STATE CONTROL OF THE DISCRETE TIME-DELAY SYSTEMS

Dušan Krokavec and Anna Filasová

Technical University of Košice, Faculty of Electrical Engineering and Informatics,
Department of Cybernetics and Artificial Intelligence,
Letná 9, 042 00 Košice, Slovak Republic
Tel.: +421 55 602 2564, 4389 Fax: +421 55 625 3574
e-mail: dusan.krokavec@tuke.sk, anna.filasov@.tuke.sk

Abstract: The purpose of this paper is to present an improved version of the time-delay system state feedback control design method. Based on the Lyapunov-Krasovskii functional the delayed-independent stability condition is derived using the linear matrix inequality techniques. The results obtained with a numerical example are presented to compare limitations in the system structure. Since the presented method is based on the convex optimization techniques it is computationally enough efficient.

Keywords: Linear matrix inequality, discrete time-delay systems, Lyapunov-Krasovskii functional**,** state control, asymptotic stability.

1 INTRODUCTION

Control systems are used in many industrial applications, where time delays can take a deleterious effect on both the stability and the dynamic performance in open and closed-loop systems. Therefore the stability and control of the dynamical systems involving time-delayed states is a problem of large theoretical and practical interest where intensive activity are done to eliminate the fixed time delays, to compensate for the uncertain ones or to develop control for the time-delay systems stabilization, especially for uncertain time-delay systems.

During the last decades, considerable attention has been devoted to the problem of stability analysis and controller design for the time-delay systems. The existing stabilization results for the time-delay systems can be delay independent or delay-dependent. The delay-dependent stabilization is concerned with the size of the delay and usually provides an upper bound of the delay such that the closed loop system is stable for any delay less than the upper bound. On the other side, the delay-independent stabilization provides such controller, which can stabilize given system irrespective of the size of the delay.

The use of Lyapunov method for the stability analysis of the time delay systems has been ever growing subject of interest, starting with the pioneering works of Krasovskii [Krasovskii 1956, 1963]. Usually now for the stability issue of the time delay systems some modified Lyapunov-Krasovskii functionals are used (e.g. see [Friedman 2001]) to obtain the delay-independent stabilization and the results based on these functionals are applied to controller synthesis and observer design. This time-delay independent methodology and the bounded inequality techniques are sources of a conservatism that can cause higher norm of the state feedback gain. Progres review in this research field can be found e.g. in [Zhong 2006], and the references therein.

The presented LMI approach is based on the Lyapunov-Krasovskii functional to eliminate some dead-time dependent terms. Since Lyapunov-Krasovskii functional is used, the sufficient conditions for exponential stability can be obtained to set the derivative of this functional staying negative along all the system's trajectories.

2 PROBLEM STATEMENT

Through this paper the task is concerned with the computation of the state feedback $\boldsymbol{u}(i)$ which control the time-delay linear dynamic system given by the set of equations

$$\boldsymbol{q}(i+1)=\boldsymbol{F}\boldsymbol{q}(i)+\boldsymbol{F}_d\boldsymbol{q}(i-h)+\boldsymbol{G}\boldsymbol{u}(i) \tag{1}$$

$$\boldsymbol{y}(i)=\boldsymbol{C}\boldsymbol{q}(i) \tag{2}$$

with the initial condition

$$\boldsymbol{q}(\theta)=\varphi(\theta), \quad \vee\,\theta\in\langle -h,-h+1,\cdots,0\rangle \tag{3}$$

where $k,h\in\mathbb{Z}^+$, $\mathbb{Z}^+$ is the set of positive integers, $h\Delta t$ is unknown time delay in general case, Δt is the sampling period, $\boldsymbol{q}(i)\in\mathbb{R}^n$, $\boldsymbol{u}(i)\in\mathbb{R}^r$, and $\boldsymbol{y}(i)\in\mathbb{R}^m$ are vectors of the state, input and measurable output variables, respectively, given system matrices $\boldsymbol{F},\boldsymbol{F}_d\in\mathbb{R}^{n\times n}$, $\boldsymbol{G}\in\mathbb{R}^{n\times r}$, $\boldsymbol{C}\in\mathbb{R}^{m\times n}$ are real matrices.

