MODE DECOUPLING AND STATIC SYSTEM CROSS-DECOPLING IN CONTINUOUS-TIME MIMO SYSTEM CONTROL DESIGN

Pavol Kocsis and Dušan Krokavec

Technical University of Košice, Faculty of Electrical Engineering and Informatics,
Department of Cybernetics and Artificial Intelligence,
Letná 9, 042 00 Košice, Slovak Republic
Tel.: +421 55 602 4214, 2564 Fax: +421 55 625 3574
e-mail: pavol.kocsis@tuke.sk, dusan.krokavec@tuke.sk

Abstract: The general problem of assigning the system matrix eigenstructure using the state feedback control combining with the static decoupling is considered in this paper. With pole assignment algorithms the exposition of the problem is generalized here for the closed-loop state variables mode decoupling and the system interaction static decoupling. This handles the optimized structure of the right eigenvectors set for desired eigenvalues spectrum and the decoupling conditions to make use of freedom in the state feedback control design for MIMO systems.

Keywords: Mode decoupling, static decoupling, singular value decomposition, eigenstructure assignment.

1 INTRODUCTION

The static and the dynamic pole placement problem belongs to the prominent design problems of modern control theory, and, although its practical usefulness has been continuously in dispute, it is one of the most intensively investigated in control system design. It sems that the state-feedback pole assignment in control system design is one from the preferred techniques. In the single-input case the solution to this problem, when it exists, is unique. In the multi-input multi output (MIMO) case various solutions may exist, and, to determine a specific solution, additional conditions have to be supplied in order to eliminate the extra degrees of freedom in design strategy.

In recent years significant progress has been achieved in this field, coming in its formulation closest to the algebraic geometric nature of the pole placement problem. The reason for the discrepancy in opinions about the conditioning of the pole assignment problem is that one has to distinguish amoung three aspects of the pole placement problem, the computation of the memoryless feedback control law matrix gain, the computation of the closed loop system matrix eigenvalues spectrum and the supressing of the cross-coupling effect, where one manipulated input variable cause change in more outputs variables. Since a desirable property of any system design is that pole should be insensitive to perturbations in the coefficient matrices, this criterion may be used in adition to restrict the degrees of freedom in design.

Using algorithms for the pole assignment based on the Singular Value Decomposition (SVD), the exposition of the problem is generalized here to handle the specified structure of the right eigenvector set in state feedback control design for linear systems and the static decoupling techniques to obtain cross-decoupling [Wang, 2003]. Extra freedom, which makes dependent the closed-loop eigenvalues spectrum, is used only for closed-loop state variables mode decoupling. The integrated procedure provides methodology usable in the linear control system design techniques when designing state controller for the state-space control structures is defined by [Sobel and Lallman, 1989].

2 PROBLEM STATEMENT

Any linear time-invariant dynamic system with n degree-of-freedom can be modeled by the continuous-time state-space equations

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, and $\boldsymbol{y}(t) \in \mathbb{R}^m$ are vectors of the state, input, and output variables, respectively, matrices $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$, $\boldsymbol{D} \in \mathbb{R}^{m\times r}$ are real matrices. Generally, for controllable time-invariant linear MIMO system (1), (2), the linear state feedback regulator control law is defined as

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{q}(t) + \mathbf{L}\mathbf{w}(t) \tag{3}$$

where $\boldsymbol{K} \in \mathbb{R}^{r\times n}$ is a constant matrix and $\boldsymbol{L} \in \mathbb{R}^{r\times m}$ is a gain matrix of the desired control signal $\mathbf{w}(t) \in \mathbb{R}^m$. This control gives rise to the closed-loop system

$$\dot{\boldsymbol{q}}(t) = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{L}\boldsymbol{u}(t) = \boldsymbol{A}_c\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{L}\boldsymbol{u}(t) \tag{4}$$

It obvious, the closed loop poles are the eigenvalues of the matrix $\boldsymbol{A_c} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$, $\boldsymbol{A}_c \in \mathbb{R}^{n\times n}$.

3 BASIC PRELIMINARIES

3.1. The static decoupling problem

If $\boldsymbol{q}(0) = \boldsymbol{0}$ and $m = r$ the state space description is related by the matrix transfer function

$$\boldsymbol{G}_c(s) = ((\boldsymbol{C} - \boldsymbol{D}\boldsymbol{K})(s\boldsymbol{I}_n - \boldsymbol{A}_c)^{-1}\boldsymbol{B} + \boldsymbol{D})\boldsymbol{L}, \qquad \boldsymbol{A_c} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K} \tag{5}$$

This function is said to be coupled if any individual input influences more than one output. Since $m = r$ the matrix trunsfer function is a square matrix function.

