ROBUST GUARANTEED COST PID CONTROLLER DESIGN FOR NETWORKED CONTROL SYSTEMS (NCSs)

Thuan, Nguyen Quang , Vojtech Vesely

Slovak University of Technology, Faculty of Electrical Engineering and Information Technology
Ilkovičova 3, 812 19 Bratislava, Slovak Republic
Tel.: +421 2 60291111 Fax: +421 2 60291111
e-mail: thuan.quang@stuba.sk

Abstract: The paper addresses the problem of output feedback guaranteed cost controller design for NCSs with time-delay and polytopic uncertainties. By constructing a new parameter-dependent Lyapunov functional and applying the free-weighting matrices technique, the parameter-dependent, delay-dependent design method will be obtained to synthesize a PID controllers achieving a guaranteed cost such that the NCSs can be stabilized for all admissible uncertainties and time-delays. Finally, numerical examples are given to illustrate the effectiveness of the proposed method.

Keywords: PID controller, output feedback, Networked Control Systems (NCSs), polytopic system, parameter-dependent quadratic stability, time-delay system.

1 INTRODUCTION

Feedback control systems wherein the loops are closed through real-time networks are called Networked Control Systems (NCSs) (Ray and Halevi, 1988; Nilson, 1998; Walsh, Ye, Bushnell, 1999; Zhang, Branicky and Philips, 2001). Advantages of using NCSs in the control area include simplicity, cost-effectiveness, ease of system diagnosis and maintenance, increased system agility and testability. However, integration of communication real-time networks into feedback control loops inevitable leads to some problems. As a result, it leads to a network-induced delay in networked control closed-loop system. The existence of such kind of delay in a network-based control loop can induce instability or poor performance of control systems (Jiang and Han, 2008).

In the recent years, the stability analysis and controller synthesis for systems with time-delay are important in theory and practice (Basin, Perez and Martinez-Zuniga, 2006; Boukaz and Al-Muthairi, 2006). In the time domain, there are two approaches for controller design and studying of stability of closed-loop systems: Razumikhin theorem and Lyapunov-Krasovskii functional (LKF) approach. It is well know that the LKF approach can provide less conservative results than Razumikhin theorem (Friedman and Niculescu, 2008; Richard, 2003; Kharintonov and Melchor-Aquilar, 2000) and references therein. Existing criteria for asymptotic stability of time-delay systems can be classified into categories: delay-independent criteria and delay-dependent. And it is also know that the delay-dependent criteria make use of information on the length of delays, they are less conservative than the delay-independent ones, even if the time delays are very small. On the other hand, a wide class of uncertainty types studied in the system and control literature fall into the polytopic perturbations. For the time-delay system with polytopic-type uncertainties, the parameter-dependent stability condition is of less conservativeness than quadratic stability condition. Recently, free-weighting matrices method or slack-variable method and cross term bounding method was developed to obtain less conservative condition (Mondie, Kharitonov, Santos, 2005; Y. He, Q. G. Wang, L. Xie and C. Lin, 2007) and reference therein.

The guaranteed cost control approach has been extended to the uncertain time-delay systems, for the state feedback case, see (Yu and Chu, 1999; Lee and GyuLee, 1999; Zhang, Boukas and Haidar, 2008) and for output feedback (Chen, Guan, and Lu, 2004). In the paper Chen, Guan and Lu, 2004 the authors consider the full order strictly proper dynamic output feedback controller. However, it seems that there is no previous result on delay-dependent guaranteed cost control via PID output feedback.

Motivated by the above observation, in this article, the parameter-dependent, delay-dependent design method will be studied to design a robust output feedback PID controller achieving a guaranteed cost such that the NCSs can be stabilized for all admissible polytopic-type uncertainties and time-delays. Sufficient condition for existence of a guaranteed cost output feedback controller is established in term of matrix inequalities.

This paper is organized as follows. Section 2 gives the problem formulation. Section 3 explains main results of the paper. And in section 4 numerical examples are presented to show the effectiveness of the proposed method.

