

Non-linear model order reduction using input to state Hammerstein structures

Omar Naeem,
Adrie Huesman and
Okko Bosgra.

DCSC, TU Delft

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1



Content

- Introduction
 - ▲ Context
 - ▲ Methodology
- Theory and background
 - ▲ Taylor expansion
 - ▲ Input to state Hammerstein (first order)
 - ▲ Improvement in accuracy
 - ▲ Input to state Hammerstein (second order)
- Results
 - ▲ High purity distillation column
 - ▲ Input design
 - ▲ Results of implementation
- Conclusions
- Future developments
- Questions/Discussion

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2



Introduction; Context

▲ Context

- The goal is to achieve a reduced model which has low computational load.
- There are various perspectives for the model reduction;
 - a. Linear system theory
 - b. Time scale based
 - c. Projection based
 - d. Using structured models (example: Hammerstein)

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3



Introduction; Methodology -I

1. Prerequisites

- i. A NL model available in DAE/ODE format in an environment (gPROMS, MATLAB, Simulink) with access to states (x) and Jacobians (J).
- ii. Input-Design: Supply inputs which are expected to occur in online applications.
- iii. The model is “smooth”, given

$$\dot{x} = f(x, u, y) \quad 0 = g(x, u, y)$$

It implies that f and g are differentiable, in other words the Jacobians exist!

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4



2. Approach

- i. To approximate the NL model by certain ODE structure (Hammerstein).
- ii. Structure is important for reduction, because it gives insight with respect to calculations and manipulations. It provides “handles” for reduction.

3. Reduction in computational effort

- i. DAE is replaced by ODE, so the algebraic computations are “removed”.
- ii. Exploitation of the structure:
 - Order reduction.
 - Efficient approximation of the steady state behavior.
 - Efficient approximation of the dynamic behavior.
- iii. Efficient implementation, for example: Matlab → C → executable.

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5

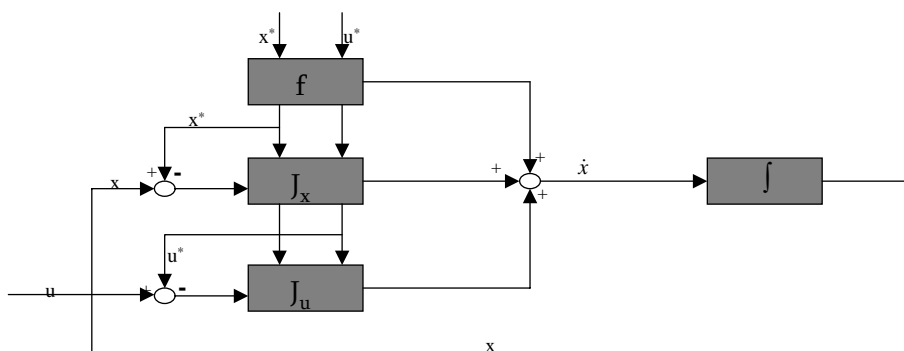


Theory; Taylor expansion for linearization

Given the ODE $\dot{x} = f(x, u)$ A first order Taylor expansion around the point (x^*, u^*) is given by:

$$f(x, u) = f(x^*, u^*) + \left. \frac{\partial f}{\partial x} \right|_{x^*, u^*} (x - x^*) + \left. \frac{\partial f}{\partial u} \right|_{x^*, u^*} (u - u^*) \rightarrow A$$

This can also be represented as a block diagram...