Considering

$$\lim_{t\to\infty} \boldsymbol{y}(\boldsymbol{t}) = \lim_{t\to\infty} \boldsymbol{w}(\boldsymbol{t}) \Rightarrow \lim_{s\to 0} s\ \tilde{\boldsymbol{y}}(s) = \lim_{s\to 0} \boldsymbol{G}_c(s)s\tilde{\boldsymbol{w}}(s) \tag{6}$$

it is evident that

$$\lim_{s\to 0} \boldsymbol{G}_c(s) = [(\boldsymbol{C}\text{ - }\boldsymbol{D}\boldsymbol{K})(\text{-}\boldsymbol{A}_c)^{-1}\boldsymbol{B} + \boldsymbol{D}]\boldsymbol{L} = \boldsymbol{I}_m \tag{7}$$

and if $\boldsymbol{G}_c(0)$ is non-singular and $\boldsymbol{A_c}$ is stable then

$$\boldsymbol{L} = ((\boldsymbol{C}\text{-}\boldsymbol{D}\boldsymbol{K})(\text{-}\boldsymbol{A}_c)^{-1}\boldsymbol{B} + \boldsymbol{D})^{-1} \tag{8}$$

and (8) results static system decoupling.

Proposition 1. *The static decoupling problem by state beedback is solvable if len only if*

(i) ($\boldsymbol{A}$,$\boldsymbol{B}$) *is stabilizable; and*

(ii) $\operatorname{rank}\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = n + m$

Proof. (e.g. see [Wang, 2003]) If (A,B) is stabilizable, it is possible to find K such that $\boldsymbol{A_c}$ is stable. Assuming that for such $\boldsymbol{K}$ is

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \text{-}\boldsymbol{K} & \boldsymbol{I}_m \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \boldsymbol{A}\text{-}\boldsymbol{BK} & \boldsymbol{B} \\ \boldsymbol{C}\text{-}\boldsymbol{DK} & \boldsymbol{D} \end{bmatrix} \tag{9}$$

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{A}\text{-}\boldsymbol{BK} & \boldsymbol{B} \\ \boldsymbol{C}\text{-}\boldsymbol{DK} & \boldsymbol{D} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \boldsymbol{I}_n & \boldsymbol{0} \\ \text{-}(\boldsymbol{C}\text{-}\boldsymbol{DK})(\boldsymbol{A}\text{-}\boldsymbol{BK})^{-1} & \boldsymbol{I}_m \end{bmatrix}\begin{bmatrix} \boldsymbol{A}\text{-}\boldsymbol{BK} & \boldsymbol{B} \\ \boldsymbol{C}\text{-}\boldsymbol{DK} & \boldsymbol{D} \end{bmatrix} \tag{10}$$

$$\operatorname{rank}\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \boldsymbol{A}\text{-}\boldsymbol{BK} & \boldsymbol{B} \\ \boldsymbol{0} & \text{-}(\boldsymbol{C}\text{-}\boldsymbol{DK})(\boldsymbol{A}\text{-}\boldsymbol{BK})^{-1}\boldsymbol{B}+\boldsymbol{D} \end{bmatrix} = n+m \tag{11}$$

respectively, since rank $\boldsymbol{A}_c = n$ and the preposition (ii) implies

$$\operatorname{rank}((\boldsymbol{C}\text{-}\boldsymbol{DK})(\text{-}\boldsymbol{A}_c)^{-1}\boldsymbol{B}+\boldsymbol{D}) = m \tag{12}$$

Thus, chosing $\boldsymbol{L}$ as in (8) then $\boldsymbol{G}_c(0) = \boldsymbol{I}_m$ is obtained, i.e. static decoupling is possible.

Converselly,

$$\operatorname{rank}(\boldsymbol{L}) = \operatorname{rank}((\boldsymbol{C}\text{-}\boldsymbol{DK})(\text{-}\boldsymbol{A}_c)^{-1}\boldsymbol{B}+\boldsymbol{D})^{-1} = m \tag{13}$$

requires that both $\boldsymbol{L}$ and $(\boldsymbol{C}\text{-}\boldsymbol{DK})(\text{-}\boldsymbol{A}_c)^{-1}\boldsymbol{B}+\boldsymbol{D}$ are non-singular. This gives again (11) which implies the necessity from. This concludes the proof.