Notation: Throughout this paper, for real matrix M, the notation $M \geq 0$ (respectively $M > 0$) means that matrix M is symmetric and positive semi-definite (respectively positive definite); “* “denotes a block that is readily inferred by symmetry; Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider the following linear time-delay system described

$$\begin{aligned} \dot{x}(t) &= A(\xi)x(t) + A_d(\xi)x(t-\tau) + B(\xi)u(t) \\ y(t) &= Cx(t) \\ x(t) &= \varphi(t), t \in [-\tau_M, 0] \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^l$ is the controlled output (measured output). The matrices $A(\xi), A_d(\xi), B(\xi) \in S$ belong to convex hull, and S is a polytope with N vertices $S_1, S_2, .., S_N$ which can formally defined as:

$$S := \left\{ \begin{array}{l} A(\xi), A_d(\xi) \in R^{n \times n}, B(\xi) \in R^{n \times m} : A(\xi) = \sum_{i=1}^{N} \xi_i A_i \ , \ A_d(\xi) = \sum_{i=1}^{N} \xi_i A_{di} \ , \\ B(\xi) = \sum_{i=1}^{N} \xi_i B_i \ , \ \sum_{i=1}^{N} \xi_i = 1 \ , \ \xi_i \geq 0 \end{array} \right\} \quad (2)$$

where A_i, A_{di}, B_i are constant matrices with appropriate dimensions and ξ_i is time-invariant uncertainty; τ_M is the upper bound of time delay and $\varphi(t)$ is a continuously differentiable initial function. Note S is a convex and bounded domain.

We assume that a real-time communication network is integrated into feedback control loops of system (1), and the network induced delay in NCS is given by $0 < \tau \leq \tau_M$ and $\dot{\tau} \leq \mu \leq 1$.

For system (1), we consider the following PID control algorithm

$$u(t) = K_P y(t-\tau) + K_I \int_0^t y(t-\tau)dt + K_D \frac{d}{dt} y(t-\tau) \quad (3)$$

Consider $z(t) = \int y(t-\tau)dt$, $\frac{d}{dt} y(t-\tau) = C_d \dot{x}(t-\tau)$ where C_d is output matrix for derivative output feedback, and then by using Newton-Leibniz formulas $x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(s)ds$ and $\dot{x}(t-\tau) = \dot{x}(t) - \int_{t-\tau}^{t} \ddot{x}(s)ds$, the PID control algorithm (3) can be written as

$$u(t) = FC_n X(t) + F_D C_D \dot{X}(t) - F_P C_P \int_{t-\tau}^{t} \dot{X}(s)ds - F_D C_D \int_{t-\tau}^{t} \ddot{X}(s)ds \tag{4}$$

where
$$X(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T$$
$$F = \begin{bmatrix} K_P & K_I \end{bmatrix},\ F_P = \begin{bmatrix} K_P & 0 \end{bmatrix},\ F_D = \begin{bmatrix} K_D & 0 \end{bmatrix}$$
$$C_n = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix},\ C_P = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix},\ C_D = \begin{bmatrix} C_d & 0 \\ 0 & 0 \end{bmatrix}$$

Consider $\dot{z}(t) = C_i x(t-\tau) = C_i x(t) - C_i \int_{t-\tau}^{t} \dot{x}(s)ds$ where C_i is output matrix for integral output feedback, the system (1) can be expanded in the following form

$$\dot{X}(t) = A_n(\xi)X(t) + B_n(\xi)u(t) - A_{dn}(\xi) \int_{t-\tau}^{t} \dot{X}(s)ds \tag{5}$$

where

$$A_n(\xi) = \sum_{i=1}^{N} \xi_i A_{ni} \quad ,\ A_{ni} = \begin{bmatrix} A_i + A_{di} & 0 \\ C_i & 0 \end{bmatrix}$$
$$B_n(\xi) = \sum_{i=1}^{N} \xi_i B_{ni} \quad ,\ B_{ni} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}$$
$$A_{dn}(\xi) = \sum_{i=1}^{N} \xi_i A_{dni} \ ,\ A_{dni} = \begin{bmatrix} A_{di} & 0 \\ C_i & 0 \end{bmatrix}$$