Problem of the interest is to design asymptotically stable closed-loop system with the linear memoryless state feedback controller of the form.

$$\boldsymbol{u}(i)=-\boldsymbol{K}\boldsymbol{q}(i) \tag{4}$$

where the matrix $\boldsymbol{K}\in\mathbb{R}^{r\times n}$ is the controller gain matrix.

3 BASIC PRELIMINARIES

Proposition 1. (Schur Complement) *Let* $\boldsymbol{Q}>\boldsymbol{0}$, $\boldsymbol{R}>\boldsymbol{0}$, $\boldsymbol{S}$ *are real matrices of appropriate dimensions, then the next inequalities are equivalent*

$$\begin{bmatrix}\boldsymbol{Q} & \boldsymbol{S}\\ \boldsymbol{S}^T & \boldsymbol{R}\end{bmatrix}>0 \quad\Leftrightarrow\quad \begin{bmatrix}\boldsymbol{Q}-\boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{R}\end{bmatrix}>0 \quad\Leftrightarrow\quad \boldsymbol{Q}-\boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T>0,\ \boldsymbol{R}>0 \tag{5}$$

Proof. Let the linear matrix inequality takes form

$$\begin{bmatrix}\boldsymbol{Q} & \boldsymbol{S}\\ \boldsymbol{S}^T & \boldsymbol{R}\end{bmatrix}<0 \tag{6}$$

then using Gauss elimination it yields

$$\begin{bmatrix}\boldsymbol{I} & -\boldsymbol{S}\boldsymbol{R}^{-1}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{Q} & \boldsymbol{S}\\ \boldsymbol{S}^T & -\boldsymbol{R}\end{bmatrix}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\\ -\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{I}\end{bmatrix}=\begin{bmatrix}\boldsymbol{Q}-\boldsymbol{S}\boldsymbol{R}^{-1}\boldsymbol{S}^T & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{R}\end{bmatrix},\quad \det\begin{bmatrix}\boldsymbol{I} & -\boldsymbol{S}\boldsymbol{R}^{-1}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}=1 \tag{7}$$

and it is evident that this transform doesn't changed positivity of (6), and so (7) implies (5). This concludes the proof.

Proposition 2. (Lyapunov-Krasovskii Inequality) *The autonomous linear time delay system of the form (1) is asymptotically stable if there exists a symmetric positive definite matrix* $\boldsymbol{P}>0$ *such that*

$$\begin{bmatrix}\boldsymbol{Q}-\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{F}^T\boldsymbol{P}\\ * & -\boldsymbol{Q} & \boldsymbol{F}_d^T\boldsymbol{P}\\ * & * & -\boldsymbol{P}\end{bmatrix}<0 \tag{8}$$

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof. Defining Lyapunov-Krasovskii functional candidate as follows

$$v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \sum_{j=1}^{h} \boldsymbol{q}^T(i-j)\boldsymbol{Q}\boldsymbol{q}(i-j) > 0 \tag{9}$$

where $\boldsymbol{P} = \boldsymbol{P}^T > 0$, $\boldsymbol{Q} = \boldsymbol{Q}^T > 0$, $\boldsymbol{P}, \boldsymbol{Q} \in \mathbb{R}^{n \times n}$. The forward difference along the solution of the autonomous system (1) is

$$\begin{gathered}\Delta v(\boldsymbol{q}(i)) = v(\boldsymbol{q}(i+1)) - v(\boldsymbol{q}(i)) = \\ = \boldsymbol{q}^T(i+1)\boldsymbol{P}\boldsymbol{q}(i+1) - \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \sum_{j=1}^{h} \boldsymbol{q}^T(i+1-j)\boldsymbol{Q}\boldsymbol{q}(i+1-j) - \sum_{j=1}^{h} \boldsymbol{q}^T(i-j)\boldsymbol{Q}\boldsymbol{q}(i-j) < 0\end{gathered} \tag{10}$$