3.2. Controlability and observability of modes

Proposition 2. *Given system eigenstructure with distinct eigenvalues and* $\boldsymbol{D} = \boldsymbol{0}$*, then*

(i) *the k-th mode ($s - s_k$) is unobserved from the l-th output if the l-th row of the output matrix* $\boldsymbol{C}$ *is orthogonal to the k-th eigenvector of the closed-loop system system, i.e.*

$$\boldsymbol{c}_l^T\boldsymbol{n}_k = \boldsymbol{n}_j^T\boldsymbol{n}_k = 0, \quad j \neq k, \quad j,k \in \{1,2,\ldots n\}, l \in \{1,2,\ldots m\}, \quad \boldsymbol{C}^T = \begin{bmatrix}\boldsymbol{c}_1 & \cdots & \boldsymbol{c}_m\end{bmatrix} \tag{14}$$

(ii) *the k-th mode ($s - s_k$) is uncontrolled from the l-th input if the l-th column of the input matrix* $\boldsymbol{B}$ *is orthogonal to the k-th eigenvector transposition of the closed-loop system, i.e.*

$$\boldsymbol{n}_k^T\boldsymbol{b}_l = \boldsymbol{n}_k^T\boldsymbol{n}_j = 0, \quad j \neq k, \quad j,k \in \{1,2,\ldots n\}, l \in \{1,2,\ldots r\}, \quad \boldsymbol{B} = \begin{bmatrix}\boldsymbol{b}_1 & \cdots & \boldsymbol{b}_r\end{bmatrix} \tag{15}$$

Proof. (e.g. see [Krokavec and Filasová, 2007]) Let $\mathbf{n}_h$ is the k-th right eigenvector correspon-ding to the eigenvalue s_h, i.e.

$$\boldsymbol{A}_c\boldsymbol{n}_h = (\boldsymbol{A}\text{-}\boldsymbol{BK})\boldsymbol{n}_h = \boldsymbol{s}_h\boldsymbol{n}_h, \quad h = 1,2,\ldots,n \tag{16}$$

By definition, the closed-loop system resolvent kernel is

$$\Upsilon = (s\boldsymbol{I}_n - \boldsymbol{A}_c)^{-1} \tag{17}$$

If all eigenvalues of the closed-loop system are distinct, (16) can be written in the compact form

$$\boldsymbol{A}_c\begin{bmatrix}\boldsymbol{n}_1 \cdots \boldsymbol{n}_n\end{bmatrix} = \begin{bmatrix}\boldsymbol{n}_1 \cdots \boldsymbol{n}_n\end{bmatrix}\operatorname{diag}\begin{bmatrix}s_1 \cdots s_n\end{bmatrix} \tag{18}$$

$$\boldsymbol{A}_c\boldsymbol{N} = \boldsymbol{NS}, \quad \boldsymbol{S} = \operatorname{diag}\begin{bmatrix}s_1 & \cdots & s_n\end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix}\boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n\end{bmatrix}, \quad \boldsymbol{N}^{-1} = \boldsymbol{N}^T \tag{19}$$

respectively. Using the property of orthogonality given in (19), the resolvent kernel of the system takes form

$$\Upsilon = (s\boldsymbol{N}\boldsymbol{N}^{-1} - \boldsymbol{N}\boldsymbol{A}_c\boldsymbol{N}^{-1})^{-1} = (\boldsymbol{N}(s\boldsymbol{I}-\boldsymbol{S})\boldsymbol{N}^{-1})^{-1} = \boldsymbol{N}(s\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{N}^T \tag{20}$$

$$\Upsilon = \begin{bmatrix} \boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n \end{bmatrix} \mathrm{diag}\begin{bmatrix} (s-s_1)^{-1} & \cdots & (s-s_n)^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{n}_1 & \cdots & \boldsymbol{n}_n \end{bmatrix}^T = \sum_{h=1}^{n} \frac{\boldsymbol{n}_h \boldsymbol{n}_h^T}{s-s_h} \tag{21}$$

respectively, and the closed loop matrix transfer function of the system (1), (2) takes form

$$\boldsymbol{G}(s) = ((\boldsymbol{C} - \boldsymbol{DK})(s\boldsymbol{I} - \boldsymbol{A}_c)^{-1}\boldsymbol{B} + \boldsymbol{D})\boldsymbol{L} = \sum_{h=1}^{n} \frac{(\boldsymbol{C} - \boldsymbol{DK})\boldsymbol{n}_h \boldsymbol{n}_h^T \boldsymbol{BL}}{s-s_h} + \boldsymbol{DL} \tag{22}$$

It is obvious if $\boldsymbol{D} = \boldsymbol{0}$ that (22) implies this preposition. This concludes the proof.

