ROBUST PI CONTROLLERS FOR SYSTEMS WITH TRANSPORT DELAY

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Abstract: The paper presents robust controller design for systems with uncertain transport delays. Robust PI controllers are designed using approach, which combines the method based on plotting the stability boundary locus in the (k_p, k_i)-plane with the pole-placement method. The approach enables to assure robust stability of the closed loop as well as the quality of the control response prescribed by the choice of the closed loop poles or the relative damping or the natural undamped frequency of the control response.

Keywords: transport delay, interval uncertainty, robust control, pole-placement

1. INTRODUCTION

Presence of transport delays in the input-output relations is a common property of many technological processes. Plants with transport delays can often not be controlled using usual controllers designed without a consideration of a transport delay. Such controllers tend to destabilize the closed-loop system.

There are several approaches for transport delay treatment. One of them is based on using transport delay compensators based on Smith predictor, see e.g. Majhi and Atherton (1998). The other approach for compensation of transport delay is based on approximation of the transport delay term in the transfer function of the controlled processes. After the approximation, the modified transfer function of the controlled process is obtained and this transfer function does not contain the term representing the transport delay. Then, classical approaches to controller design can be used, as e.g. polynomial approach (Dostál et al. (2000) and many others) or robust control approach (Zhong (2006) and many others).

The method for controller design for systems with transport delay and parametric uncertainty is presented in this paper. The method combines several known approaches. The term representing a transport delay in the transfer function of the controlled system is approximated by Pade expansion (Dostál et al. (2000)) at first. The method for robust PI controller design is applied for obtaining the robust stability regions for parameters of PI controllers (Tan and Kaya (2003), Závacká et al. (2007)). In the last step, the PI controllers from robust stability regions are selected using the pole-placement method (Mikleš and Fikar (2008)).

2. CONTROLLED SYSTEM

Consider the controlled system in the form of a transfer function

$$G_s(s) = \frac{K}{(Ts+1)^n} e^{-[D_{\min}, D_{\max}]s} \tag{1}$$

where K is the gain, T is the time constant, n is the order of the system and D_{min}, D_{max} are the minimal and maximal transport delays of the system. In this paper, we consider $n = 1$.

The transfer function (1), for $n = 1$, is modified by approximation of the transport delay. The term representing the transport delay in (1) is substituted by the linear part of its Pade expansion. The transfer function of the controlled system has after the transport delay approximation following form

$$G_s(s) = \frac{-K[D_{\min}, D_{\max}]/2s \quad + K}{(T[D_{\min}, D_{\max}]/2)s^2 + (T + [D_{\min}, D_{\max}]/2)s + 1} \tag{2}$$

with interval polynomials in the numerator and denominator.

3. ROBUST CONTROLLER SYNTHESIS

3.1. Description of an uncertain system

Consider a system with real parametric uncertainty described by the transfer function

$$G(s,q) = \frac{b(s,q)}{a(s,q)} \tag{3}$$

where q is a vector of uncertain parameters and b, a are polynomials in s with coefficients which depend on the parameter q.

An uncertain polynomial

$$a(s,q) = \sum_{i=0}^{n} a_i(q) s^i \tag{4}$$

is said to have an independent uncertainty structure if each component q_i of q enters into only one coefficient.

A family of polynomials

$$A = \{a(s,q) : q \in Q\} \tag{5}$$

is said to be an interval polynomial family if *a(s, q)* has an independent uncertainty structure, each coefficient depends continuously on q and Q is a box. An interval polynomial family A arises from the uncertain polynomials described by *a(s, q)* with uncertainty bounds $| q_i |\leq 1$ for $i = 0,\ldots,n$. When dealing with an interval family, the shorthand notation

$$a(s,q) = \sum_{i=0}^{n} [q_i^-, q_i^+] s^i \tag{6}$$

may be used with $[q_i^-, q_i^+]$ denoting the bounding interval for the i^{th} of uncertainty component of uncertainty q_i.