$$\Delta v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i+1)\boldsymbol{P}\boldsymbol{q}(i+1) - \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \sum_{j=0}^{h-1} \boldsymbol{q}^T(i-j)\boldsymbol{Q}\boldsymbol{q}(i-j) - \sum_{j=1}^{h} \boldsymbol{q}^T(i-j)\boldsymbol{Q}\boldsymbol{q}(i-j) < 0 \tag{11}$$

$$\Delta v(\boldsymbol{q}(i)) = \boldsymbol{q}^T(i+1)\boldsymbol{P}\boldsymbol{q}(i+1) - \boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \boldsymbol{q}^T(i)\boldsymbol{Q}\boldsymbol{q}(i) - \boldsymbol{q}^T(i-h)\boldsymbol{Q}\boldsymbol{q}(i-h) \tag{12}$$

respectively. Therefore, inserting from (1) into (12) gives

$$\begin{aligned}\Delta v(\boldsymbol{q}(i)) = (\boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{F}_d\boldsymbol{q}(i-h))^T \boldsymbol{P}(\boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{F}_d\boldsymbol{q}(i-h)) - \\ -\boldsymbol{q}^T(i)\boldsymbol{P}\boldsymbol{q}(i) + \boldsymbol{q}^T(i)\boldsymbol{Q}\boldsymbol{q}(i) - \boldsymbol{q}^T(i-h)\boldsymbol{Q}\boldsymbol{q}(i-h) < 0\end{aligned} \tag{13}$$

$$\begin{bmatrix}\boldsymbol{q}^T(i) & \boldsymbol{q}^T(i-j)\end{bmatrix}\begin{bmatrix}\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F} - \boldsymbol{P} + \boldsymbol{Q} & \boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F}_d \\ * & \boldsymbol{F}_d^T\boldsymbol{P}\boldsymbol{F}_d - \boldsymbol{Q}\end{bmatrix}\begin{bmatrix}\boldsymbol{q}(i) \\ \boldsymbol{q}(i-j)\end{bmatrix} < 0 \tag{14}$$

respectively. It is evident that (14) is negative if the next inequality

$$\begin{bmatrix}\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F} - \boldsymbol{P} + \boldsymbol{Q} & \boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F}_d \\ * & \boldsymbol{F}_d^T\boldsymbol{P}\boldsymbol{F}_d - \boldsymbol{Q}\end{bmatrix} < 0 \tag{15}$$

is satified. Thus, (15) can be rewritten as

$$\begin{bmatrix}\boldsymbol{Q} - \boldsymbol{P} & \boldsymbol{0} \\ * & -\boldsymbol{Q}\end{bmatrix} + \begin{bmatrix}\boldsymbol{F}^T \\ \boldsymbol{F}_d^T\end{bmatrix}\boldsymbol{P}\begin{bmatrix}\boldsymbol{F} & \boldsymbol{F}_d\end{bmatrix} < 0 \tag{16}$$

and using Schur complement property (7) it is possible to write

$$\begin{bmatrix}\boldsymbol{F}^T \\ \boldsymbol{F}_d^T\end{bmatrix}\boldsymbol{P}\begin{bmatrix}\boldsymbol{F} & \boldsymbol{F}_d\end{bmatrix} \geq 0 \Leftrightarrow \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}^T \\ * & \boldsymbol{0} & \boldsymbol{F}_d^T \\ * & * & -\boldsymbol{P}^{-1}\end{bmatrix} \geq 0 \Leftrightarrow \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}^T\boldsymbol{P} \\ * & \boldsymbol{0} & \boldsymbol{F}_d^T\boldsymbol{P} \\ * & * & -\boldsymbol{P}\end{bmatrix} \geq 0 \tag{17}$$

Combining (16), (17) gives

$$\begin{bmatrix}\boldsymbol{Q} - \boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0} \\ * & -\boldsymbol{Q} & \boldsymbol{0} \\ * & * & \boldsymbol{0}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}^T\boldsymbol{P} \\ * & \boldsymbol{0} & \boldsymbol{F}_d^T\boldsymbol{P} \\ * & * & -\boldsymbol{P}\end{bmatrix} < 0 \tag{18}$$

and it is obvious that (18) implies (8). This concludes the proof.