Applying the PID control algorithm (4) to system (5) will result in the closed-loop system

$$M_d(\xi)\dot{X}(t) + A_c(\xi)X(t) + A_{dc}(\xi) \int_{t-\tau}^{t} \dot{X}(s)ds + A_{dd}(\xi) \int_{t-\tau}^{t} \ddot{X}(s)ds = 0 \tag{6}$$

where
$$M_d(\xi) = \sum_{i=1}^{N} \xi_i M_{di} \ ,\ M_{di} = I - B_{ni} F_D C_D$$
$$A_c(\xi) = \sum_{i=1}^{N} \xi_i A_{ci} \quad ,\ A_{ci} = -\left(A_{ni} + B_{ni} F C_n\right)$$
$$A_{dc}(\xi) = \sum_{i=1}^{N} \xi_i A_{dci} \ ,\ A_{dci} = A_{dni} + B_{ni} F_P C_P$$
$$A_{dd}(\xi) = \sum_{i=1}^{N} \xi_i A_{ddi} \ ,\ A_{ddi} = B_{ni} F_D C_D$$

Given positive definite symmetric matrices Q, R and S, we will consider the cost function

$$J = \int_0^\infty J(t)dt \tag{7}$$

where $J(t) = X^T(t)QX(t) + u^T(t)Ru(t) + \dot{X}^T(t-\tau)S\dot{X}(t-\tau)$

Consider $\eta(t) = \begin{bmatrix} \dot{X}^T(t) & X^T(t) & \int_{t-\tau}^{t} \dot{X}^T(s)ds & \int_{t-\tau_M}^{t-\tau} \dot{X}^T(s)ds & \int_{t-\tau}^{t} \ddot{X}^T(s)ds \end{bmatrix}^T$ and by substituting $u(t)$ from (4) to $u^T(t)Ru(t)$ we obtain

$$u^T(t)Ru(t) = \eta^T(t)K^T RK\eta(t)$$

where $K = \begin{bmatrix} F_D C_D & FC_n & -F_P C_P & 0 & -F_D C_D \end{bmatrix}$

Using $\eta(t)$, $J(t)$ can be rewritten as follows

$$J(t) = \eta^T(t) M_Q(\xi)\eta(t) \tag{8}$$

where

$$M_Q(\xi) = \begin{bmatrix} C_D^T F_D^T RF_D C_D + S & C_D^T F_D^T RFC_n & -C_D^T F_D^T RF_P C_P & 0 & -C_D^T F_D^T RF_D C_D - S \\ * & C_n^T F^T RFC_n + Q & -C_n^T F^T RF_P C_P & 0 & -C_n^T F^T RF_D C_D \\ * & * & C_P^T F_P^T RF_P C_P & 0 & C_P^T F_P^T RF_D C_D \\ * & * & * & 0 & 0 \\ * & * & * & * & C_D^T F_D^T RF_D C_D + S \end{bmatrix}$$

Associated with the cost, the guaranteed cost controller is defined as follows:

Definition 1.

Consider the uncertain system (1). If there exist a controller of form (3) and a positive scalar J_0 such that for all uncertainties (2), the closed-loop system (6) is asymptotically stable and closed-loop value of the cost function (7) satisfies $J \le J_0$ then J_0 is said to be a guaranteed cost and the controller (2) is said to be guaranteed cost controller.

Finally we introduce the well known results from LQ theory.

Lemma 1.

Consider the continuous-time delay system (5) with control algorithm (3). The control algorithm (3) is the guaranteed cost control for system (5) if and only if there exists LKF $V(\xi, t)$ such that the following condition holds:

$$\frac{d}{dt}V(\xi, t) + J(t) \le 0 \tag{9}$$

The objective of this paper is to develop a procedure to design a robust PID controller of form (4) which ensure parameter-dependent the closed-loop system stability and guaranteed cost.

3 MAIN RESULTS

The following theorem provides robust parameter-dependent quadratic stability and robust performance results for the closed-loop system (6).

Theorem 1.

Consider the uncertain linear time-delay system (1) with network-induced delay τ satisfying $0 < \tau \le \tau_M$, $\dot{\tau} \le \mu \le 1$ and the cost function (7). If there exist a PID controller of form (3), scalar J_0, and matrices $P_i > 0$, $G_i > 0$, $G_{1i} > 0$, $G_{2i} > 0$, $G_{3i} > 0$ $(i = 1,\dots,N)$, N_1, N_2, N_3, N_4, and N_5 that satisfy the following matrix inequality

$$W_i = \begin{bmatrix} w_i^{11} & w_i^{12} & w_i^{13} & w_i^{14} & w_i^{14} \\ * & w_i^{22} & w_i^{23} & w_i^{24} & w_i^{25} \\ * & * & w_i^{33} & w_i^{34} & w_i^{35} \\ * & * & * & w_i^{44} & w_i^{45} \\ * & * & * & * & w_i^{55} \end{bmatrix} \le 0 \qquad \textbf{(10)}$$

