A_2 x(\xi_2) + Bu(\xi) + B_2\mathcal{W}. \quad (5)$$

The input force Γ, supplied to the RSS actuator, now satisfies the following differential equation:

$$\Gamma' = -\frac{b_0}{b_1}\Gamma + \frac{1}{b_1}u. \quad (6)$$

For the filter (6) to be asymptotically stable, b_0/b_1 needs to be positive. The latter assumption is only violated for rather high values of the composite parameter $\eta\Pi$ (e.g. high weight-on-bit Π), see Perneder (2013), which are not under consideration here.

The control input u is decomposed as $u(\xi) = v(\xi) + u_r(\xi)$, where u_r is a model-based feedforward signal and v is the control input used for feedback. The influence of the quasi-constant gravity disturbance $\mathcal{W}$ in (3) will not be taken into account in the feedforward design. Hence, the feedforward input u_r is obtained by solving the inverse dynamics:

$$Bu_r(\xi) = x_r'(\xi) - A_0 x_r(\xi) - A_1 x_r(\xi_1) - A_2 x_r(\xi_2). \quad (7)$$

To obtain $u_r(\xi)$ satisfying (7), it suffices to ensure the satisfaction of the first scalar equation of the vector equation (7), since by definition if $\Theta(\xi) = \Theta_r(\xi)$ then $\langle\Theta\rangle_1(\xi) = \langle\Theta_r\rangle_1(\xi)$ and $\langle\Theta\rangle_2(\xi) = \langle\Theta_r\rangle_2(\xi)$. The solution for u_r that solves (7) is thus given as: $u_r(\xi) = B^T(x_r'(\xi) - A_0 x_r(\xi) - A_1 x_r(\xi_1) - A_2 x_r(\xi_2))$.

Next, we propose a feedback control strategy (for v) that consists of a model-based observer with integral action in combination with a dynamic state-feedback controller including a low-pass filter and integral action.

The observer provides a state estimate $\hat{x}$ to be employed by the state-feedback controller. The following observer is proposed:

$$\begin{aligned} \hat{x}' =& A_0\hat{x}(\xi) + A_1\hat{x}(\xi_1) + A_2\hat{x}(\xi_2) + L(y_m - \hat{y}_m) \\ &+ B(q+u), \\ q' =& \zeta_o[l_1, l_2](y_m - \hat{y}_m), \\ \hat{y}_m =& C\hat{x} + D\Gamma, \end{aligned} \tag{8}$$

where a 'hat' is now used to denote an estimate. The observer gain matrix L is defined as:

$$L = \begin{bmatrix} l_1 & l_2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{9}$$

This structure in L ensures that the estimates of the average inclinations $\langle\hat{\Theta}\rangle_1$ and $\langle\hat{\Theta}\rangle_2$ are simply given by integration of the inclination estimate $\hat{\Theta}$ (i.e. are obtained purely by model-based prediction). This choice reduces the number of observer parameters that needs to be designed. The strength of the weak integral action is determined by the parameter ζ_o in (8). This integral action is included to ensure convergence of the observer error to zero in the presence of the gravitational disturbance $\mathcal{W}$ acting on the system (which is not modeled in (8)).

The dynamic state-feedback controller is designed as follows:

$$\begin{aligned} z_1{}' &= \zeta[k_1, 0, 0](\hat{x} - x_r) \\ z_2{}' &= -\gamma z_2 + \gamma(z_1 + K(\hat{x} - x_r)) \\ v &= z_2, \end{aligned} \tag{10}$$

where the control gain matrix $K = [k_1, k_2, k_3]$. Weak integral action is included in order to remove the influence constant of disturbances (such as gravitational effects) on the steady-state tracking error. The cut-off frequency of the weak integral action is determined by the control parameter ζ. Moreover, the controller contains a low-pass filter to reduce oscillations in the transient borehole inclination response (to further reduce borehole oscillations). The controller parameter γ determines the cut-off frequency of the low-pass filter. Note that indeed the observer-controller combination (8), (10) (and in particular the inclusion of the low-pass and integrals actions) aims at addressing the additional performance aspects discussed in Section 3: 1) improving the transient response in an attempt to reduce undesired borehole oscillations and 2) robustness to quasi-constant (gravity-related) disturbances.