4 PARAMETER DESIGN

Theorem 1. *For system (1), (2) the sufficient condition for the stable nominal control (4) is that there exist positive definite symmetric matrices $\boldsymbol{X} > 0$, $\boldsymbol{Y} > 0$, $\boldsymbol{X}, \boldsymbol{Y} \in \mathbb{R}^{n \times n}$ and a matrix $\boldsymbol{Z} \in \mathbb{R}^{r \times n}$ such that the following LMIs are satisfied*

$$\boldsymbol{X} = \boldsymbol{X}^T > 0 \tag{19}$$

$$\boldsymbol{Y} = \boldsymbol{Y}^T > 0 \tag{20}$$

$$\begin{bmatrix} \boldsymbol{Y} - \boldsymbol{X} & \boldsymbol{0} & \boldsymbol{X}\boldsymbol{F}^T - \boldsymbol{Z}^T\boldsymbol{G}^T \\ * & -\boldsymbol{Y} & \boldsymbol{X}\boldsymbol{F}_d^T \\ * & * & -\boldsymbol{X} \end{bmatrix} < 0 \tag{21}$$

The control law gain matrix is then given as

$$\boldsymbol{K} = \boldsymbol{Z}\boldsymbol{X}^{-1} \tag{22}$$

Proof. The linear state feedback control law, defined in (4) gives rise to the closed-loop system as follows

$$\boldsymbol{q}(i+1) = (\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})\boldsymbol{q}(i) + \boldsymbol{F}_d\boldsymbol{q}(i-h) \tag{23}$$

Substituting in (8) results in

$$\begin{bmatrix} \boldsymbol{Q} - \boldsymbol{P} & \boldsymbol{0} & (\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})^T\boldsymbol{P} \\ * & -\boldsymbol{Q} & \boldsymbol{F}_d^T\boldsymbol{P} \\ * & * & -\boldsymbol{P} \end{bmatrix} < 0 \tag{24}$$

Defining the congruence transform matrix

$$\boldsymbol{T} = \mathrm{diag}\begin{bmatrix} \boldsymbol{P}^{-1} & \boldsymbol{P}^{-1} & \boldsymbol{P}^{-1} \end{bmatrix} \tag{25}$$

and premultiplying (24) from the right and the left side by (25) gives results

$$\begin{bmatrix} \boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{P}^{-1} - \boldsymbol{P}^{-1} & \boldsymbol{0} & \boldsymbol{P}^{-1}\boldsymbol{F}^T - \boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{G}^T \\ * & -\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{P}^{-1} & \boldsymbol{P}^{-1}\boldsymbol{F}_d^T \\ * & * & -\boldsymbol{P}^{-1} \end{bmatrix} < 0 \tag{26}$$

With notation

$$\boldsymbol{P}^{-1} = \boldsymbol{X} > 0, \quad \boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{P}^{-1} = \boldsymbol{Y} > 0, \quad \boldsymbol{K}\boldsymbol{P}^{-1} = \boldsymbol{Z} \tag{27}$$

(26) implies (21). This concludes the proof.