3.2. Analysis of robust stability

In order to use the Kharitonov theorem (Barmish (1994)) for robust stability analysis, polynomials associated with an interval polynomial family A have to be defined at first. In the definition below the polynomials are fixed in the sense that only the bounds q_i^- and q_i^+ enter into the description but not the q_i themselves. The number of polynomials is four and they are independent on the degree of *a(s, q)*. Associated with the interval polynomial family (6) are four fixed Kharitonov polynomials (Barmish (1994))

$$\begin{aligned} K_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + q_4^- s^4 + q_5^- s^5 + \ldots \\ K_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + q_4^+ s^4 + q_5^+ s^5 + \ldots \\ K_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + q_4^+ s^4 + q_5^- s^5 + \ldots \\ K_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + q_4^- s^4 + q_5^+ s^5 + \ldots \end{aligned} \tag{7}$$

The interval polynomial family A with invariant degree is robustly stable if and only if its four Kharitonov polynomials (7) are stable.

3.3. Description of PI controller synthesis

The method of a robust PI controller synthesis is based on finding and plotting the stability boundary locus in the (k_p, k_i)-plane and then determining stabilizing PI controllers (Neymark (1978), Tan and Kaya (2003)). The method is used to find all parameters of a PI controller, which stabilizes a control system with an interval plant family. The stability boundary locus divides the parameter plane ((k_p , k_i) - plane) into stable and unstable regions. The stable region, which contains the values of stabilizing k_p and k_i parameters can be determined by choosing a test point within each region.

Consider the control system in Fig. 1, where $G_s(s)$ is a controlled process with the transfer function (8)

$$G_S(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \tag{8}$$

where

$b_1 = [b_1^-; b_1^+]$ $b_0 = [b_0^-; b_0^+]$, $a_2 = [a_2^-; a_2^+]$, $a_1 = [a_1^-; a_1^+]$, $a_0 = [a_0^-; a_0^+]$

and $C(s)$ is a feedback stabilizing PI controller (9)

$$C(s) = k_p + \frac{k_i}{s} \tag{9}$$

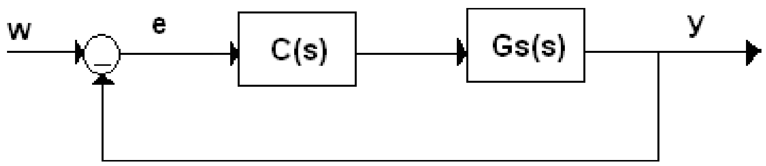

Figure 1: Control system

According to (7), for the controlled system in the form of the transfer function (8) with interval uncertainty, the Kharitonov polynomials $N_i(s)$, $i = 1, 2, 3, 4,$ for the numerator and $D_j(s)$, $j = 1, 2, 3, 4,$ for the denominator can be created, as it is seen in (10), (11)

$$\begin{aligned} N_1(s) &= b_1^- s + b_0^- \\ N_2(s) &= b_1^+ s + b_0^+ \\ N_3(s) &= b_1^- s + b_0^+ \\ N_4(s) &= b_1^+ s + b_0^- \end{aligned} \tag{10}$$

$$\begin{aligned} D_1(s) &= a_2^+ s^2 + a_1^- s + a_0^- \\ D_2(s) &= a_2^- s^2 + a_1^+ s + a_0^+ \\ D_3(s) &= a_2^- s^2 + a_1^- s + a_0^+ \\ D_4(s) &= a_2^+ s^2 + a_1^+ s + a_0^- \end{aligned} \tag{11}$$

Sixteen Kharitonov plants can be obtained using polynomials (10) and (11) for the systems (8).

The closed loop characteristic equation of the control system from Fig. 1, where e. g.

$$G_S(s) = \frac{b_1^- s + b_0^-}{a_2^- s^2 + a_1^- s + a_0^+} \tag{12}$$

can be written by substituting $s = j\omega$ as

$$d(j\omega) = \left[-\left(a_1^- + b_1^- k_p\right)\omega^2 + b_0^- k_i\right] + j\left[-a_2^-\omega^3 + \left(b_0^- k_p + b_1^- k_i + a_0^+\right)\omega\right] = 0 \tag{13}$$

The PI controller can be easily obtained by equating the real and the imaginary parts of the characteristic equation (13) to zero, for details see (Závacká et al. (2007)). Equating the real and imaginary parts of $d(j\omega)$ to zero gives following expressions for calculating of k_p, k_i in the dependence on the frequency ω

$$\begin{aligned} b_0^- k_i - b_1^- k_p \omega^2 &= a_1^- \omega^2 \\ b_1^- k_i \omega + b_0^- k_p &= a_2^- \omega^3 - a_0^+ \omega \end{aligned} \tag{14}$$

Plotting the dependence k_i on k_p for a certain range of ω gives the stability boundary locus in the (k_p, k_i)-plane. The stability boundary splits the plane into the stable and unstable regions. The parameters of the PI controller are then chosen from stable regions.