where

$$\begin{aligned}
w_i^{11} &= N_1 M_{di} + M_{di}^T N_1^T + \tau_M G_{1i} + \mu G_{3i} + C_D^T F_D^T R F_D C_D + S \\
w_i^{12} &= N_1 A_{ci} + M_{di}^T N_2^T + P_i + C_D^T F_D^T R F C_n \\
w_i^{13} &= N_1 A_{dci} + M_{di}^T N_3^T - C_D^T F_D^T R F_P C_P \\
w_i^{14} &= M_{di}^T N_4^T \\
w_i^{15} &= N_1 A_{ddi} + M_{di}^T N_5^T + (1-\mu)\mu G_{3i} - C_D^T F_D^T R F_D C_D - S \\
w_i^{22} &= N_2 A_{ci} + A_{ci}^T N_2^T + \mu G_i + C_n^T F^T R F C_n + Q \\
w_i^{23} &= N_2 A_{dci} + A_{ci}^T N_3^T + (1-\mu) G_i + G_{2i} - C_n^T F^T R F_P C_P \\
w_i^{24} &= A_{ci}^T N_4^T + G_{2i} \\
w_i^{25} &= N_2 A_{ddi} + A_{ci}^T N_5^T - C_n^T F^T R F_D C_D \\
w_i^{33} &= N_3 A_{dci} + A_{dci}^T N_3^T - (1-\mu) G_i - \frac{1}{\tau_M} G_{1i} - G_{2i} + C_P^T F_P^T R F_P C_P \\
w_i^{34} &= A_{dci}^T N_4^T - G_{2i} \\
w_i^{35} &= N_3 A_{ddi} + A_{dci}^T N_5^T + C_P^T F_P^T R F_D C_D \\
w_i^{44} &= -G_{2i} - \frac{1}{\tau_M} G_{1i} \\
w_i^{45} &= N_4 A_{ddi} \\
w_i^{55} &= N_5 A_{ddi} + A_{ddi}^T N_5^T - (1-\mu) G_{3i} + C_D^T F_D^T R F_D C_D + S
\end{aligned}$$

Then the uncertain system (1) with controller (3) is parameter-dependent quadratically-asymptotically stable and the cost function (7) satisfies the following bound

$$J \le J_0 = \sqrt{\lambda_{MP}^2 + \lambda_{MG}^2 + \lambda_{MG1}^2 + \lambda_{MG2}^2 + \lambda_{MG3}^2} * J_M \qquad \textbf{(11)}$$

where

$$\lambda_{MP} = \underset{i=1..N}{Max}(Max(Eigenvalue(P_i))), \ \lambda_{MG} = \underset{i=1..N}{Max}(Max(Eigenvalue(G_i))),$$

$$\lambda_{MG1} = \underset{i=1..N}{Max}(Max(Eigenvalue(G_{1i}))) \,,\; \lambda_{MG2} = \underset{i=1..N}{Max}(Max(Eigenvalue(G_{2i})))$$

$$\lambda_{MG3} = \underset{i=1..N}{Max}(Max(Eigenvalue(G_{3i})))$$

$$J_M = \sqrt{\|x_0\|^4 + \left(\int_{-\tau}^{0}\|\varphi(s)\|^2 ds\right)^2 + \left(\int_{-\tau}^{0} d\theta\int_{\theta}^{0}\|\dot\varphi(s)\|^2 ds\right)^2 + \left(\int_{-\tau_M}^{-\tau}\|\varphi(s)\|^2 ds\right)^2 + \left(\int_{-\tau}^{0}\|\dot\varphi(s)\|^2 ds\right)^2}$$

Proof.