Remark 1. Analyzing (26) it is evident that $\boldsymbol{Q}$ cannot be chosen to be equal $\boldsymbol{P}$. Subsequently, if $\boldsymbol{Q}$ is not a variable but any given matrix, (26) has to be rewritten as

$$\begin{bmatrix} -\boldsymbol{P}^{-1} & \boldsymbol{P}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{P}^{-1}\boldsymbol{F}^T - \boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{G}^T \\ * & -\boldsymbol{Q}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{0} & \boldsymbol{P}^{-1} & \boldsymbol{P}^{-1}\boldsymbol{F}_d^T \\ * & * & * & \boldsymbol{Q}^{-1} & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{P}^{-1} \end{bmatrix} < 0 \tag{28}$$

which implies

$$\begin{bmatrix} -\boldsymbol{X} & \boldsymbol{X} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{X}\boldsymbol{F}^T - \boldsymbol{Z}^T\boldsymbol{G}^T \\ * & -\boldsymbol{Q}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{0} & \boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}_d^T \\ * & * & * & \boldsymbol{Q}^{-1} & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{X} \end{bmatrix} < 0 \tag{29}$$

5 UNIFIED ALGEBRAIC APPROACH

Theorem 2. *For system (1), (2) the sufficient condition for the stable nominal control (3) is that there exist positive definite symmetric matrices* $\boldsymbol{X} > 0$, $\boldsymbol{Y} > 0$, $\boldsymbol{X}, \boldsymbol{Y} \in \mathbb{R}^{n \times n}$ *such that the following LMIs are satisfied*

$$\boldsymbol{X} = \boldsymbol{X}^T > 0 \tag{30}$$

$$\boldsymbol{Y} = \boldsymbol{Y}^T > 0 \tag{31}$$

$$\begin{bmatrix} \boldsymbol{Y} - \boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}^T\boldsymbol{G}^{\perp T} \\ * & \boldsymbol{G}^{\perp}\boldsymbol{X}\boldsymbol{G}^{\perp T} \end{bmatrix} < 0 \tag{32}$$

$$-\boldsymbol{Y} - \boldsymbol{X} + \boldsymbol{X}\boldsymbol{F}_d^T + \boldsymbol{F}_d\boldsymbol{X} < 0 \tag{33}$$

The control law gain matrix $\boldsymbol{K}$ *is then given as a solution of the inequality*

$$\begin{bmatrix} -\boldsymbol{M}\boldsymbol{R}^{-1}\boldsymbol{M}^T - \boldsymbol{S} & \boldsymbol{M}\boldsymbol{R}^{-1} + \boldsymbol{N}^T\boldsymbol{K}^T \\ * & -\boldsymbol{R}^{-1} \end{bmatrix} < 0 \tag{34}$$

where

$$\boldsymbol{S} = -\begin{bmatrix} \boldsymbol{Y} - \boldsymbol{X} & \boldsymbol{0} & \boldsymbol{X}\boldsymbol{F}^T \\ * & -\boldsymbol{Y} & \boldsymbol{X}\boldsymbol{F}_d^T \\ * & * & -\boldsymbol{X} \end{bmatrix} + \varepsilon \boldsymbol{I}_{3n} > 0, \quad \boldsymbol{M} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{G} \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} -\boldsymbol{X} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}^T \tag{35}$$

and $0 < \boldsymbol{R} \in \mathbb{R}^{r \times r}$, *and* $0 < \varepsilon \in \mathbb{R}$ *are arbitrary design parameters.*

Proof. Now, with notation (27) inequality (26) can be written as

$$\begin{bmatrix} \boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} - \boldsymbol{X} & \boldsymbol{0} & \boldsymbol{X}\boldsymbol{F}^T \\ * & -\boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}_d^T \\ * & * & -\boldsymbol{X} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{X} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{K}^T \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{G}^T \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{G} \end{bmatrix} \boldsymbol{K} \begin{bmatrix} -\boldsymbol{X} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} < 0 \tag{36}$$

