DESIGN-ORIENTED IDENTIFICATION BASED ON SINE WAVE SIGNAL AND ITS ADVANTAGES FOR TUNING OF THE ROBUST PID CONTROLLERS

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Abstract: For tuning of PID controllers, modern engineering methods use frequently design-oriented identification of the controlled plant (Visoli, 2006). The main idea of the design oriented-identification consists in a purposeful formulation of the identification method supporting the fulfillment of control objectives (Ingimundarson, 2004). The proposed paper shows that a reliable choice of the sine wave signal frequency used for identification of the uncertain plant and application of the proposed frequency-domain engineering method guarantee achieving required performance in terms of specified settling time and maximum overshoot. The paper deals with setting up and implementation of the proposed modern engineering method to control benchmark examples (Åsröm, 2000) with prespecified performance. Robustness properties are studied on a physical DC motor model.

Keywords: design-oriented identification, sine wave excitation signal, identification levels

1 INTRODUCTION

Engineering methods are two-step procedures consisting of identification of certain characteristic data of the plant followed by tuning PID controller coefficients depending directly on aquired information about the plant (Yu, 2006). These popular tools combine the field experience with analytical control engineering approaches into attractive design methods. Their widespread use and lot of variations are due to the possibility to specify performance requirements in advance and build directly into design algorithms. The widespread practical use of these methods is due to both simplicity of their algorithtms and quick controller synthesis (Veselý, 2003).

2 ALGORITHM OF THE PROPOSED ENGINEERING METHOD

This section briefly describes the design-oriented identification algorithm based on sine wave excitation and the presentation of the PID controller tuning approach including verification on benchmark examples. A generalization of the experience from the PID controller synthesis is provided yielding emprical expressions and plots to be used to achieve required closed-loop performance.

2.1 Formulation of engineering performance requirements

It is obvious that effective closed loop operation depends both on suitable PID controller adjustment and reasonably performed identification. The proposed engineering method guarantees performance specified in terms of the following measures:

- *maximum overshoot* η_{max},
- *settling time* t_{reg}.

However, we ask the question: what is the reliable way for transforming the above-mentioned engineering requirements into frequency domain specifications applicable to PID controller coefficients tuning?

2.2 Transformation of the the engineering performance requirements to control objectives

It is a well-known fact in the frequency-domain control theory that maximum overshoot η_{max} can be estimated from the desired phase margin ϕ_M and similarly, the settling time can be estimated from the open-loop gain crossover frequency ω_a. Analytical dependences $\eta_{max}=f(\phi_M)$ and $t_{reg}=f(\omega_a)$ derived for the 2[nd] order closed-loop transfer functions are according to Reinisch (Hudzovič, 1989)

$$\eta_{max} = -0.91\phi_M + 64.55 \quad \text{for } \phi_M \in \langle 38°, 71° \rangle, \tag{1}$$

$$\eta_{max} = -1.53\phi_M + 88.46 \quad \text{for } \phi_M \in \langle 12°, 38° \rangle, \tag{2}$$

$$\frac{\pi}{\omega_a} < t_{reg} < \frac{4\pi}{\omega_a}. \tag{3}$$

Hitherto, the Reinisch´s formulae are useful tolls to express the desired closed-loop behavior in classical analytical design procedures. However, with increasing order of the closed-loop transfer function, validity of expressions (1)-(3) fails. Moreover, in case of engineering methods, it is impossible to employ the considered expressions without the knowledge the mathematical model of the plant.

Design-oriented identificiation used in the proposed engineering method allows determining the frequency ω_n of the sinusoidal excitation signal from the settling time t_{reg} given by the process technologist. The control objective is to guarantee the fulfillment of required phase margin ϕ_M under frequency ω_n, according to expected engineering requirements on t_{reg} and η_{max} as performance. To prescribe the phase margin ϕ_M for the design stage and excitation frequency ω_n for the identification and design stage, respectively, empirical dependences $\eta_{max}=f(\phi_M)$ and $t_{reg}=(\phi_M)$ were constructed. These dependences obtained experimentally by observing performance on a batch of benchmark examples under PID controller for various phase margins and excitation frequencies provide a helpful tool for the operator in choosing suitable control objectives to achieve his engineering requirements.The above-considered empirical dependences are mentioned in Section 3, after deriving the identification and design procedures, respectively.

2.3 Performance requirements and control objectives

Control objective of the proposed engineering method including design-oriented sine wave identification of the controlled plant with unknown mathematical model (further only engineering method) is to guarantee the desired phase margin ϕ_M under the frequency ω_n. Parameters ω_n and ϕ_M adjusted in advance conform to engineering requirements express in terms of t_{reg} and η_{max}. Thus, design objectives are expressed as the couple $\{\omega_n, \phi_M\}$.

2.4 Design-oriented identification of the controlled plant using the sine wave excitation

The relay feedback is a wide-spread technique used for identification of the controlled plant in classical engineering PID tuning methods (Yu, 2006). This chapter presents advantages of the sine wave identification of the plant with unknown mathematical model.