Remark 1. The dynamics of the directional drilling model exhibits three essential length scales: 1) short-range, $\xi = O(10^{-1})$, related to fast inclination changes (borehole kinking) induced by changes in the RSS force, 2) medium range, $\xi = O(10^0 - 10^1)$, related to borehole oscillations, and 3) long-range, $\xi = O(10^2 - 10^3)$, related to the steady-state inclination behavior, see Perneder (2013). The structural design of the controller proposed above targets these different length scales in the following way:

- Short-range: the low-pass filtering properties in the feedback controller (10) ensure that excitation of the short-range (boundary layer) dynamics is avoided, therewith avoiding severe borehole kinking.
- Medium-range: The design of both the observer in (8) and the controller in (10) aims at the stabilization of the medium-range dynamics (through design of the gains K and L to be addressed in Section 4.3), therewith guaranteeing the absence of instabilities related to borehole oscillations.
- Long-range: The inclusion of integral action in both the observer in (8) and the controller in (10) ensure the long-range tracking error to be zero in the presence of (e.g. gravity-related) disturbances.

4.2 Error dynamics

In support of the optimization-based tuning of the controller and observer parameters, we now construct the tracking and observer error dynamics. We define the tracking error as $e := x - x_r$ and the observer error as $\delta := x - \hat{x}$. Applying the control decomposition $u = u_r + v$ and observer-based controller (8), (10) to (5), we obtain the following closed-loop error dynamics:

$$\begin{aligned} \begin{bmatrix} e' \\ z_1' \\ z_2' \\ \delta' \\ q' \end{bmatrix} =& \begin{bmatrix} A_0 & 0 & B & 0 & 0 \\ \zeta[k_1,0,0] & 0 & 0 & -\zeta[k_1,0,0] & 0 \\ \gamma K & \gamma & -\gamma & -\gamma K & 0 \\ 0 & 0 & 0 & A_0 - LC & -B \\ 0 & 0 & 0 & \zeta_0[l_1,l_2]C & 0 \end{bmatrix} \begin{bmatrix} e(\xi) \\ z_1(\xi) \\ z_2(\xi) \\ \delta(\xi) \\ q(\xi) \end{bmatrix} \\ &+ \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e(\xi_1) \\ z_1(\xi_1) \\ z_2(\xi_1) \\ \delta(\xi_1) \\ q(\xi_1) \end{bmatrix} \\ &+ \begin{bmatrix} A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e(\xi_2) \\ z_1(\xi_2) \\ z_2(\xi_2) \\ \delta(\xi_2) \\ q(\xi_2) \end{bmatrix}. \end{aligned} \tag{11}$$

Note that the quasi-constant disturbance $\mathcal{W}$ of the gravity is neglected in (11), since the error dynamics are constructed to support the design of a stabilizing controller. The effect of gravity-induced disturbances on the closed-loop response is further investigated in Section 5. The origin (corresponding to zero tracking and observer errors) is an asymptotically equilibrium point of (11) if all poles of these closed-loop dynamics are located in the open left-half complex plane. As mentioned before, the n poles at zero, caused by the inclusion of the average inclination states in the state-space description, can be disregarded for the stability analysis of the system. Note that due to the block-diagonal structure of the system matrices in (11), the separation principle holds. This means that the poles of the closed-loop system are given by the union of the poles of the 'tracking error' subsystem, with state $[e, z_1, z_2]^T$ and $[\delta, q]^T$ as input, and the 'observer error' subsystem, with state $[\delta, q]^T$. This allows the controller parameters K, ζ, γ and the observer parameters l_1, l_2, ζ_0 to be designed separately, such that the poles of the respective subsystems are properly placed in the left-half complex plane and stabilization is achieved.