Defining the orthogonal complement approximation as follows

$$\boldsymbol{G}^{\circ\perp} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{G} \end{bmatrix}^{\perp} = \mathrm{diag}\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{G}^{\perp} \end{bmatrix} \tag{37}$$

where $\boldsymbol{G}^{\perp}$ is the orthogonal complement to $\boldsymbol{G}$, and premultiplying (36) from the left side by (37) and from the right side by its transposition gives

$$\begin{bmatrix} \boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} - \boldsymbol{X} & \boldsymbol{0} & \boldsymbol{X}\boldsymbol{F}^T\boldsymbol{G}^{\perp T} \\ * & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{G}^{\perp}\boldsymbol{X}\boldsymbol{G}^{\perp T} \end{bmatrix} < 0 \quad \Leftrightarrow \quad \begin{bmatrix} \boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} - \boldsymbol{X} & \boldsymbol{X}\boldsymbol{F}^T\boldsymbol{G}^{\perp T} \\ * & \boldsymbol{G}^{\perp}\boldsymbol{X}\boldsymbol{G}^{\perp T} \end{bmatrix} < 0 \tag{38}$$

Subsequently defining the orthogonal complement

$$\boldsymbol{X}^{\circ\perp} = \begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}^{\perp} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} \end{bmatrix} \tag{39}$$

then premultiplying (36) from the left and right side by (39) results in

$$-\boldsymbol{X}\boldsymbol{Q}\boldsymbol{X} - \boldsymbol{X} + \boldsymbol{X}\boldsymbol{F}_d^T + \boldsymbol{F}_d\boldsymbol{X} < 0 \tag{40}$$

With notation

$$\boldsymbol{XQX} = \boldsymbol{Y} > 0, \quad \boldsymbol{X} = \boldsymbol{P}^{-1} > 0 \tag{41}$$

(38), (40) implies (32), (33), respectively.

Let for $\boldsymbol{S} > 0$ a matrix $\boldsymbol{K}$ satysfies the inequality

$$\boldsymbol{MKN} + \boldsymbol{N}^T\boldsymbol{K}^T\boldsymbol{M}^T - \boldsymbol{S} < 0 \tag{42}$$

Since there exists a matrix $\boldsymbol{R} > 0$ such that

$$\boldsymbol{MKN} + \boldsymbol{N}^T\boldsymbol{K}^T\boldsymbol{M}^T - \boldsymbol{S} + \boldsymbol{N}^T\boldsymbol{K}^T\boldsymbol{RKN} < 0 \tag{43}$$

completing (43) to square it can be obtained

$$(\boldsymbol{MR}^{-1} + \boldsymbol{N}^T\boldsymbol{K}^T)\boldsymbol{R}(\boldsymbol{MR}^{-1} + \boldsymbol{N}^T\boldsymbol{K}^T)^T - \boldsymbol{MR}^{-1}\boldsymbol{M}^T - \boldsymbol{S} < 0 \tag{44}$$

$$\begin{bmatrix} -\boldsymbol{MR}^{-1}\boldsymbol{M}^T - \boldsymbol{S} & \boldsymbol{MR}^{-1} + \boldsymbol{N}^T\boldsymbol{K}^T \\ * & -\boldsymbol{R}^{-1} \end{bmatrix} < 0 \tag{45}$$

respectively. Thus, using notation (35) inequality (45) implies (34). This concludes the proof.

6 ILLUSTRATIVE EXAMPLE

To demonstrate the algorithm properties it was assumed that system is given by (1), (2), where

$$\boldsymbol{F} = \begin{bmatrix} -0.0057 & 0.2255 & 0.0344 \\ -0.4433 & 1.0287 & -0.0979 \\ -0.0612 & 0.1643 & 0.0382 \end{bmatrix}, \quad \boldsymbol{F}_d = \begin{bmatrix} -0.0015 & 0.0064 & -0.1486 \\ 0.0010 & -0.0061 & -1.0355 \\ -0.0062 & -0.0006 & -0.1178 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} 0.0689 & 0.1177 \\ 0.1714 & 0.0064 \\ 0.0517 & 0.0338 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \Delta t = 0.1\, s$$