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6

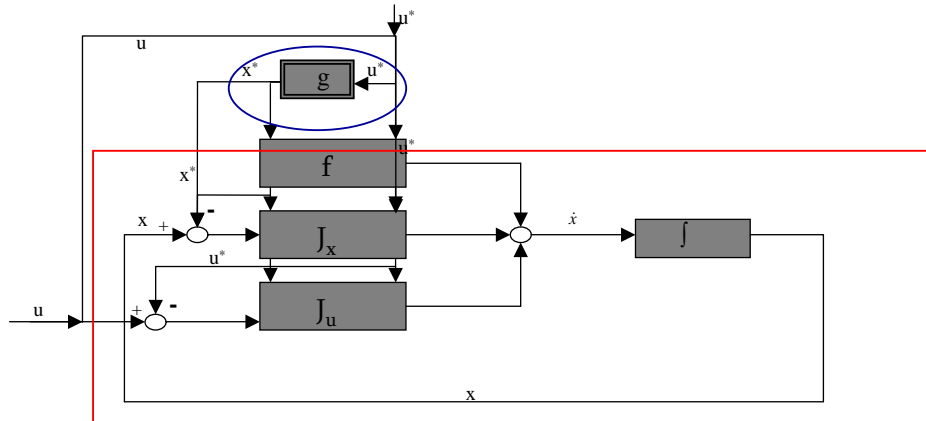


Theory; Taylor expansion to Hammerstein structure

We now choose the point (x^*, u^*) :

- Suppose u^* leads (finally) to a steady state x_{ss} , it is assumed that $x_{ss} = g(u)$. We set $x^* = x_{ss}$, and as a result the output of block f becomes zero.
- We set $u = u^*$, so the input to block J_u becomes zero and as a result the output of this block zero as well.

Rearrangement of the blocks reveals an input to state Hammerstein structure.



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7



Theory; Accuracy improvement by higher order approximation

The accuracy of the approximation can be improved by extending Taylor series to higher order.

Taylor series around the point (x^*, u^*) is given by:

$$f(x, u) = f(x^*, u^*) + \left. \frac{\partial f}{\partial x} \right|_{x^*, u^*} (x - x^*) + \mathcal{O}(x - x^*)^2 \rightarrow 1$$

Taylor series around the point (x, u) is given by:

$$f(x^*, u^*) = f(x, u) + \left. \frac{\partial f}{\partial x} \right|_{x, u} (x^* - x) + \mathcal{O}(x^* - x)^2 \rightarrow 2$$

Adding equation 1 and 2 and rearranging leads to:

$$f(x, u) = \frac{1}{2} \left(\left. \frac{\partial f}{\partial x} \right|_{x^*, u^*} + \left. \frac{\partial f}{\partial x} \right|_{x, u} \right) (x - x^*) \rightarrow 3$$

Equation 3 gives as second order approximation for function $f(x, u)$ by evaluating the Jacobian at the steady-state and the current state.

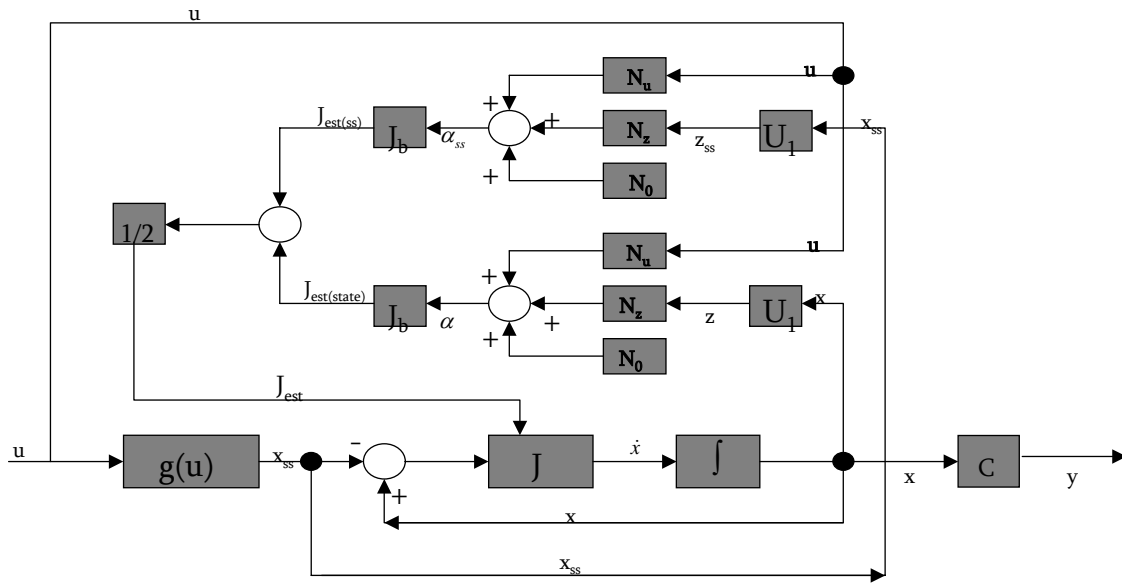
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8