After assemblying the multifunction control loop shown in Fig. 1a and turning the swith SB to position ”2“, the excitation variable injected into the unknown plant G(s) is a sine wave signal (Fig. 1b)

$$u(t) = U_n \sin(\omega_n t), \tag{4}$$

where U_n is magnitude of the excitation signal u(t) and ω_n is excitation frequency of the sine wave generator.

The principal advantage of using the sine wave method is that the output signal of the process y(t) is also of sine wave type with the same frequency ω_n as the frequency of the excitation signal u(t). The ouput signal y(t) can be described as

$$y(t) = Y_n \sin(\omega_n t + \varphi), \tag{5}$$

where Y_n is magnitude of the output signal and φ is the phase lag with respect to the excitation signal u(t). After reading off the values Y_n a φ from the recorded signals u(t) and y(t), the particular point of the plant frequency characteristics

$$G(j\omega_n) = |G(j\omega_n)| e^{j \arg G(\omega_n)} = \frac{Y_n}{U_n}(j\omega_n) e^{j\varphi(\omega_n)} = A_n(j\omega_n) e^{j\varphi(\omega_n)} \tag{6}$$

corresponding to the excitation frequency ω_n output is plotted in the compex plane (Fig. 1b). Another advantage of the sine wave identification is that the sine wave signal magnitude Y_n of the output y(t) can be affected by the magnitude U_n of the excitation sine signal generated by the sine wave generator. Characteristic data of the controlled plant obtained by performing the sine wave identification test are represented by the triple $\{\omega_n, A(j\omega_n), \varphi(\omega_n)\}$.

2.5 Control law of the sine wave engineering method for tuning PID controllers

The block diagram of the control loop for application of the proposed engineering method is in Fig. 1a. The control law is easy to derive from the closed-loop characteristic equation (7) when the swith SB is turned into „1“ and the controller is adjusted to manual regime.

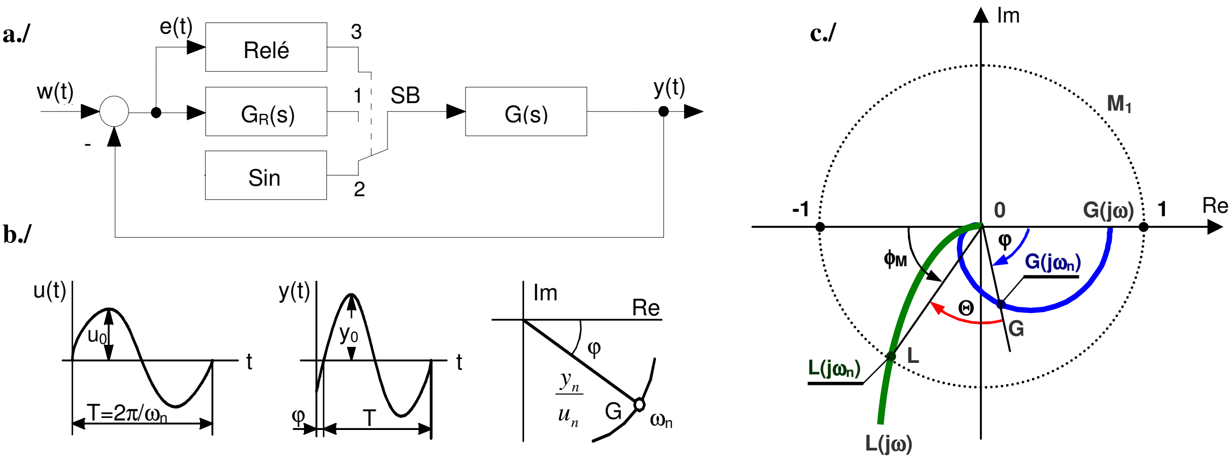

Fig. 1: a./ Feedback loop for the sine wave experimental method, b./ Identification procedure using the sine wave method, c./ grafical representation of the PID controller synthesis in the complex plane

In Fig. 1a, G(s) is the plant transfer function with unknown mathematical model and $G_R(s)$ the PID controller transfer function. The closed-loop characteristic equation

$$1 + L(j\omega_n) = 1 + G(j\omega_n) G_R(j\omega_n) = 0, \tag{7}$$

can be easily broken down into the magnitude and phase conditions

$$|G(j\omega_n)||G_R(j\omega_n)| = 1, \; \arg G(\omega_n) + \arg G_R(\omega_n) = -180° + \phi_M, \tag{8}$$

where ϕ_M is the required phase margin, L(jω) is the open-loop transfer function. Graphical interpretation of the conditions (8) is depicted in Fig. 1c.