Applying the Matlab function *svd*(.), the orthogonal complement were obtained as

$$\boldsymbol{G}^{\perp} = \begin{bmatrix} -0.2622 & -0.1806 & 0.9480 \end{bmatrix}$$

and using Self-Dual-Minimization (SeDuMi) package for Matlab [Paeucelle at all. 2002] the control design problem was solved as feasible with

$$\boldsymbol{X} = \begin{bmatrix} 0.7074 & 0.3158 & 0.2559 \\ 0.3158 & 0.1410 & 0.1142 \\ 0.2559 & 0.1142 & 0.0925 \end{bmatrix}, \quad \boldsymbol{Y} = 10^{-3}\begin{bmatrix} 0.8129 & 0.3631 & 0.2940 \\ 0.3631 & 0.1622 & 0.1313 \\ 0.2940 & 0.1313 & 0.1063 \end{bmatrix}$$

For that $\boldsymbol{X}$, $\boldsymbol{Y}$ upon some computation was found

$$\boldsymbol{M} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.0689 & 0.1177 \\ 0.1714 & 0.0064 \\ 0.0517 & 0.0338 \end{bmatrix}, \quad \boldsymbol{N}^T = \begin{bmatrix} -0.7074 & -0.3158 & -0.2559 \\ -0.3158 & -0.1410 & -0.1142 \\ -0.2559 & -0.1142 & -0.0925 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{S}_0 = \begin{bmatrix} -0.7066 & -0.3154 & -0.2556 & & & & 0.0760 & -0.0138 & 0.0184 \\ -0.3154 & -0.1408 & -0.1141 & & & & 0.0339 & -0.0062 & 0.0082 \\ -0.2556 & -0.1141 & -0.00924 & & & & 0.0275 & 0.0050 & 0.0066 \\ & & & -0.0008 & -0.0004 & -0.0003 & -0.0371 & -0.2662 & -0.0348 \\ & & & -0.0004 & -0.0002 & -0.0001 & -0.0165 & -0.1188 & -0.0155 \\ & & & -0.0003 & -0.0001 & -0.0001 & -0.0134 & -0.0963 & -0.0126 \\ 0.0760 & 0.0339 & 0.0275 & -0.0371 & -0.0165 & -0.0134 & -0.7074 & -0.3158 & -0.2599 \\ -0.0138 & -0.0062 & -0.0050 & -0.2662 & -0.1188 & -0.0963 & -0.3158 & -0.1410 & -0.1142 \\ 0.0184 & 0.0082 & 0.0066 & -0.0348 & -0.0155 & -0.0126 & -0.2559 & -0.1142 & -0.0925 \end{bmatrix}$$

$$\boldsymbol{S} = \boldsymbol{S}_0 + \varepsilon \boldsymbol{I}_9 > 0, \quad \varepsilon = 1.3$$

Solving (34) with $\boldsymbol{R} = \boldsymbol{I}_2$ the final result was

$$\boldsymbol{K} = \begin{bmatrix} -0.0182 & -0.0081 & -0.0066 \\ -0.0194 & -0.0087 & -0.0070 \end{bmatrix}$$

$$\boldsymbol{F}_c = \boldsymbol{F} \text{ - } \boldsymbol{GK} = \begin{bmatrix} -0.0022 & 0.2271 & 0.0357 \\ -0.4401 & 1.0302 & -0.0967 \\ -0.0596 & 0.1650 & 0.0388 \end{bmatrix}, \quad \rho(\boldsymbol{F}_c) = \begin{bmatrix} 0.8995 \\ 0.1262 \\ 0.0411 \end{bmatrix}$$

which gives the stable closed-loop eigenvalues spectrum $\rho(\boldsymbol{F}_c)$ lying in the stable unit circle.

7 CONCLUDING REMARKS

The method uses the standard LMI numerical optimization procedures to manipulate the system feedback gain matrix as the direct design variable. The manipulation is accomplished in that manner that produces the desired closed-loop system asymptotical stability. Finally the design method is illustrated by a nontrivial example.

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