The over described approach is then applied onto all Kharitonov plants and the robust stability region is obtained as the intersection of stability regions of all Kharitonov plants (Tan and Kaya (2003), Závacká et al. (2007)).

4. POLE -PLACEMENT METHOD

The pole-placement control design belongs to the class of well-known analytical methods where transfer function of the controlled process is known (Mikleš and Fikar (2008)). In this method, only the closed-loop denominator that assures stability is specified. The advantage of this approach is its usability for a broad range of systems. If the controller is of PID structure then the characteristic equation can be one of the following

$$s + \omega_0 = 0 \tag{15}$$

$$s^2 + 2\xi\omega_0 s + \omega_0^2 = 0 \tag{16}$$

$$(s + c_1)\left(s^2 + 2\xi\omega_0 s + \omega_0^2\right) = 0 \tag{17}$$

or the combination of (16) and (17), where ξ is the relative damping, ω_0 the natural undamped frequency, and $-c_1$ is a closed-loop pole. Specifying suitable values of parameters ξ, ω_0, c_1 in (15)-(17) leads to the controller which can assure the desired quality of the control response. To obtain unique solution, the system of equations for calculation of controller parameters has to be the system with zero degree of freedom. If higher order characteristic polynomial is considered, then any of parameters ξ, ω_0 or c_1 can be added to unknown variables.

5. RESULTS

5.1. Application of robust controller synthesis and pole-placement method

The main aim is to find parameters of PI controller (9) such that the stability of the feedback closed loop with the system (8) is assured and at the same time the closed loop response has certain quality, which can be prescribed by the choice of closed loop poles, the relative damping or the natural undamped frequency.

The area of all controller parameters, which are able to assure the robust stability of the feedback closed loop, is found by the method, which is based on plotting the stability boundary locus in the (k_p, k_i)-plane (Tan, Kaya (2003), Závacká et al. (2007), Vaneková et al. (2009)).

Then the pole-placement method is used to specify those controller parameters, which assure the control performance. The close loop characteristic equation for the system (8) and PI controller (9) can be express in the following forms

$$s^3 + \frac{a_1 + b_1 k_p}{a_2} s^2 + \frac{a_0 + b_1 k_i + b_0 k_p}{a_2} s + \frac{b_0 k_i}{a_2} = 0 \tag{18}$$

$$s^3 + (c_1 + 2\xi\omega_0) s^2 + \left(\omega_0^2 + 2\xi\omega_0 c_1\right) s + c_1 \omega_0^2 = 0 \tag{19}$$

where

$$b_1 = -K \frac{D_{\max}}{2}, \quad b_0 = K, \quad a_2 = T \frac{D_{\max}}{2}, \quad a_1 = T + \frac{D_{\max}}{2}, \quad a_0 = 1$$

and c_1 is one pole of the feedback control system, ξ is relative damping and ω_0 is the natural undamped frequency. The quality of the control response can be assured by the choice of these three parameters.

By comparison of the coefficients in (18) and (19), we obtain the system of equations for calculation of PI controller parameters in the form

$$\begin{aligned} (c_1 + 2\xi\omega_0) &= \frac{a_1 + b_1 k_p}{a_2} \\ \left(\omega_0^2 + 2\xi\omega_0 c_1\right) &= \frac{a_0 + b_1 k_i + b_0 k_p}{a_2} \\ c_1 \omega_0^2 &= \frac{b_0 k_i}{a_2} \end{aligned} \tag{20}$$

It is clear that the unique solution is obtained when there are three unknown variables in (20). Two of them are PI controller parameters and the third is one of c_1, ξ or ω_0.