Introduce the following substitutions

$$\varphi = arg\, G(\, \omega_n \,)\,,\ \Theta = arg\, G_R(\, \omega_n \,)\,. \tag{9}$$

Consider the interacting form of ideal PID controller

$$G_R(\, s \,) = K\left[1 + \frac{1}{T_i s} + T_d s\right], \tag{10}$$

where K is proportional gain, and T_i and T_d are integral and derivative time constants, respectively. A frequency-domain comparison of the right-hand side of equation (10)

$$G_R(\, j\omega_n \,) = K + jK\left[T_d \omega_n - \frac{1}{T_i \omega_n}\right] \tag{11}$$

with the right-hand side of the PID controller in polar form

$$G_R(\, j\omega_n \,) = \left|G_R(\, j\omega_n \,)\right| e^{j\Theta} = \left|G_R(\, j\omega_n \,)\right| cos\,\Theta + j\left|G_R(\, j\omega_n \,)\right| sin\,\Theta \tag{12}$$

yields a complex equality

$$K + jK\left[T_d \omega_n - \frac{1}{T_i \omega_n}\right] = \frac{cos\,\Theta}{\left|G(\, j\omega_n \,)\right|} + j\frac{sin\,\Theta}{\left|G(\, j\omega_n \,)\right|} \tag{13}$$

from which it is possible to obtain PID controller parameters using the substitution

$$\left|G_R(\, j\omega_n \,)\right| = \frac{1}{\left|G(\, j\omega_n \,)\right|} \tag{14}$$

resulting from the magnitude condition (8a). The controller gain K can be expressed directly from the complex equation (14)

$$K = \frac{cos\,\Theta}{\left|G(\, j\omega_n \,)\right|} \tag{15}$$

and the derivative time constant T_d can be specified from the quadratic equation with respect to $T_d\omega_n$

$$K\left[T_d \omega_n - \frac{1}{\beta T_d \omega_n}\right] = \frac{sin\,\Theta}{\left|G(\, j\omega_n \,)\right|}\,. \tag{16}$$

The ratio of integral and derivative time constants is set by an appropriate choice of coefficient β

$$T_i = \beta T_d \tag{17}$$

according to the most frequently used empirical methods β=4. Substituting (15) into (16) yields the following modificated quadratic equation with respect to $T_d\omega_n$

$$\left[T_d \omega_n - \frac{1}{\beta T_d \omega_n}\right] = tg\,\Theta \tag{18}$$

that can simply be modified as follows

$$T_d^2 \omega_n^2 - T_d \omega_n tg\,\Theta - \frac{1}{\beta} = 0\,. \tag{19}$$

The expression for calculation of the derivative time constant T_d results directly from the solution of (19)

$$T_d = \frac{tg\Theta}{2\omega_n} \pm \frac{1}{\omega_n}\sqrt{\frac{tg^2\Theta}{4} + \frac{1}{\beta}} \,. \tag{20}$$

Hence, PID controller parameters are calculated according to the following expressions

$$K = \frac{cos\Theta}{|G(j\omega_n)|} = \frac{U_n}{Y_n}cos\Theta\,,\; T_i = \beta T_d\,,\; T_d = \frac{tg\Theta}{2\omega_n} \pm \frac{1}{\omega_n}\sqrt{\frac{tg^2\Theta}{4} + \frac{1}{\beta}}\,, \tag{21}$$

where the angle Θ is obtained from the phase condition (8b)

$$\Theta = -180° + \phi_M - arg\, G(\omega_n) = -180° + \phi_M - \varphi\,. \tag{22}$$

The identified point G of the plant frequency response G(jω) with co-ordinates according to (6) is moved into the point L of the open-loop frequency response lying on the unit circle M_1 by the designed PID controller. Thus, the identified point G of the plant frequency response G(jω) determines the gain crossover point L of the open-loop L(jω)

$$L \equiv L(j\omega_n) = \left[|L(j\omega_n)|, arg\, L(\omega_n)\right] = \left[|L(j\omega_n)|, \phi_M\right], \tag{23}$$

in which the designed PID controller guarantees the required phase margin ϕ_M. Therefore under the excitation frequency ω_n holds $|L(j\omega_n)|=1$. Mutual position of the points $G(j\omega_n)$ and $L(j\omega_n)$ is shown in Fig.1c.

It is advantageous to derive the frequency of the sine wave generator from the *plant ultimate frequency* ω_c that can be determined by the well-known relay experiment according Rotač (Rotač, 1984). The experiment is carried out by switching the switch in the block diagram in Fig. 1a to „3“. It is useful to choose the sine wave generator frequency from the interval

$$\omega_n \in \langle 0.2\omega_c, 0.95\omega_c \rangle, \tag{24}$$

obtained from observing results of the proposed engineering method applied to control a batch of benchmark examples. As the choice of the frequency ω_n influences the closed-loop dynamics, the interval (24) enables to shape the closed-loop step response.

3 EXPERIMENTAL RESULTS OBTAINED ON BENCHMARK EXAMPLES

Proposed engineering method based on design-oriented sine wave identification was used to identify and tune PID controllers on the following benchmark examples (Åström, 2000)

$$G_A(s) = \frac{e^{-Ds}}{Ts+1}\,,\; G_B(s) = \frac{1}{(s+1)(\alpha s+1)(\alpha^2 s+1)(\alpha^3 s+1)}\,, \tag{25}$$

$$G_C(s) = \frac{1}{(Ts+1)^n}\,,\; G_D(s) = \frac{1}{s^5 + 2.19s^4 + 6.5s^3 + 6.3s^2 + 5.24s + 1}\,. \tag{26}$$

Control objectives for benchmark systems with transfer functions (25), (26) are to guarantee achieving the following phase margins

$$\phi_{Mj} = \{20°, 30°, 40°, 50°, 60°, 70°, 80°, 90°\}, \quad \text{j=1... 8.} \tag{27}$$

It can be expected that lower phase margin values ϕ_M will cause increase of the maximum overshoot η_{max} of the closed-loop step response.