Theory; Input-state Hammerstein structure-I

The block diagram for input to state Hammerstein structure is (the Jacobian is already approximated)...



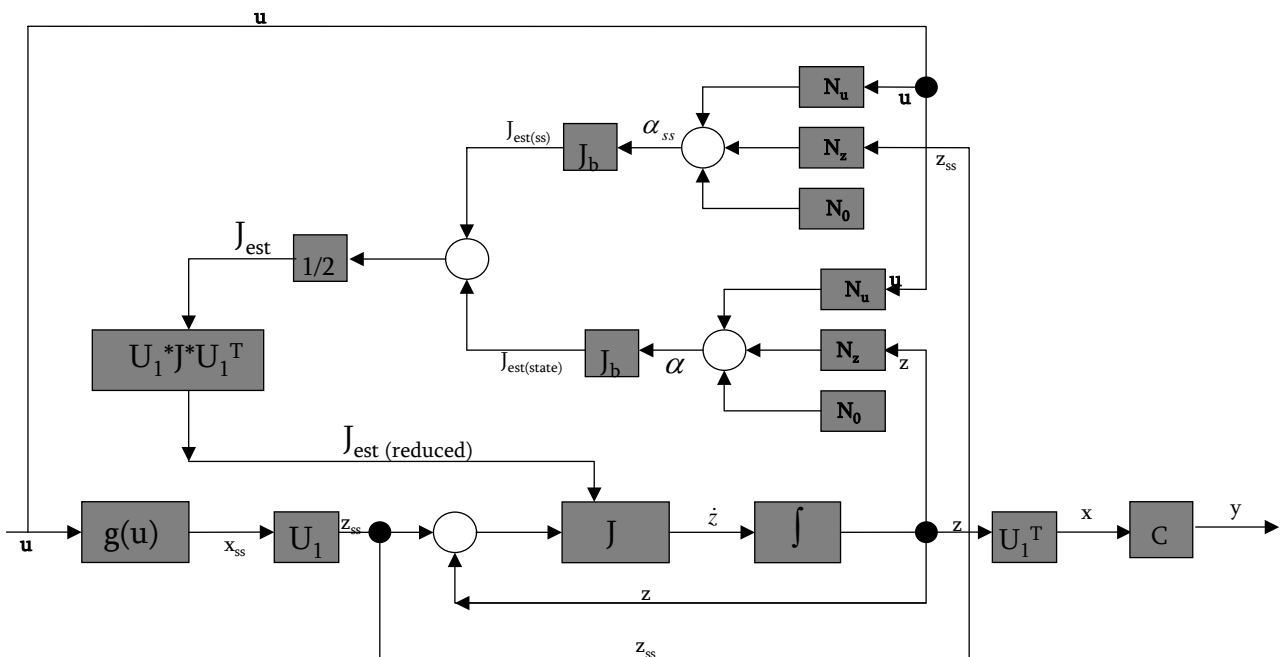
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9



Theory; Input-state Hammerstein structure (reduced)-II

The block diagram for reduced order input to state Hammerstein structure is...