4.3 Optimization-based tuning for stabilization

To guarantee asymptotic stability of the closed-loop system, the controller and observer parameters need to be tuned such that the poles of the error dynamics are located in the left-half complex plane. An optimization-based approach is taken to design such stabilizing controller and observer parameters. Herein, we aim to minimize the real part of the right-most pole of both the closed-loop tracking error dynamics and the observer error dynamics. By the grace of the separation principle, mentioned above, if these right-most poles have negative real part, then the origin is a globally asymptotically stable equilibrium point of the error dynamics. Moreover, this eigenvalue- and optimization-based approach towards controller and observer tuning also aims to improve transient performance in order to limit transient borehole oscillations. Here, we only optimize the controller gain matrix K and the observer gains l_1, l_2 in (9). The parameters ζ, γ and ζ_o, corresponding to the properties of the dynamic low-pass and integrating filters in (8), (10), are designed a priori as 1) these effectuate the desired controller properties at different length scales (see Remark 1) and 2) this reduces the number of parameters that need to be optimized. Since the separation principle holds, the synthesis method consists of separately minimizing the objective function $\alpha_c(K)$, that describes the right-most pole of the tracking error dynamics, and minimizing the objective function $\alpha_o(L)$, that describes the right-most pole of the observer error dynamics. This allows us to place the poles of the observer error dynamics further into left-half complex plane, such that the observer error δ converges to zero faster than the inclination error e. These objective functions are given as:

$$\alpha_c(K) = \sup_{i\in[1,2,...,\infty]} \{\Re(p_{Ci}(K)\}, \tag{12}$$

$$\alpha_o(L) = \sup_{i\in[1,2,...,\infty]} \{\Re(p_{Oi}(L)\}, \tag{13}$$

where $p_{Ci}(K)$ indicates poles relating to the tracking error dynamics for controller gain K and $p_{Oi}(L)$ indicates the poles corresponding to the observer error dynamics for observer gain L. As shown in Michiels and Niculescu (2007), the objective functions in (12), (13) are typically non-convex and non-smooth. Because of this fact, standard optimization tools such as gradient-based optimization tools are not suitable for this optimization problem. Instead, the gradient sampling method Burke et al. (2005) can be used to support the optimization of these non-smooth objective functions.

5. ILLUSTRATIVE BENCHMARK STUDY

In this section, a simulation study is performed in order to confirm that the proposed control strategy indeed solves the tracking (borehole generation) problem. Here, we consider a particular BHA with two stabilizers and characterized by geometric properties listed in Table 1 (The inner and outer radius Ir and Or of the BHA are needed to compute the scaled BHA weight Υ and its second moment of area I). It is assumed that the entire BHA is made of steel with Young's modulus $E = 2e11$ N/m^2 and density $\rho = 7800$ Kg/m^3. Moreover, $\Pi = 0.0093$, $\eta = 30$ (i.e. $\eta\Pi = 0.279$) and $\chi = 0.1$.

Table 1. Geometry of the benchmark BHA.

λ_1	$\varkappa_1$	λ_2	$\varkappa_2$	Λ	Ir	Or
3.66 m	1	6.10 m	$\frac{2}{1.2}$	$\frac{1}{6}$	0.053 m	0.086 m

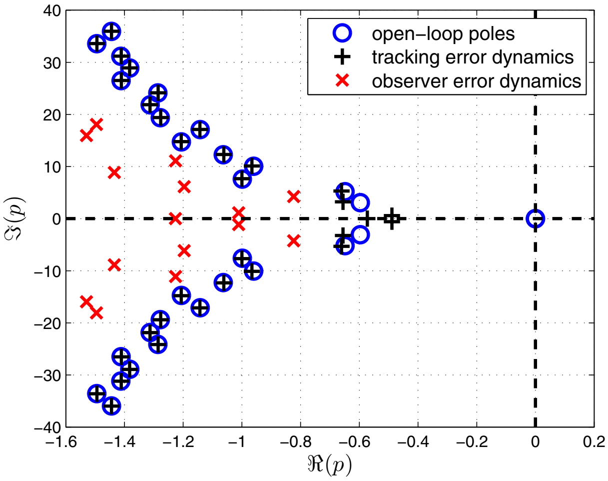

Fig. 3. Comparison of the poles of the open-loop dynamics, tracking error dynamics and observer error dynamics.