The main objective of engineering method is to guarantee fulfillment of magnitude and phase conditions (8) at the same frequency ω_n.

Notice that according to the proposed PID design technique the gain crossover frequency ω_a and sine wave frequency are identical (e.g. $\omega_a \equiv \omega_n$).

It can be expected that changes in identification frequency will affect considerably the settling time t_{reg}.

Let us split the interval (24) into five linear segments with the segment width $\Delta\omega_n = 0.15\omega_c$. The described segmentation yields a set of excitation frequencies given by

$$\omega_{nk} = \{0.2\omega_c, 0.35\omega_c, 0.5\omega_c, 0.65\omega_c, 0.8\omega_c, 0.95\omega_c\} = \{\sigma_k \omega_c\}, \text{ k=1...6.} \tag{28}$$

Either element from this set represents a different identification level ω_{nk}. Synthesis of PID controllers on benchmark examples $G_A(s)$ … $G_D(s)$ with performance requirements expressed by a carthesian product $\omega_{nk} \times \phi_{Mj}$ of sets (27) and (28) for j=1...8 and k=1...6, followed by the analysis of acquired settling times t_{reg} and maximum overshoots η_{max} of closed-loop step responses yielded empirical dependences $\eta_{max}=f(\phi_M)$; $t_{reg}=(\phi_M)$ obtained for individual identification levels ω_{nk} (Fig. 2).

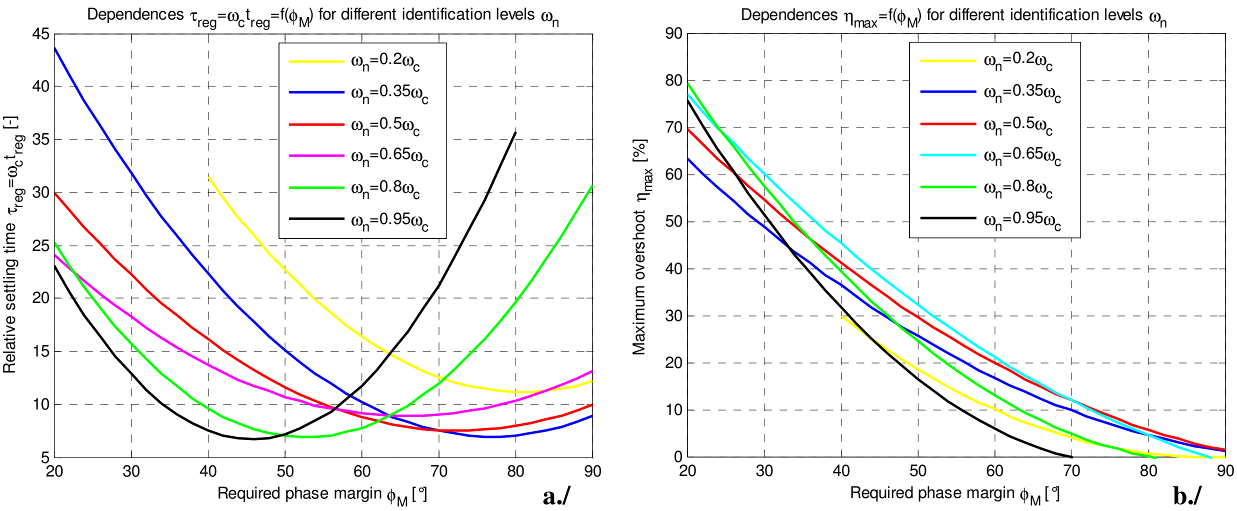

Fig. 2: Dependences: a./ $\tau_{reg}=\omega_c t_{reg} f(\phi_M)$, b./ $\eta_{max}=f(\phi_M)$ for individual identification levels ω_{nk}, k=1,2,3,4,5,6

Considering the identity of frequencies $\omega_a \equiv \omega_n$ yielded from moving the identified plant frequency response point $G(j\omega_n)$ into the open-loop gain crossover point $L(j\omega_n)$ in Fig. 2c, it is straightforward to define the settling time t_{reg} as

$$t_{reg} = \frac{\gamma\pi}{\omega_n}, \tag{29}$$

which is similar to (3) from the Reinisch´s formulae (Hudzovič, 1982); γ represents the shape factor of the closed loop step response. Its value in Reinisch´s relation for a 2nd order closed-loop system usually ranges within the values from 1 to 4, according to damping coefficient specifications (Hudzovič, 1982).