3.2. System model canonical form

Let the original state description of the system is

$$\dot{\boldsymbol{q}}_0(t) = \boldsymbol{A}_0 \boldsymbol{q}_0(t) + \boldsymbol{B}_0 \boldsymbol{u}(t) \tag{23}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}_0 \boldsymbol{q}_0(t) \tag{24}$$

and $r = m$.

Proposition 3. *If* rank($\boldsymbol{C}_0\boldsymbol{B}_0$) *= m then there exists a coordinates change in which (**A**, **B**, **C**) takes structure*

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{0} \end{bmatrix} \tag{25}$$

where $\boldsymbol{A}_{11} \in \mathbb{R}^{m\times m}$, $\boldsymbol{B}_1 \in \mathbb{R}^{m\times m}$ *is regular and* $\boldsymbol{C}_1 \in \mathbb{R}^{m\times m}$ *is orthogonal.*

Proof. Defining the transform matrix $\boldsymbol{T}_1$ such that

$$\boldsymbol{C}_1 = \boldsymbol{C}_0 \boldsymbol{T}_1 = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{T}_1^{-1} = \begin{bmatrix} \boldsymbol{C}_0 \\ \boldsymbol{I}_{n-m} \quad \boldsymbol{0} \end{bmatrix} \tag{26}$$

then using (1), (2) yields

$$\boldsymbol{A}_1 = \boldsymbol{T}_1^{-1} \boldsymbol{A}_0 \boldsymbol{T}_1, \qquad \boldsymbol{B}_1 = \boldsymbol{T}_1^{-1} \boldsymbol{B}_0 = \boldsymbol{T}_1^{-1} \begin{bmatrix} \boldsymbol{B}_{01} \\ \boldsymbol{B}_{02} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{11} \\ \boldsymbol{B}_{12} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_0 \boldsymbol{B}_0 \\ \boldsymbol{B}_{02} \end{bmatrix} \tag{27}$$

If $\boldsymbol{C}_0 \boldsymbol{B}_0 = \boldsymbol{B}_{11}$ is a regular matrix then the second transform matrix $\boldsymbol{T}_2$ can be defined as follows

$$\boldsymbol{T}_2^{-1} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{B}_{12}\boldsymbol{B}_{11}^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix}, \quad \boldsymbol{T}_2 = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{B}_{12}\boldsymbol{B}_{11}^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix} \tag{28}$$

This results in

$$\boldsymbol{B} = \boldsymbol{T}_2^{-1} \boldsymbol{B}_1 = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{B}_{12}\boldsymbol{B}_{11}^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{B}_{11} \\ \boldsymbol{B}_{12} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_{11} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_0 \boldsymbol{B}_0 \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{0} \end{bmatrix} \tag{29}$$

and with

$$\boldsymbol{T}_c^{-1} = \boldsymbol{T}_2^{-1} \boldsymbol{T}_1^{-1} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{B}_{12}\boldsymbol{B}_{11}^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{C}_0 \\ \boldsymbol{I}_{n-m} \quad \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ -\boldsymbol{B}_{02}(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix} \begin{bmatrix} \boldsymbol{C}_0 \\ \boldsymbol{I}_{n-m} \quad \boldsymbol{0} \end{bmatrix} \tag{30}$$

it yields

$$\boldsymbol{C} = \boldsymbol{C}_0 \boldsymbol{T}_c = \boldsymbol{C}_1 \boldsymbol{T}_2 = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{B}_{12}\boldsymbol{B}_{11}^{-1} & \boldsymbol{I}_{n-m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix} \tag{31}$$

as well as

$$\boldsymbol{A}=\boldsymbol{T}_2^{-1}\boldsymbol{T}_1^{-1}\boldsymbol{A}_0\boldsymbol{T}_1\boldsymbol{T}_2=\boldsymbol{T}_c^{-1}\boldsymbol{A}_0\boldsymbol{T}_c=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix},\ \boldsymbol{B}=\boldsymbol{T}_2^{-1}\boldsymbol{T}_1^{-1}\boldsymbol{B}_0=\boldsymbol{T}_c^{-1}\boldsymbol{B}_0,\ \boldsymbol{C}=\boldsymbol{C}_0\boldsymbol{T}_1\boldsymbol{T}_2=\boldsymbol{C}_0\boldsymbol{T}_c \quad (32)$$

Thus, (29), (31), (32) implies (23). This concludes the proof.