Consider the Lyapunov-Krasovskii functional as follows

$$V(\xi,t) = \sum_{i=1}^{5} V_i(\xi,t) \tag{12}$$

$$V_1(\xi,t) = X^T(t)\,P(\xi)\,X(t)\;; V_2(\xi,t) = \int_{t-\tau}^{t} X^T(s)\,G(\xi)\,X(s)ds\;; V_4(\xi,t) = \int_{t-\tau_M}^{t} X^T(s)\,G_2(\xi)\,X(s)ds$$

$$V_3(\xi,t) = \int_{-\tau}^{0} d\theta \int_{t+\theta}^{t} \dot X^T(s)\,G_1(\xi)\,\dot X(s)ds\;;\; V_5(\xi,t) = \int_{t-\tau}^{t} \dot X^T(s)\,G_3(\xi)\,\dot X(s)ds$$

Differentiating $V(\xi,t)$ with respect to t and using Newton-Leibniz formula $x(t-\tau) = x(t) - \int_{t-\tau}^{t}\dot x(s)ds$, we obtain

$$\dot V_1(\xi,t) = 2X^T(t)\,P(\xi)\,\dot X(t)$$

$$\dot V_2(\xi,t) \le \eta_1^T(t)\begin{bmatrix}\mu G(\xi) & (1-\mu)G(\xi)\\ * & -(1-\mu)G(\xi)\end{bmatrix}\eta_1(t) \;,\; \eta_1^T(t) = \begin{bmatrix}X^T(t) & \int_{t-\tau}^{t}\dot X^T(s)ds\end{bmatrix}$$

$$\dot V_3(\xi,t) \le \tau_M \dot X^T(t) G_1(\xi)\dot X(t) - \frac{1}{\tau_M}\int_{t-\tau}^{t}\dot X^T(s)ds.G_1(\xi).\int_{t-\tau}^{t}\dot X(s)ds$$

$$\dot V_4(\xi,t) \le \eta_2^T(t)\begin{bmatrix}0 & G_2(\xi) & G_2(\xi)\\ * & -G_2(\xi) & -G_2(\xi)\\ * & * & -G_2(\xi)\end{bmatrix}\eta_2(t) \;,\; \eta_2^T(t) = \begin{bmatrix}X^T(t) & \int_{t-\tau}^{t}\dot X^T(s)ds & \int_{t-\tau_M}^{t-\tau}\dot X^T(s)ds\end{bmatrix}$$

$$\dot V_5(\xi,t) \le \eta_3^T(t)\begin{bmatrix}\mu G_3(\xi) & (1-\mu)G_3(\xi)\\ * & -(1-\mu)G_3(\xi)\end{bmatrix}\eta_3(t) \;,\; \eta_3^T(t) = \begin{bmatrix}X^T(t) & \int_{t-\tau}^{t}\ddot X^T(s)ds\end{bmatrix}$$

Applying the free-weighting matrices technique, the equation (8) is represented in the following equivalent form

$$\alpha(t) = 2\eta^T(t)\begin{bmatrix}N_1^T & N_2^T & N_3^T & N_4^T & N_5^T\end{bmatrix}^T\begin{bmatrix}M_d(\xi) & A_c(\xi) & A_{dc}(\xi) & 0 & A_{dd}(\xi)\end{bmatrix}\eta(t) = 0$$

After manipulation of the above equation, we obtain

$$\alpha(t) = \eta^T(t) M_\alpha(\xi)\eta(t) = 0 \tag{13}$$

where

$$M_\alpha(\xi)=\begin{bmatrix} N_1M_d(\xi)+M_d^T(\xi)N_1^T & N_1A_c(\xi)+M_d^T(\xi)N_2^T & N_1A_{dc}(\xi)+M_d^T(\xi)N_3^T & M_d^T(\xi)N_4^T & N_1A_{dd}(\xi)+M_d^T(\xi)N_5^T \\ * & N_2A_c(\xi)+A_c^T(\xi)N_2^T & N_2A_{dc}(\xi)+A_c^T(\xi)N_3^T & A_c^T(\xi)N_4^T & N_2A_{dd}(\xi)+A_c^T(\xi)N_5^T \\ * & * & N_3A_{dc}(\xi)+A_{dc}^T(\xi)N_3^T & A_{dc}^T(\xi)N_4^T & N_3A_{dc}(\xi)+A_{dc}^T(\xi)N_5^T \\ * & * & * & 0 & N_4A_{dd}(\xi) \\ * & * & * & * & N_5A_{dd}(\xi)+A_{dd}^T(\xi)N_5^T \end{bmatrix}$$