Unlike this, when using the proposed engineering method, γ changes more significantly within the empirical interval (0.5;16) strongly depending on the phase margin ϕ_M. i.e. $\gamma=f(\phi_M)$ under the given excitation frequency ω_n. To explore settling times of closed-loops with different dynamics it is useful to define a new performance measure, a so-called relative settling time

$$t_{reg}\omega_n = \pi\gamma(\phi_M). \tag{30}$$

Substituting $\omega_n=\sigma\omega_c$ into (30) we can define the *relative settling time* $\tau_{reg}=t_{reg}\omega_c$ as follows

$$t_{reg}\omega_c = \frac{\pi}{\sigma}\gamma(\phi_M) \tag{31}$$

relating the settling time t_{reg} to the ultimate frequency ω_c of the plant. Advantage of using the plant ultimate frequency ω_c consists in that the left-hand side of (31) is independent from the excitation frequency ω_n. This empirical dependence for different identification levels ω_{nk} is plotted in Fig. 2a showing that when increasing the desired phase margin ϕ_M, the relative settling time first drops and after achieving its optimal value τ_{reg_opt} grows again quadratically.

For example for ω_c=1rad.s^{-1}, the same settling time t_{reg}=10s is achieved for both ϕ_{M1}=38° and ϕ_{M2}=66° with corresponding maximum overshoots η_{max1}=40% and η_{max2}=7%, respectively, at the identification level ω_{n5}=0.8rad.s^{-1} according to green plots in Fig. 2b. The maximum overshoot η_{max}=10% can be obtained either on the identification level ω_{n2}=0.35rad.s^{-1} for the phase margin ϕ_{M1}=70° with settling time t_{reg1}=7s (blue plots), or on the identification level ω_{n1}=0.2rad.s^{-1} for the phase margin ϕ_{M2}=62° with more than double settling time t_{reg2}=16s (yellow plots).

Dependences shown is Fig. 2 provide the tool for choosing the excitation signal frequency ω_n and phase margin ϕ_M such that the settling time t_{reg} and maximum overshoot η_{max} requirements are met.

Closed-loop time responses under PID controllers tuned for both phase margins ϕ_{M1}=50° and ϕ_{M2}=70° on different identification levels ω_{n1}=0.2ω_c, ω_{n3}=0.5ω_c, ω_{n5}=0.8ω_c a ω_{n6}=0.95ω_c are shown in Fig. 3. Significant differences between dynamics of individual control loops can be observed. Designs were performed for the benchmark plant $G_B(s)$ and the parameter α=0.5.

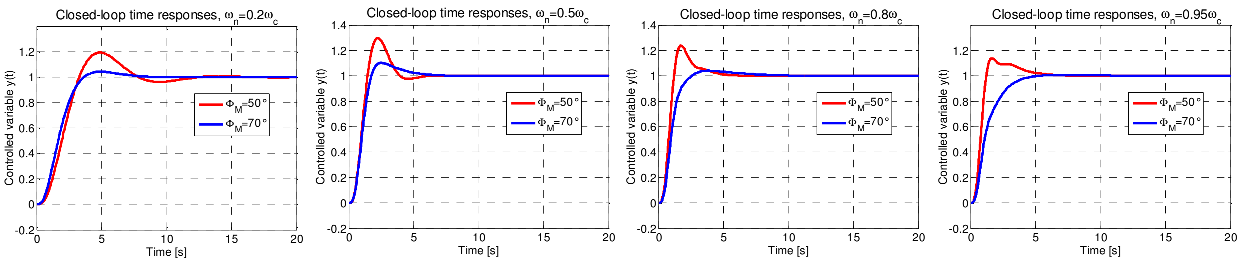

Fig. 3: Closed-loop step responses for both ϕ_{M1}=50° and ϕ_{M2}=70° on several identification levels ω_{nk}

4 ROBUSTIFICATION OF THE PROPOSED ENGINEERING METHOD

The main idea of the design-oriented identification of the uncertain plant consists in repeating the sine wave identification for individual uncertainty changes using the excitation signal frequency ω_n yielding a set of identified points G_i, i=1,2,...,N of the uncertain plant frequency responses

$$G_i = G_i(j\omega_n) = |G_i(j\omega_n)| e^{j\arg G_i(\omega_n)}. \tag{32}$$

Plant parameter changes are reflected in magnitude and phase changes $|G_i(j\omega_n)|$ and $\arg G_i(\omega_n)$, respectively, of identified points $G_i(j\omega_n)$ plotted in the complex plane. If the multiple identification of individual points G_i of the uncertain plant frequency characteristics is performed using sine wave excitation signals with the same frequency ω_n in each identification experiment, then each identified point G_i in the complex plane corresponds to a different frequency characteristics from the set of plant models and simultaneously, each identified point G_i in the complex plane corresponds to the same angular frequency ω_n for i=1,2,...,N; where N=p+1 is the number of identification experiments and p is the number of varying technological quantities of the plant (Veselý, 2008). Location of identified points $G_i(j\omega_n)$ of the unknown uncertain plant in the complex plane can be expressed in the standard form complex number

$$G_i(j\omega_n) = a_i + jb_i, \text{ i=1,2,...,N.} \tag{33}$$

The real and imaginary parts of the nominal plant model $G_0(j\omega_n)$ are obtained as mean values of real and imaginary parts of identified points according to

$$G_0(j\omega_n) = a_0 + jb_0 = \frac{1}{i}\sum_i^N a_i + j\frac{1}{i}\sum_i^N b_i, \text{ i=1,2,...,N,} \tag{34}$$

where the magnitude $|G_0(j\omega_n)|$ and the phase $\varphi_0(\omega_n)=\arg G_0(\omega_n)$ are calculated as follows

$$G_0(j\omega_n) = |G_0(j\omega_n)|e^{j\varphi_0(\omega_n)}, \; |G_0(j\omega_n)| = \sqrt{a_0^2 + b_0^2}, \; \varphi_0(\omega_n) = arctg\frac{b_0}{a_0}. \tag{35}$$