Note, the structure of $\boldsymbol{T}_1^{-1}$ is not unique and the others can be obtained by permutations of *n-m* rows in the structure defined in (25).

4 EIGENSTRUCTURE ASSIGNMENT

4.1. Eigenvalues spectrum

In the pole assignment problem, a feedback gain matrix $\boldsymbol{K}$ is sought so that the closed-loop system has a prescribed eigenvalues spectrum $\Omega(\boldsymbol{A}_c)=\{s_h : \mathrm{Re}(s_h) < 0,\ h = 1,2, ..., n\}$. Note that spectrum $\Omega(\boldsymbol{A}_c)$ is closed under complex conjugation and the observability and controlability of modes is determined by the closed-loop eigenstructure.

Noting the same assumption as above (16) can be rewritten as

$$\begin{bmatrix} s_h\boldsymbol{I} - \boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}=\boldsymbol{L}_h\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}=\boldsymbol{0},\quad h=1,2,\ldots,n \quad (33)$$

Subsequently, singular value decomposition (SVD) of $\mathbf{L}_h$ gives

$$\begin{bmatrix}\boldsymbol{u}_1^T\\ \vdots \\ \boldsymbol{u}_n^T\end{bmatrix}\boldsymbol{L}_h\begin{bmatrix}\boldsymbol{v}_{h1} & \cdots & \boldsymbol{v}_{hn} & \boldsymbol{v}_{h,n+1} & \cdots & \boldsymbol{v}_{h,n+r}\end{bmatrix}=\begin{bmatrix}\sigma_{h1} & & & & \\ & \ddots & & \boldsymbol{0}_{n+1} & \cdots & \boldsymbol{0}_{n+r}\\ & & \sigma_{hn} & & \end{bmatrix} \quad (34)$$

where $\{\boldsymbol{u}_{hl},\ l = 1,2,\ldots,n\}$, $\{\boldsymbol{v}_{hl},\ l = 1,2,\ldots,n+r\}$ are sets of the left and the right singular vectors, respectively and $\{\sigma_{hl},\ l = 1,2,\ldots,n\}$ is a set of the singular values of $\boldsymbol{L}_h$.

It is evident that all column vectors $\{\mathbf{v}_{hl},\ l = n+1,n+2,\ldots,n+r\}$ satisfy (33), i.e.

$$\boldsymbol{L}_h\boldsymbol{v}_{hl}=\begin{bmatrix} s_h\boldsymbol{I} - \boldsymbol{A} & \boldsymbol{B}\end{bmatrix}\boldsymbol{v}_{hl}=\boldsymbol{0},\quad h=1,2,\ldots,n,\ l=n+1,n+2,\ldots,n+r \quad (35)$$

The set of column vectors $\{\mathbf{v}_{hl},\ l = n+1,n+2,\ldots,n+r\}$ is a non-trivial solution of (33), and results the null space of $\mathbf{L}_h$

$$\begin{bmatrix}\boldsymbol{n}_h\\ \boldsymbol{K}\boldsymbol{n}_h\end{bmatrix}\in\mathcal{N}\begin{bmatrix} s_h\boldsymbol{I} - \boldsymbol{A} & \boldsymbol{B}\end{bmatrix},\quad h=1,2,\ldots,n \quad (36)$$

The null space (36) consists of the orthonormal vector set. Any combination of these vectors (the span of given null space) will provide a vector $\boldsymbol{n}_h$ which used as an eigenvector produces the desired eigenvalue s_h in the closed-loop system.