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10



Results; Distillation column

- Implementation on a benchmark distillation (High purity 2-cut splitter, 74 trays)

Model of the system:

- Assumptions :
- a) On each tray liquid and vapor phases are well mixed & in thermodynamic equilibrium ($\alpha = 1.33$)
 - b) Liquid molar hold up is constant, Vapor hold ups are negligible
 - c) Pressure is constant and uniform;
 - d) Equimolal overflow ($L_n = L_{n+1}$ & $V_{n-1} = V_n$)

Over all mass balance :

$$F = D + B$$

Mass Balance on trays:

$$\frac{dx_1}{dt} = (L_{in}x_{1_{in}} + V_{in}y_{1_{in}} - L_{out}x_{1_{out}} - V_{out}y_{1_{out}} + F * z_f) / M_1$$

Equilibrium equation:

$$y_1 = \frac{\alpha * x_1}{1 + (\alpha - 1) * x_1}$$

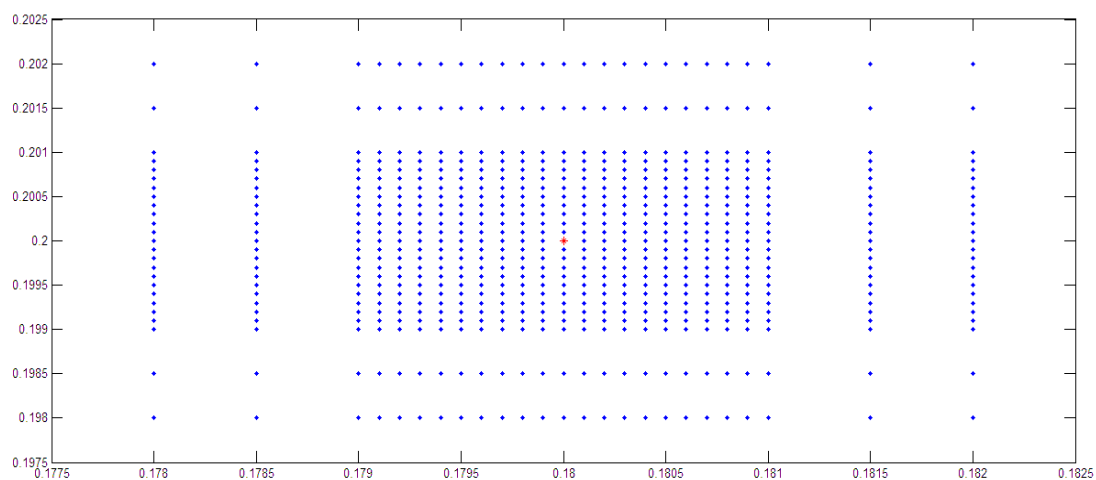
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11



Results; Input design

The operating domain of the application is shown. The operating domain is chosen by keeping in view the input and/or output constraints (input design).