The points G_i representing plant uncertainties can be enclosed in the circle M_G centered in $G_0(j\omega_n)$ with the radius $R_G \equiv R_G(\omega_n)$ obtained as a maximum distance between the i-th identified point $G_i(j\omega_n)$ and the nominal point $G_0(j\omega_n)$

$$R_G = \max_i\left\{\sqrt{(a_i - a_0)^2 + (b_i - b_0)^2}\right\}, \text{ i=1,2,...,N.} \tag{36}$$

The dispersion circle M_G centered in the nominal point G_0 with the radius R_G encircles all identified points G_i of the uncertain plant; Fig. 4 illustrates the situation for N=3 identifications.

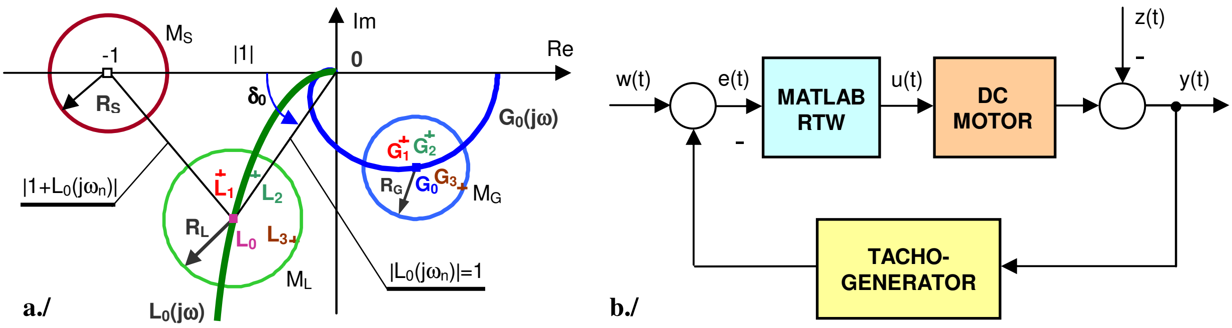

Fig. 4: a./ Dispersion circles M_G, M_L and the prohibited area delimited by the circle M_s, b./ Feedback loop of the DC motor

The proposed control law generated by the robust controller $G_{Rrob}(s)$ designed for the nominal point $G_0(j\omega_n)$ actually carries out the following transformation of the set of identified points $G_i(j\omega_n)$ encircled by M_G with the radius R_G into the set of points $L_i(j\omega_n)$ delimited by M_L

$$\Re : \{R_G \rightarrow R_L : R_L = |G_{Rrob}|R_G\} \tag{37}$$

and also calculates the radius $R_L \equiv R_L(\omega_n)$ of the dispersion circle M_L corresponding to the points $L_i(j\omega_n)$ of the Nyquist plot so as to guarantee fulfillment of the robust stability condition.

The robust PID controller is designed using the proposed sine wave method described in section 2; the input data for the nominal model $G_0(j\omega_n)$ are its following coordinates

$$A_0 = |G_0(j\omega_n)|, \; \varphi_0 = \arg G_0(\omega_n). \tag{38}$$

Substituting coordinates of the nominal model $G_0(j\omega_n)$ into (21) following expressions for calculating robust PID controller parameters are obtained

$$K_{rob} = \frac{\cos\Theta_0}{A_0}, T_{irob} = \beta T_{drob}, T_{drob} = \frac{tg\Theta_0}{2\omega_n} \pm \frac{1}{\omega_n}\sqrt{\frac{tg^2\Theta_0}{4} + \frac{1}{\beta}}. \tag{39}$$

For the parameter Θ_0 and the excitation frequency ω_n the modified phase condition holds

$$\Theta_0 = -180° + \delta_0 - \varphi_0 . \tag{40}$$

Thus, δ_0 is a modified phase margin and at the same time a robust PID controller tuning parameter appearing in (39) and (40) for calculation of its parameters that guarantee the phase margin required for guaranteeing robust stability; δ_0 does not influence the radius of the dispersion circle M_L, just the distance between L_0 and the critical point (-1,0). This enables to draw the circle M_L apart from the critical point (-1,0) thus improving robust closed-loop performance. If the nominal open-loop

$$L_0(s) = G_R(s)G_0(s) \tag{41}$$

is stable, then according to the Nyquist stability criterion the closed-loop with the uncertain plant will be stable if the distance between L_0 and the point (-1,j0), i.e. $|1+L_0(j\omega_n)|$ is greater than the radius $R_L(\omega_n)$ of the dispersion circle M_L centered in L_0 (Veselý, 2008)