Presented results were obtained for the choice $\xi = 1$ and $\omega \in [0; 0.56]$, and calculated PI controller parameters lie in Fig. 2 on the cyan line. The robust stability area for PI controller parameters was found as the intersection of stability areas obtained by plotting the stability boundary locus in the (k_p, k_i)-plane for 8 Kharitonov plants obtained for the system (2) and PI controller (9) (Závacká et al. (2007). It is clear from Fig. 2 that using the pole-placement method leads to the reasonable choice of PI controller parameters.

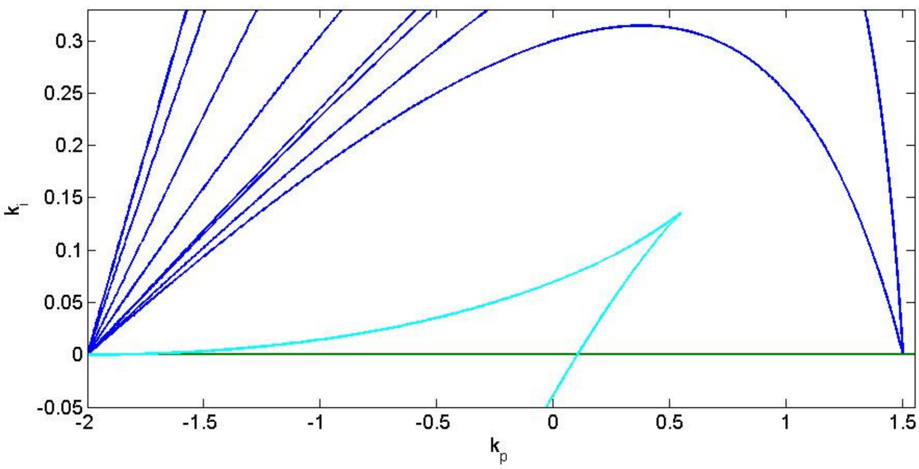

Figure 2: Designed PI controllers for $\xi = 1$

Then we used the choice $\xi = 1.2$, $\xi = 0.8$ and $\xi = 0.5$ and calculated PI controller parameters are in Fig. 3. From all designed PI controllers we used for simulation experiments controllers shown in Tab. 1 and Fig. 3 (blue stars).

5.2. Simulation results

Robust PI controllers (Tab. 1) designed by the described approach for various systems with uncertain transport delay were tested by simulations and by verification of correctness of the relative damping.

For verification of correctness of the relative damping we consider the worst case of transport delay (D_{max}). In this case, since we know right hand sides in (20), parameters c_1, ξ and ω_0 can be computed. These values are shown in Tab. 2 for every designed controller.

Simulations parameters of the system (1) were $K = 0.5$, $T = 2$, $D_{min} = 2$, $D_{max} = 8$ and nominal value of transport delay was taken as average of D_{min} and D_{max}, so $D_{nom} = 5$.

Control responses of the process obtained using PI controllers *reg1*-*reg8* (Tab. 1) for three different values of transport delay are shown in Figs. 4, 5, 6 and 7.

Here, w is the setpoint, *reg* is the controlled output for the minimal (min), maximal (max) and nominal (nom) transport delays of the controlled system and for controller taken from Tab. 1, respectively.

The simulation results confirm that the approach leads to the design of robust PI controllers which assure both, the robust stability of the feedback closed loop and the robust control performance with prescribed quality.