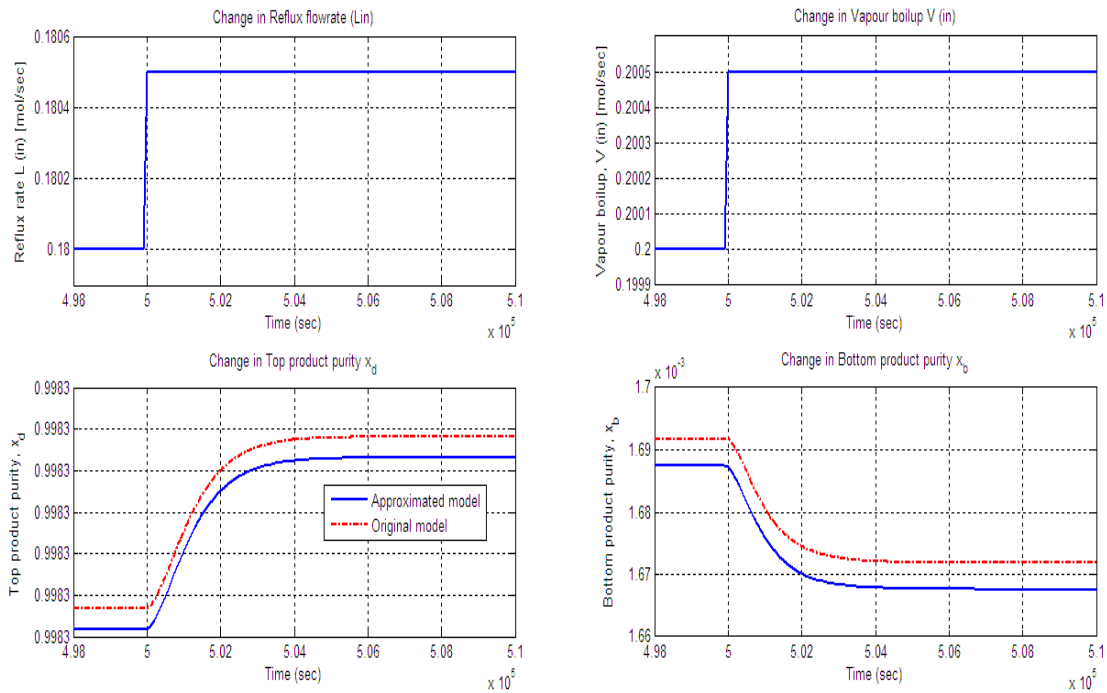


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12



Results; Separation index

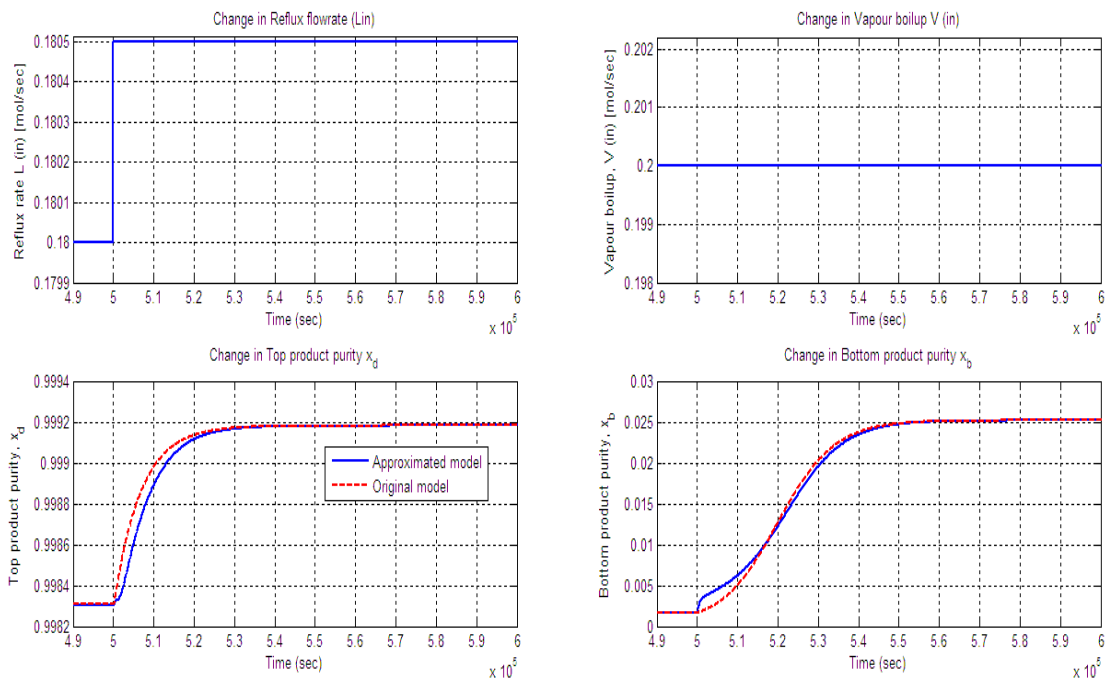


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13



Results; Effective cut point- I

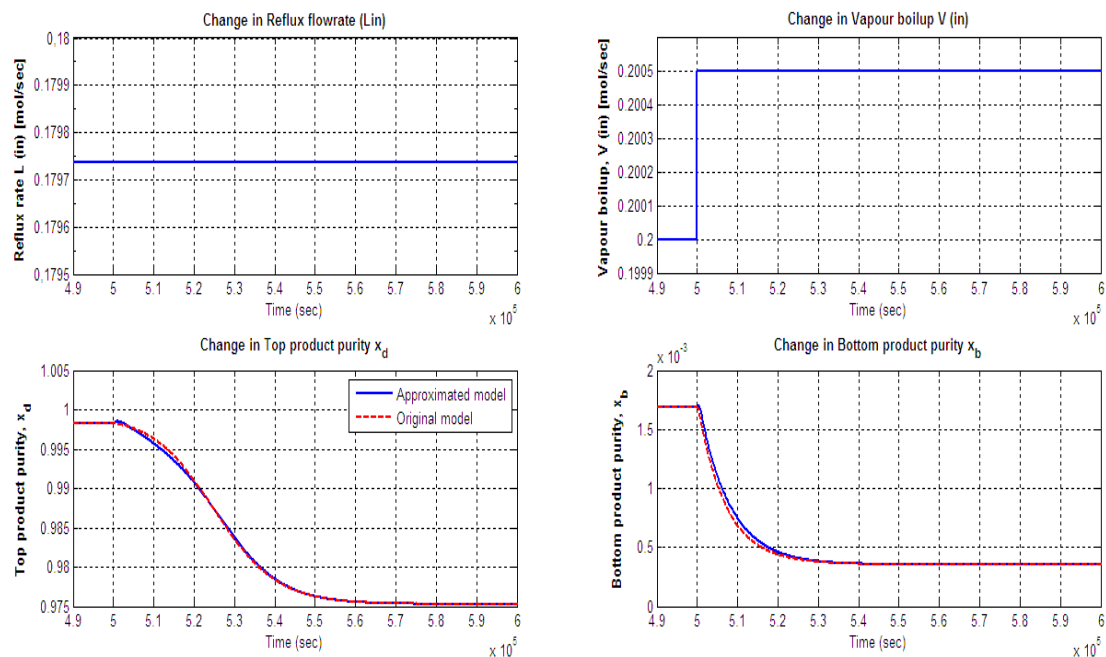


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14



Results; Effective cut point- II

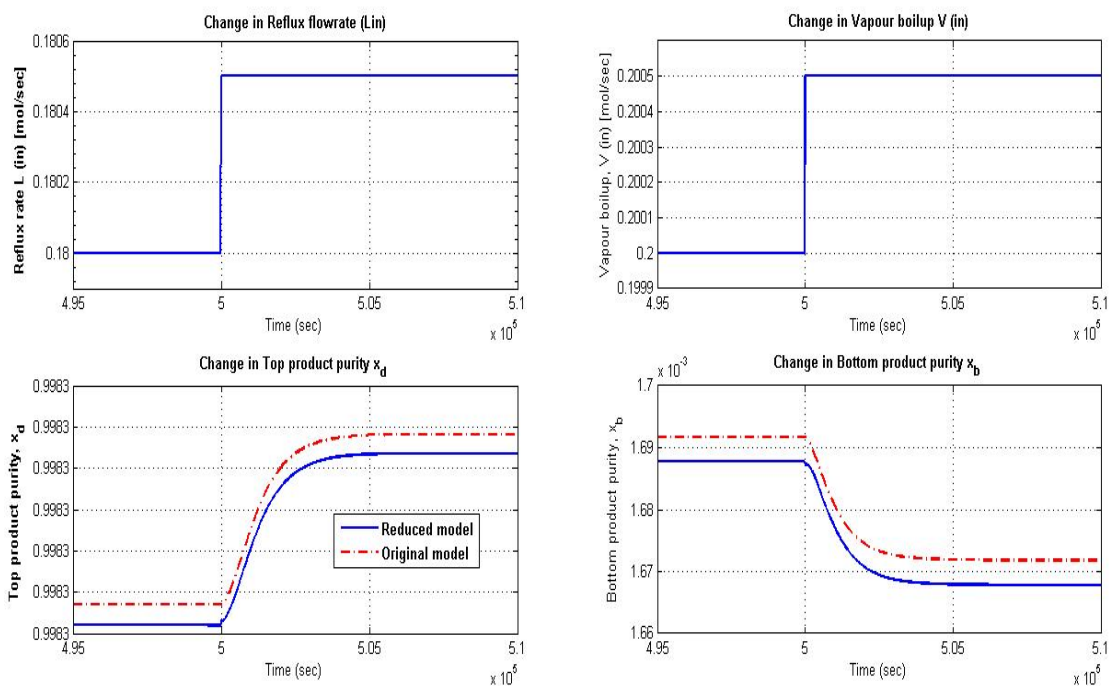


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15



Results; Separation index reduced model

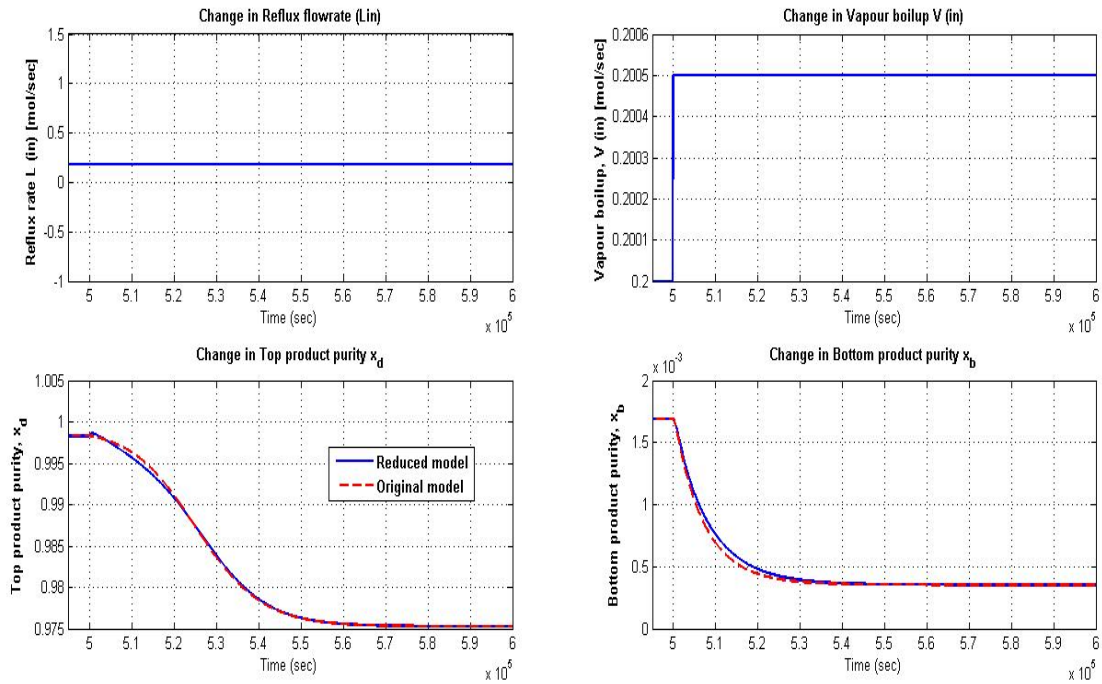


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16



Results; Effective cut point- I (reduced model)



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17



Conclusions;

1. Input to state Hammerstein structure can be derived from a Taylor expansion under certain assumptions.
2. It has been shown that the input to state Hammerstein structure can achieve second order accuracy.
3. The input to state Hammerstein structure approximates the high purity distillation column very well.
4. The reduced order input to state Hammerstein structure also approximates the high purity distillation column very well. A reduction to 30% of the original states was achieved with sufficient output accuracy.

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18



Future work;

- Further work will focus on:
 1. Is further state reduction possible?
 2. The reduction in computational effort so:
 - Efficient approximation of the steady state map.
 - Efficient approximation of Jacobian.
 - Others ways to increase computational efficiency.
 - Efficient implementations so Matlab \rightarrow C \rightarrow executable.
 3. Comparison with original model in gPROMS and other reduction techniques.
 4. Application to other test cases.

October 31, 2008

19



Discussion/Questions

October 31, 2008

20