$$R_L(j\omega_n) < \left|1 + L_0(j\omega_n)\right| , \tag{42}$$

where ω_n is the sine wave generator frequency. The distance between the point (-1,0) and the open-loop Nyquist plot with the nominal model L_0 can be calculated according to Fig. 4a by applying the cosine rule to the triangle (-1,0,L_0)

$$\left|1 + L_0\right|^2 = \left|1\right|^2 + \left|L_0\right|^2 - 2.1.\left|L_0\right| cos\,\delta_0 , \tag{43}$$

where δ_0 is the modified phase margin. According to the robust stability condition the distance

$$\left|1 + L_0\right| = \sqrt{1 + \left|L_0\right|^2 - 2\left|L_0\right| cos\,\delta_0} , \tag{44}$$

has to be greater than the radius R_L of the dispersion circle centered in L_0, i.e. the following inequality has to be satisfied

$$R_L \overset{!}{<} \left|1 + L_0\right| . \tag{45}$$

Substituting the distance (44) into (45) yields the robust stability condition in the form

$$R_L \overset{!}{<} \sqrt{1 + \left|L_0\right|^2 - 2\left|L_0\right| cos\,\delta_0} . \tag{46}$$

From the principles of the proposed empirical sine wave PID controller tuning method results, that the robust controller shifts the nominal point of the plant frequency response G_0 to the point L_0 of the unit circle at frequency ω_n. Thus ω_n becomes open loop gain crossover frequency. As the point L_0 is lying on the circle M_1, the magnitude $|L_0(j\omega_n)|$ equals one, i.e. $|L_0|=|G_0||G_R|=1$, yielding the transformation ratio $|G_R|=|G_0|^{-1}$ between the radii R_G and R_L of the circles M_G and M_L, respectively. The radius R_L of the dispersion circle M_L can be expressed as

$$R_L = R_G\left|G_R\right| = R_G\left|G_0\right|^{-1} . \tag{47}$$

Substituting (47) in (46) yields the robust stability condition in the following form

$$R_G^2\left|G_R^2\right|^{-1} \overset{!}{<} 2 - 2\,cos\,\delta_0 . \tag{48}$$

After minor manipulations, condition for calculating the angle δ_0 is obtained

$$cos\,\delta_0 < \left(1 - \frac{1}{2}\frac{R_G^2}{\left|G_0^2\right|}\right). \tag{49}$$

According to the robust stability condition the chosen value δ_0 is substituted into the phase condition (40) and afterwards parameters of the robust PID controller are calculated from (39).

5 VERIFICATION OF THE PROPOSED METHOD ON A REAL PLANT

The empirical sine wave method was applied for robust control of a physical plant – the DC motor where the controlled output y(t) is the speed and the input variable u(t) is the armature voltage generated by the control system implemented in Matlab-Realtime Workshop. To sense the output y(t) a tachogenerator (TG) is used. The disturbance affecting the motor operation is the load torque z(t) as depicted in the feedback loop in Fig. 4b. Let the performance requirements for the nominal model of the motor formulated by the process technologist are given in terms of maximum overshoots η_{max1}=25% and η_{max2}=10%, respectively.

According to empirical curves depicted in Fig. 2 the maximum overshoot η_{max1}=25% can be obtained for ϕ_{M1}=50° and η_{max2}=10% for ϕ_{M2}=70° both at identification level $\omega_{n2}=0.35\omega_c$ (blue curves). Specified nominal ultimate frequency of the DC motor is ω_{c0}=2.05 rad.s^{-1}, hence it is recommended to carry out the design-oriented identification on the „blue level" at frequency ω_{n0}=0.72rad.s^{-1}. Expected relative settling time values according to blue empirical curves are τ_{reg1}=16 and τ_{reg2}=4.5, respectively, from which $t_{reg1}=\tau_{reg1}/\omega_{c0}$=7.8s and $t_{reg2}=\tau_{reg2}/\omega_{c0}$=2.2s. Three identification experiments were carried out on the DC motor, for the minimum, medium and the maximum loads.

For both cases, robust properties were verified by applying first the reference speed step change and after the transient response died out the load step change was applied. Corresponding time responses are shown in Fig. 5.