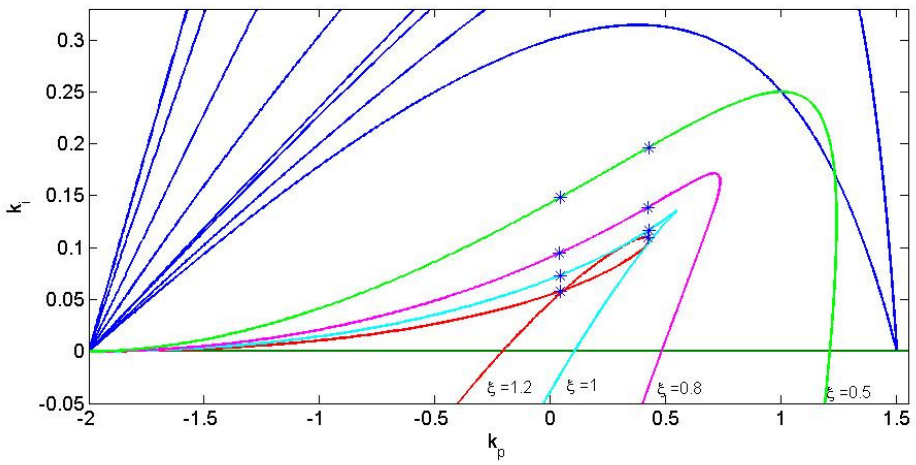

Figure 3: Designed PI controllers

Table 1: PI controllers

		k_p	k_i
ξ = 1.2	*reg1*	0.424	0.110
	reg2	0.042	0.058
ξ = 1	*reg3*	0.426	0.117
	reg4	0.042	0.073
ξ = 0.8	*reg5*	0.425	0.139
	reg6	0.040	0.095
ξ = 0.5	*reg7*	0.426	0.197
	reg8	0.041	0.148

Table 2: Verification of PI controllers

Controller	k_p	k_i	ξ	ω_0	$-c_1$
reg1	0.424	0.110	1.199	0.180	-0.212
reg2	0.042	0.058	1.199	0.082	-0.544
reg3	0.426	0.117	0.995	0.143	-0.360
reg4	0.042	0.073	1.003	0.090	-0.558
reg5	0.425	0.139	0.800	0.145	-0.411
reg6	0.040	0.095	0.798	0.101	-0.578
reg7	0.426	0.197	0.498	0.159	-0.485
reg8	0.041	0.148	0.500	0.122	-0.617