Theorem 1. *Canonical form eigenstructure optimization provides optimal eigenstructure for that model it was derived.*

Proof. Since (20) implies

$$\Upsilon_0=\boldsymbol{N}_0(s\boldsymbol{I}-\boldsymbol{S})^{-1}\boldsymbol{N}_0^{-1} \quad (37)$$

and it yields

$$(\boldsymbol{A}_0 - \boldsymbol{B}_0\boldsymbol{K}_0)\boldsymbol{n}_{0h}=(\boldsymbol{T}_c\boldsymbol{A}\boldsymbol{T}_c^{-1} - \boldsymbol{T}_c\boldsymbol{B}\boldsymbol{K}_0\boldsymbol{T}_c\boldsymbol{T}_c^{-1})\boldsymbol{n}_{0h}=\boldsymbol{T}_c(\boldsymbol{A} - \boldsymbol{B}\boldsymbol{K})\boldsymbol{T}_c^{-1}\boldsymbol{n}_{0h}=\boldsymbol{s}_h\boldsymbol{n}_{0h} \quad (38)$$

$$\boldsymbol{s_h T_c^{-1} n_{0h}} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{T}_c^{-1}\boldsymbol{n}_{0\boldsymbol{h}} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{n_h} = \boldsymbol{s_h n_h} \tag{39}$$

respectively, then it yields

$$\boldsymbol{K} = \boldsymbol{K}_0\boldsymbol{T_c}, \quad \boldsymbol{n}_{0\boldsymbol{h}} = \boldsymbol{T_c n_h}, \quad \boldsymbol{N}_0 = \boldsymbol{T_c N}, \quad \boldsymbol{N}_0^{-1} = \boldsymbol{N}^{-1}\boldsymbol{T}_c^{-1} = \boldsymbol{N}^T\boldsymbol{T}_c^{-1} \tag{40}$$

Subsequently

$$\boldsymbol{G}(s) = \boldsymbol{C}_0\boldsymbol{N}_0(s\boldsymbol{I} - \boldsymbol{S})^{-1}\boldsymbol{N}_0^{-1}\boldsymbol{B}_0\boldsymbol{L}_0 = \boldsymbol{C}_0\boldsymbol{T_c N}(s\boldsymbol{I} - \boldsymbol{S})^{-1}\boldsymbol{N}^T\boldsymbol{T}_c^{-1}\boldsymbol{B}_0\boldsymbol{L}_0 = \boldsymbol{CN}(s\boldsymbol{I} - \boldsymbol{S})^{-1}\boldsymbol{N}^T\boldsymbol{BL}_0 \tag{41}$$

$$\boldsymbol{G}(s) = \boldsymbol{C}_0(s\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{B}_0 = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} \tag{42}$$

respectively. Thus, it is evident that $\boldsymbol{L} = \boldsymbol{L}_0$ and for $\boldsymbol{D} = \boldsymbol{0}$ (22) and (41) gives

$$\boldsymbol{G}(s) = \sum_{h=1}^{n} \frac{\boldsymbol{C}_0\boldsymbol{n}_{0h}\boldsymbol{n}_{0h}^T\boldsymbol{B}_0\boldsymbol{L}_0}{s - s_h} = \sum_{h=1}^{n} \frac{\boldsymbol{C}\boldsymbol{n}_h\boldsymbol{n}_h^T\boldsymbol{BL}}{s - s_h} \tag{43}$$

It is obvious that optimizing $\boldsymbol{Cn}_h, \boldsymbol{n}_h^T\boldsymbol{B}$ it is optimized $\boldsymbol{C}_0\boldsymbol{n}_{0h}, \boldsymbol{n}_{0h}^T\boldsymbol{B}_0$, respectively, too. This concludes the proof.

Of course, optimizing one position in a vector is very simple comparing with a full structure vector optimization, and this strategy is supported by the structure of $\boldsymbol{C}$ in the canonical form. Since rank($\boldsymbol{C}$) = m, it is evident that only m modes can be decoupled (one for one single output).