Let us design a controller and an observer for this benchmark BHA system using the design strategy proposed in Section 4. The controller parameters are chosen as $\gamma = 0.8$ and $\zeta = 0.5$; the optimization for designing the controller gain K is terminated when $\alpha_c(K) < -0.5$. The observer objective function is optimized such that $\alpha_o(L) < -0.8$ with $\zeta_o = 0.45$. Optimization of both objective functions results in the controller gain matrix $K = [-2565, -742, 161]$ and observer gains $l_1 = 133$ and $l_2 = 2998$. Figure 3 shows a comparison of the open-loop poles, the poles of the tracking error subsystem and the poles of the observer error subsystem. This figure shows that the open-loop dynamics is not asymptotically stable. Moreover, it can be observed that the optimization procedure successfully places the poles of the closed-loop system, such that closed-loop stability is guaranteed and the tracking problem is solved.

The performance of this controller/observer combination can be verified by means of a simulation of the closed-loop system. Here, we consider the situation in which we transition from constant inclination borehole into a constant curvature borehole. The inclination reference trajectory is given as: $\Theta_r(\xi) = \frac{\pi}{4}$ for $\xi \in [-(\varkappa_1 + \varkappa_2), 5]$, $\Theta_r(\xi) = \frac{\pi}{4} + 0.01(\xi - 5)$ for $\xi \in [5, \infty]$.

The initial borehole inclination is given as $\Theta(s) = \frac{\pi}{4} + 0.01$, for $s \in [-(\varkappa_1 + \varkappa_2), 0]$, and the initial inclination estimate is given as $\hat{\Theta}(s) = \frac{\pi}{4}$, for $s \in [-(\varkappa_1 + \varkappa_2), 0]$, i.e. there exists both an initial inclination tracking error as well as an initial observer error. Figure 4 shows the inclination tracking error $e_\Theta := \Theta - \Theta_r$ and the observer inclination error $\delta_\Theta := \Theta - \hat{\Theta}$. Clearly, the desired borehole geometry is generated (corresponding to $e = 0$) in steady state and borehole oscillations are avoided. Note that the (non-linear) influence of the gravity disturbance ($\mathcal{W} = \Upsilon \sin\langle\Theta\rangle_1$) on the system (1) has been successfully compensated for by the integral action in both the controller and observer and as a result the steady-state errors converge to zero. Although the observer error contains some fast transients, the inclination error remains very smooth due to low-pass filter included in the controller. Due to the adopted feedforward design for u_r, no error is

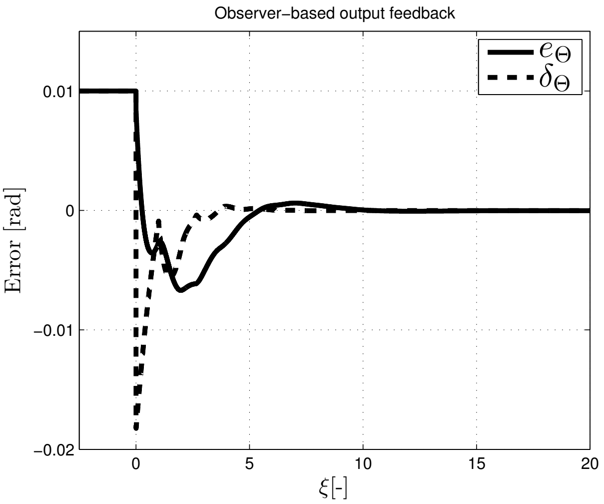

Fig. 4. Inclination and observer error for the observer with weak integrating action and the dynamic controller.

induced at the transition between the constant inclination and constant curvature borehole at $\xi = 5$. In other words, during a transition from a constant inclination section to a constant curvature section, no kinks in the borehole or borehole oscillations are induced (as would be the case with conventional constant RRS force actuation).

6. CONCLUSION

In this work, an output-feedback inclination control strategy for directional drilling has been proposed. The problem of drilling complex curved boreholes using down-hole robotic systems has been formulated as a tracking problem. The benefits of the proposed control strategy are as follows. Firstly, it guarantees the stable generation of complex curved boreholes without the occurrence of undesired borehole oscillations. Secondly, the observer-based controllers only need limited measurements of the inclination of the bottom-hole-assembly. Thirdly, the resulting closed-loop system is robust against (gravity-induced) perturbations. The effectiveness of the proposed observer-based output feedback control strategy has been illustrated by a case study.

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