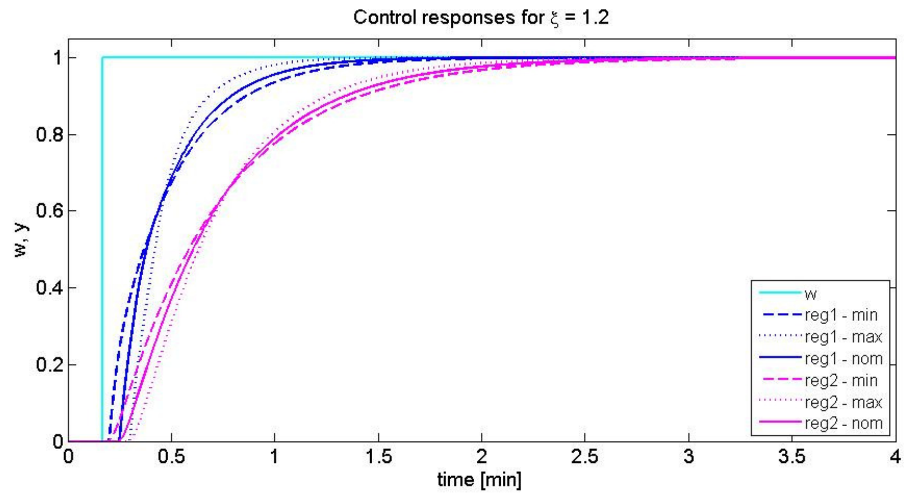

Figure 4: Control responses for ξ = 1.2

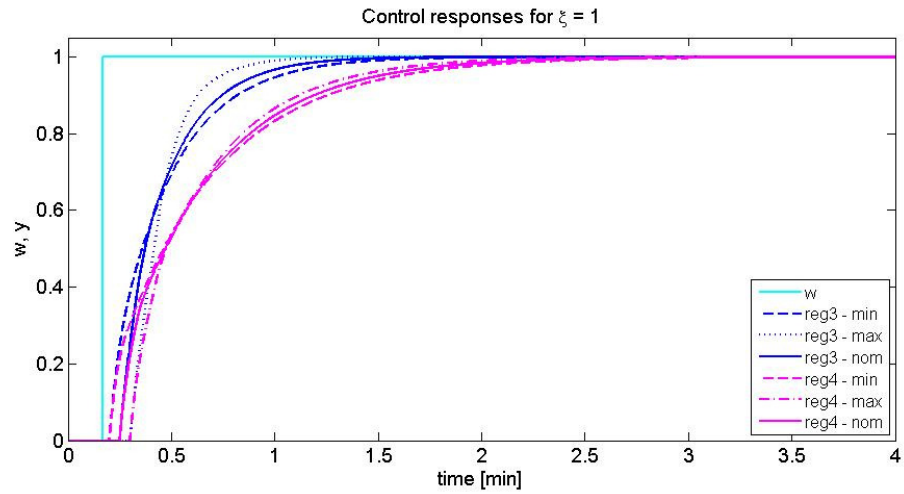

Figure 5: Control responses for ξ = 1

CONCLUSION

Robust PI controllers were designed for systems with transport delay via combination of two methods, the method for robust PI controller design based on finding the stability region for controller parameters and the pole-placement method. The pole-placement method was used for the choice of the PI controller from the stability region in such manner that the quality of control was achieved. The quality of control response was prescribed by the choice of relative damping or natural undamped frequency of the closed loop response. Designed controllers were tested by simulation experiments. Obtained results confirm that the proposed approach leads to the design of robust controllers that are suitable for control of real processes with transport delay and with uncertainty.

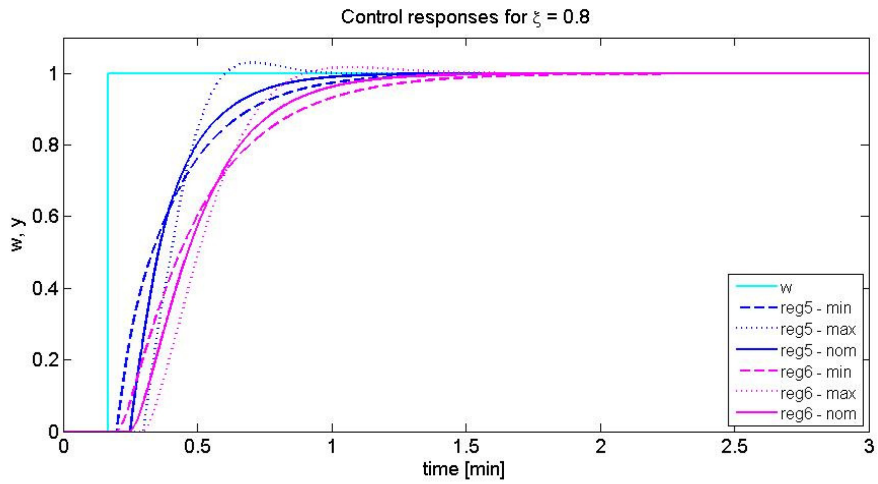

Figure 6: Control responses for $\xi = 0.8$

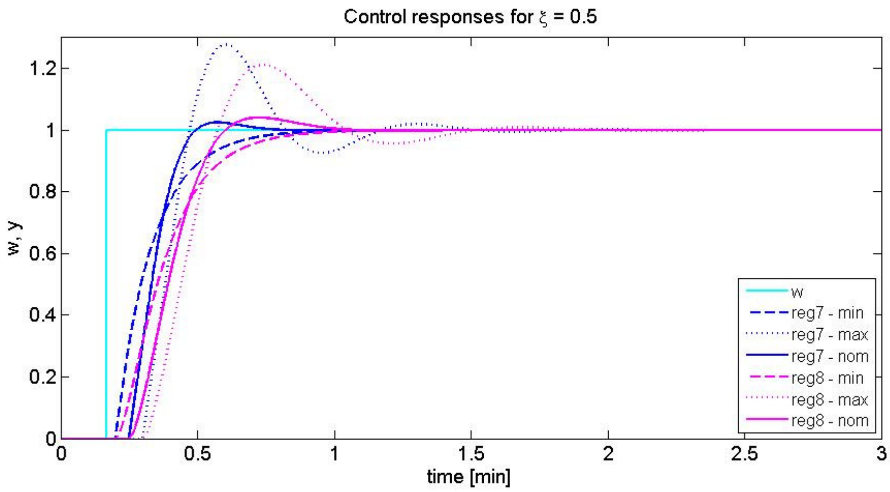

Figure 7: Control responses for $\xi = 0.5$

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