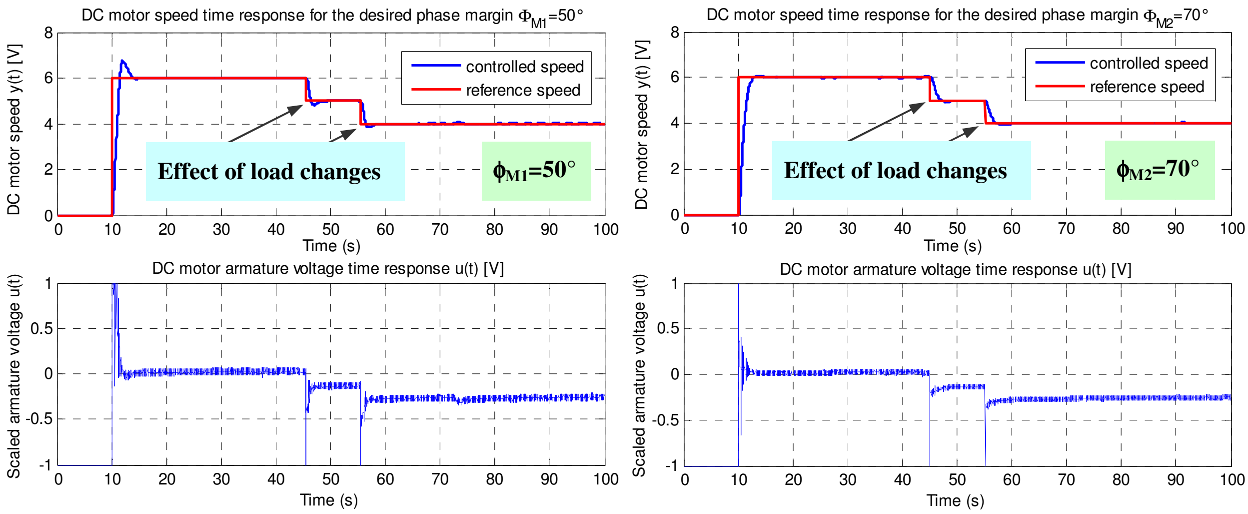

Fig. 5: DC motor closed-loop time responses for different phase margin values ϕ_M

6 CONCLUSION

Closed-loop time responses in Fig. 5 and position of the dispersion circle M_L in Fig. 6 prove robust stability and satisfaction of performance requirements under the designed PID controller. In addition, Fig. 6 shows that the design for ϕ_{M2}=70° determines a larger prohibited area in the complex plain delineated by the circle M_s than the design for ϕ_{M1}=50°. The proposed new engineering method based on sine wave type identification of the controlled plant allows successful tuning of PID controller coefficients. An important contribution is the construction of empirical curves to calculate frequency domain performance specification (in terms of phase margin) from engineering time-domain requirements specified by a process technologist (in terms of maximum overshoot and settling time).

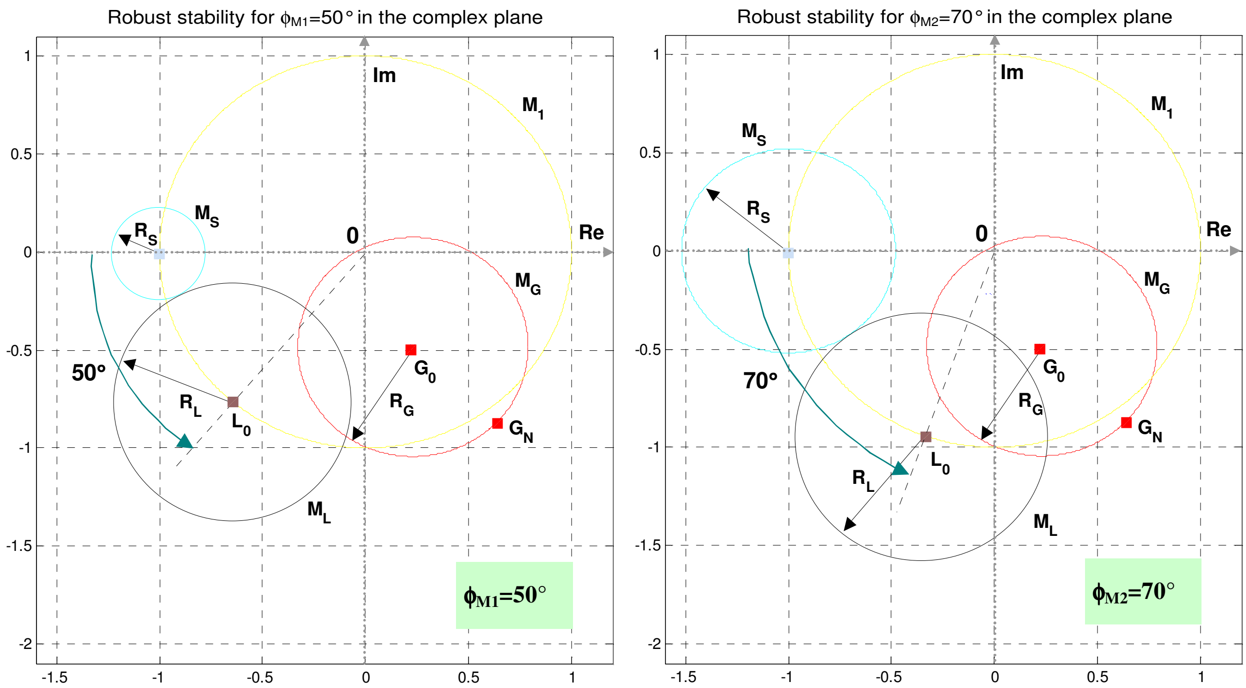

Fig. 6: Robust stability for different phase margins ϕ_M depicted in the complex plane

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