4.2. Controller gain matrix design

Set of eigenvectors of desired structure $\{\boldsymbol{n}_{dh}, h = 1,2, \ldots, p, p \le \mathrm{m}\}$ can be specified to reflect potential possibility to choose in the closed-loop structure the dynamic modes to be decoupled. This specification employs this freedom to choose a closed-loop eigenvalue such that its associated eigenvector has aproximatly equal zero value on prescribed position in given row of $\boldsymbol{C}$. This freedom is so limited by the structure of the output matrix $\boldsymbol{C}$ in the canonical model form. Computing iterative $p \le m$ column vectors with desired structure for decouple modes, and $n - p$ column vectors for rest modes (solving ever singular equalitys (33), (34) using SVD procedures), it is possible to construct a matrix $\boldsymbol{M}$ such that (see [Krokavec and Filasová, 2008])

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{v}_1 & \cdots & \boldsymbol{v}_p & \boldsymbol{v}_{p+1} & \cdots & \boldsymbol{v}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{n}_1 & \cdots & \boldsymbol{n}_p & \boldsymbol{n}_{p+1} & \cdots & \boldsymbol{n}_n \\ \boldsymbol{w}_1 & \cdots & \boldsymbol{w}_p & \boldsymbol{w}_{p+1} & \cdots & \boldsymbol{w}_n \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{W} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{KP} \end{bmatrix} \tag{44}$$

Therefore, using partition (44) the gain matrix $\boldsymbol{K}$ is given by

$$\boldsymbol{K} = \boldsymbol{WP}^{-1}, \quad \boldsymbol{P} \in \mathbb{R}^{nxn}, \boldsymbol{W} \in \mathbb{R}^{rxn}, \boldsymbol{K} \in \mathbb{R}^{rxn} \tag{45}$$

5 ILLUSTRATIVE EXAMPLE

The system under consideration was described by (1), (2), where

$$\boldsymbol{A}_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -9 & -5 \end{bmatrix}, \quad \boldsymbol{B}_0 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 2 & 5 \end{bmatrix}, \quad \boldsymbol{C}_0 = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Design task was given as decopling of one mode to be as unobserved as possible from the second output of the system. This mode couldn't be the dominamt closed-loop system mode.

At first, given system description was transformed to the canonical form, where

$$\boldsymbol{T}_1^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{T}_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 7 & 10 \\ 3 & 4 \\ 1 & 3 \end{bmatrix}, \quad \boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{T}_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2.5 & 5.5 & 1 \end{bmatrix}, \quad \boldsymbol{T}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2.5 & -5.5 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 7 & 10 \\ 3 & 4 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} -3 & -2 & 0 \\ 1 & -1 & 0 \\ 10.5 & 6 & -1 \end{bmatrix}, \quad \boldsymbol{T}_c^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 4 & 0.5 & -2.5 \end{bmatrix}, \quad \boldsymbol{T}_c = \begin{bmatrix} 2.5 & -5.5 & 1 \\ -2.5 & 6.5 & -1 \\ 3.5 & -7.5 & 1 \end{bmatrix}$$

Chosing the dominant mode as s_1 = - 0.7 then contructing iteratively matrices $\boldsymbol{L}_{sd}$ for $s_d < s_1$ and subsequently solving (34), a possible mode solution was obtained such that

$$s_d = -6, \quad \boldsymbol{L}_{sd} = \begin{bmatrix} -3 & 2 & 0 & 7 & 10 \\ -1 & -5 & 0 & 3 & 4 \\ -10.5 & -6 & -5 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{V}_{sd} = \begin{bmatrix} 0.6338 & 0.4457 & -0.4723 & 0.2344 & 0.3487 \\ 0.3127 & 0.4349 & 0.8438 & 0.0321 & -0.0027 \\ 0.2262 & 0.2963 & -0.2187 & -0.5307 & -0.7291 \\ -0.3879 & 0.4162 & -0.0986 & 0.6954 & -0.4278 \\ -0.5467 & 0.5927 & -0.0855 & -0.4228 & 0.4047 \end{bmatrix}, \boldsymbol{v}_{sd} = \begin{bmatrix} 0.3487 \\ -0.0027 \\ -0.7291 \\ -0.4278 \\ 0.4047 \end{bmatrix}, \boldsymbol{n}_{sd} \approx \begin{bmatrix} 0.3487 \\ -0.0027 \\ -0.7291 \end{bmatrix}$$

It is evident that this vector structure minimizes product

$$\boldsymbol{c}_2^T \boldsymbol{n}_{sd} \approx \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.3487 \\ -0.0027 \\ -0.7291 \end{bmatrix} = -0.0027$$

Analogously, by constructing $\boldsymbol{L}_1$, $\boldsymbol{L}_2$ for s_1 = -0.7, s_2 = -1.0, and computing associated null-spaces as introduced in (36) by SVD, there was obtained

$$\boldsymbol{v}_1 = \begin{bmatrix} -0.1372 \\ 0.2676 \\ 0.5509 \\ -0.6479 \\ 0.4316 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 0.0613 \\ -0.1073 \\ 0.9818 \\ 0.1226 \\ -0.0766 \end{bmatrix}$$