Because of $\alpha(t)=0$, thus

$$\dot V(\xi,t)=\sum_{i=1}^{5}\dot V_i(\xi,t)+\alpha(t)\le\eta^T(t)\left[M_\alpha(\xi)+M_V(\xi)\right]\eta^T(t) \tag{14}$$

where

$$M_V(\xi)=\begin{bmatrix} \tau_M G_1(\xi)+\mu G_3(\xi) & P(\xi) & 0 & 0 & (1-\mu)G_3(\xi) \\ * & \mu G(\xi) & (1-\mu)G(\xi)+G_2(\xi) & G_2(\xi) & 0 \\ * & * & -(1-\mu)G(\xi)-G_2(\xi)-\frac{1}{\tau_M}G_1(\xi) & -G_2(\xi) & 0 \\ * & * & * & -G_2(\xi)-\frac{1}{\tau_M}G_1(\xi) & 0 \\ * & * & * & * & -(1-\mu)G_3(\xi) \end{bmatrix}$$

Due to lema1, the closed-loop system (6) is robustly asymptotically stable and give an upper bound (a guaranteed cost) for the cost function (7) if

$$\dot V(\xi,t)+J(t)\le\eta^T(t)W(\xi)\eta^T(t)\le 0 \iff W(\xi)\le 0 \tag{15}$$

where $W(\xi)=\sum_{i=1}^{N}\xi_iW_i = M_\alpha(\xi)+M_V(\xi)+M_Q(\xi)$

If for each $W_i\le 0$, $i=1...N$, then $W(\xi)=\sum_{i=1}^{N}\xi_iW_i\le 0$. Therefore, $\dot V(\xi,t)\le -J(t)\le 0$ $(J(t)\ge 0)$, respectively $J(t)\le-\dot V(\xi,t)$. By integrating $J(t)\le-\dot V(\xi,t)$ we obtain

$$J\le-\int_0^\infty \dot V(\xi,t)dt=V_0=X_0^TP(\xi)X_0+\int_{-\tau}^{0}X^T(s)G(\xi)X(s)ds+\int_{-\tau}^{0}d\theta\int_{\theta}^{0}\dot X^T(s)G_1(\xi)\dot X(s)ds+$$
$$+\int_{-\tau_M}^{0}X^T(s)G_2(\xi)X(s)ds+\int_{-\tau}^{0}\dot X^T(s)G_3(\xi)\dot X(s)ds$$

Because of $X(t) = \begin{bmatrix} \varphi^T(t) & 0 \end{bmatrix}$, $\forall t \in [-\tau_M, 0]$ then

$$V_0 \le \lambda_{MP} \|x_0\|^2 + \lambda_{MG} \int_{-\tau}^{0} \|\varphi(s)\|^2 ds + \lambda_{MG1} \int_{-\tau}^{0} d\theta \int_{\theta}^{0} \|\dot{\varphi}(s)\|^2 ds + \lambda_{MG2} \int_{-\tau_M}^{0} \|\varphi(s)\|^2 ds + \lambda_{MG3} \int_{-\tau}^{0} \|\dot{\varphi}(s)\|^2 ds$$

I t is known, that for two arbitrary vectors $\overline{X}, \overline{Y}$, the following inequality hold:

$$\overline{X}^T \, \overline{Y} \le \|\overline{X}\| \|\overline{Y}\| \tag{16}$$

Consider $\overline{X} = \begin{bmatrix} \lambda_{MP}^T & \lambda_{MG}^T & \lambda_{MG1}^T & \lambda_{MG2}^T & \lambda_{MG3}^T \end{bmatrix}^T$

$$\overline{Y} = \left[\left(\|x_0\|^2 \right)^T \quad \left(\int_{-\tau}^{0} \|\varphi(s)\|^2 ds \right)^T \quad \left(\int_{-\tau}^{0} d\theta \int_{\theta}^{0} \|\dot{\varphi}(s)\|^2 ds \right)^T \quad \left(\int_{-\tau_M}^{0} \|\varphi(s)\|^2 ds \right)^T \quad \left(\int_{-\tau}^{0} \|\dot{\varphi}(s)\|^2 ds \right)^T \right]^T$$

Applying the inequality (16), to above equation the upper bound cost function (7) J_0 is obtained as (11).

The *theorem 1.* is proved.

4 EXAMPLES

In this section we present the results of numerical calculations of two examples to design a robust output feedback PID controller with guaranteed cost for NCSs with time-delay. Design procedure based on BMI inequalities (10).