Thus, for $\boldsymbol{M}$ as in (44) used method gives results

$$\boldsymbol{M} = \begin{bmatrix} 0.3487 & 0.0613 & -0.1372 \\ -0.0027 & -0.1073 & 0.2676 \\ -0.7291 & 0.9818 & 0.5509 \\ -0.4278 & 0.1226 & -0.6479 \\ 0.4047 & -0.0766 & 0.4316 \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} 0.3487 & 0.0613 & -0.1372 \\ -0.0027 & -0.1073 & 0.2676 \\ -0.7291 & 0.9818 & 0.5509 \end{bmatrix}$$

$$\boldsymbol{W} = \begin{bmatrix} -0.4278 & 0.1226 & -0.6479 \\ 0.4047 & -0.0766 & 0.4316 \end{bmatrix}, \quad \boldsymbol{K} = \boldsymbol{W}\boldsymbol{P}^{-1} = \begin{bmatrix} -1.4721 & -2.9570 & -0.1063 \\ 1.3300 & 2.1439 & 0.0732 \end{bmatrix}$$

$$\boldsymbol{K}_0 = \boldsymbol{K}\boldsymbol{T}_c^{-1} = \begin{bmatrix} -4.8542 & -5.9543 & -1.2064 \\ 3.7665 & 4.8405 & 1.1471 \end{bmatrix}$$

Finally, there was obtained

$$\boldsymbol{A}_{0c} = \begin{bmatrix} -6.4453 & -7.5671 & -2.2350 \\ 5.9419 & 7.0682 & 2.2657 \\ -14.1241 & -21.2978 & -8.3229 \end{bmatrix}, \quad \boldsymbol{L}_0 = \begin{bmatrix} -13.5881 & -2.5947 \\ 10.0981 & 2.0828 \end{bmatrix}$$

6 CONCLUDING REMARKS

The general problem of the eigenstructure assigning for the state variable mode decoupling in state feedback control design is considered in this paper. The method covers the standard SVD numerical optimization procedures to manipulate the system feedback gain matrix as a direct design variable, and to obtain the control signal gain matrix. The manipulation is accomplished in that manner to produce desired system global performance by the pole placement and to modify output variable dynamics by mode controllability optimization. With generalization of the known algorithms for pole assignment the modified exposition of the problem is presented here to handle optimized structure of the closed-loop eigenvectors.

ACKNOWLEDGMENT

The work presented in this paper was supported by VEGA, Grant Agency of Ministry of Education and Academy of Science of Slovak Republic, under Grant No. 1/0328/08. This support is very gratefully acknowledged.

REFERENCES

GOLUB, G.H., VAN LOAN, C. F. (1989): *Matrix Computations*. The John Hopkins University Press, Baltimore.

HONC, D., HABER, R., AROUSI, F. (2007): TITO decentralized PI control using static decoupling compensators, *In Proceedings of the 16th International Conference "Process Control '09"*, Štrbské Pleso, Slovak Republic, pp. 061.1-061.6.

IPSEN, I.C.S. (2009): *Numerical Matrix Analysis. Linear Systems and Least Squares*, SIAM Society for Industrial and Applied Mathematics, Philadelphia.

KOCSIS, P., KROKAVEC, D. (2008): State variables mode decoupling in state control design for linear MIMO systems, *In International conference "Cybernetics and Informatics 2008*", Ždiar, Slovak Republic, Id. 44.

KROKAVEC, D., FILASOVÁ, A. (2007): *Dynamic System Diagnosis*. Elfa, Košice (in Slovak).

KROKAVEC, D., FILASOVÁ, A. (2008): *Discrete-time Systems*. Elfa, Košice (in Slovak).

SOBEL, K.M., LALLMAN, F.J. (1989): Eigenstructure assignment for the control of highly augmented aircraft, *Journal of Guidance, Control and Dynamics* **12**, pp. 318-324.

WANG, Q.G. (2003): Decoupling Control. Springer-Verlag, Berlin.

ŽILKA, V., HALÁSZ, M., HUBA, M.: Nonlinear controllers for a fluid tank system. *In Computer Aided System Theory – EUROCAST 2009*, Springer-Verlag, Berlin, pp. 618-625.