Example 1 has been borrowed from Benton and Smith, 1999 to demonstrate the used for algorithm (10) on the problem robustly stabilizing with a guaranteed cost a vertical take-off and landing of a helicopter. The system is control through NCS with time-varying time-delay $0 < \tau \le \tau_M = 200$ $[ms]$, $\dot{\tau} \le \mu = 0.99$. Let uncertain matrices A, B, C, Ad be defined as

$$A = \begin{bmatrix} -0.036 & 0.0270 & 0.0188 & -0.4555 \\ 0.0482 & -1.010 & 0.0024 & -4.0208 \\ 0.1002 & q_1(t) & -0.707 & q_2(t) \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 4.422 & 0.1761 \\ q_3(t) & -7.59222 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}, C^T = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; A_d = 0$$

with parameters bounds for all time

$-0.6319 \le q_1(t) \le 1.3681$, $1.22 \le q_2(t) \le 1.420$, $2.7446 \le q_3(t) \le 4.3446$. The above model has been recalculated to the form (1). The respective eight vertices are calculated. Note that the matrix A is unstable with *max(real(eigenvalue(A)))=1.2675*. The results of calculation for the case $r = 1, q = 0.1, s = 0.001, r_0 = 10$ as follows

$$F = \begin{bmatrix} K_P & K_I \end{bmatrix} = \begin{bmatrix} -0.2788 & 0.0927 \\ 0.5857 & 0.4086 \end{bmatrix}, \; K_D \cong 0$$

The *max(real(eigenvalue(Close-loop)))=-0.072209*. And guaranteed cost $J_0 = 17{,}9128 \, J_M$ where $\lambda_{MP} = \lambda_{MG} = \lambda_{MG2} = 8.9794, \lambda_{MG1} = 8.8841, \; \lambda_{MG3} = 0.2319$.

Example 2 We consider the linear model of two cooperating DC motors. The problem is to design two PI controllers for a laboratory MIMO system which guarantee robust stability with a guaranteed cost. The system is control through NCS with time-varying time-delay $0 < \tau \le \tau_M = 100$ $[ms]$, $\dot{\tau} \le \mu = 0.2$. The system model is given with a time invariant matrix affine type uncertain structure, where

$$A_0 = \begin{bmatrix} 0 & -.2148 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1.014 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2605 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -.9107 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.1639 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -.8137 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.2279 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -.8251 \end{bmatrix}, \; B_0 = \begin{bmatrix} .3148 & 0 \\ .0478 & 0 \\ 0 & -.1028 \\ 0 & -.0091 \\ -.0841 & 0 \\ -.0287 & 0 \\ 0 & .3676 \\ 0 & .2448 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & -.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1395 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.0938 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2911 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .0208 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.1173 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.0625 & 0 \\ .0798 & 0 \\ 0 & -.0462 \\ 0 & -.0449 \\ .0016 & 0 \\ .0072 & 0 \\ 0 & .077 \\ 0 & -.00500 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & .0125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0594 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0116 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0308 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.0188 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.01560 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .0208 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.0333 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.0094 & 0 \\ .0151 & 0 \\ 0 & .0019 \\ 0 & -.003 \\ -0.0121 & 0 \\ -.03 & 0 \\ 0 & -.064 \\ 0 & .0189 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \; A_d = 0$$

The above model has been recalculated to the form (1). The respective four vertices are calculated. The results of calculation for the case $r = 1, q = 0.1, s = 0.001, r_0 = 20$ as follows

$$F = [K_P \quad K_I] = \begin{bmatrix} -1.8916 & 0.1128 & 0.2026 & -0.6964 \\ -0.4310 & -1.8916 & 0.8782 & -1.4985 \end{bmatrix}$$

The *max(real(eigenvalue(Close-loop)))=-0.17726*. And guaranteed cost $J_0 = 37{,}7659\, J_M$ where $\lambda_{MP} = 18.916, \lambda_{MG} = 15.0011, \lambda_{MG2} = 18.916, \lambda_{MG1} = 15.5723, \lambda_{MG3} = 15.5918$.

5 CONCLUSION

The guaranteed cost control problem is studied in this paper for a class of linear time-delay uncertain polytopic systems and a given quadratic cost function with three terms (QRS). On base of Lyapunov-Krasovskii functional, new sufficient parameter-dependent quadratic stability conditions are given for output feedback PID controller proposed design procedure in terms of bilinear matrix inequality.

The examples show the effectiveness of the proposed method.

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