CONTROL OF LABORATORY MODEL OF PENDUBOT

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Abstract: This paper presents a concept of control of laboratory model of pendubot, which is a two-link under actuated robotic mechanism. Method of obtaining a mathematical model for pendubot is presented. Further this mathematical model is used for synthesis of LQ control. The inverted pendulum problem is well suited for education in control theory as well as for research in control of nonlinear mechatronic systems with quick dynamics.

Keywords: Inverted pendulum, Pendubot, LQ control, state space model

1 INTRODUCTION AND PRELIMINARIES

The pendulum is mechatronical system which is one of the most important examples in dynamics and control and has been studied. Many important engineering systems can be approximately modelled as pendulum in order to gain insight into their dynamic behaviour and for control systems design e.g. trajectory of rocket or segway. Pendubot (Fig.1) is a two-link planar robot with an actuator at the first shoulder and no actuator at the elbow. The second arm moves freely around the first link which is driven by a motor (Mates, 2009). The control objective is to bring the mechanism to one of the unstable equilibrium positions. This paper deals with deriving of a mathematical model of the pendubot. Further the gain matrix of LQ control is obtained and the results are verified on a physical model.

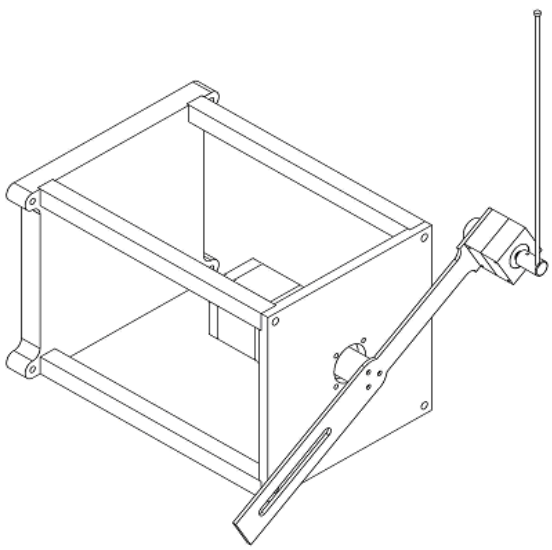

Figure 1: Pendubot construction

2 MATHEMATICAL MODEL

At first we will derive the nonlinear dynamic equations of the system using the Lagrange method of the second kind, which depends on the balance of system energy. The resulting equations can be written in closed form to allow an appropriate system analysis. After that the state space representation is created using linearization in chosen operating point (Aurelie 2006, Block 1996, Mates 2008). The following table (Table 1) lists physical parameters of our laboratory pendubot physical model, the symbols and corresponding values. The general notations are shown in figure 2.

Table 1: Parameter values

Description	Symbol	Value
Weight of arm	m_r	0,63 kg
Length of arm	l_1	0,44 m
Distance from centre of gravity of the arm to the axis of rotation	l_{g1}	0 m
Friction coefficient in arm joint	$k1$	0,08 kg.m^{2}s^{-1}
Mass moment of inertia of the arm	I_r	0,021 kg.m^2
Weight of pendulum	m_k	0,062 kg
Distance from centre of gravity of the pendulum to the axis of rotation	l_{g2}	0,2 m
Mass moment of inertia of the pendulum	I_k	0,0012 kg.m^2
Friction coefficient in pendulum joint	$k2$	0,0001 kg.m^{2}s^{-1}

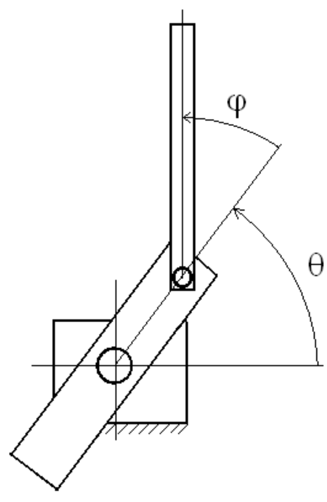

Figure 2: Measured angles

The basic form of Lagrange equations is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\left(\frac{\partial L}{\partial q_i}\right)=Q_i \tag{1}$$

Where L is the Lagrange function, q_i is the i-th generalized coordinates and Q_i is a generalized force in the direction of i-th coordinates:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right)-\left(\frac{\partial L}{\partial \varphi}\right)=\tau \tag{2}$$

and τ is the input torque of the system:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\left(\frac{\partial L}{\partial \theta}\right)=0 \tag{3}$$

The Lagrange function is expressed as the difference between kinetic and potential energy of system. For the pendubot system this is:

$$\begin{aligned} L\left(\varphi,\theta,\dot{\varphi},\dot{\theta}\right) &= \dot{\varphi}_1^2\left[\frac{1}{2}I_r+\frac{1}{2}l_1^2\right]+\dot{\theta}^2\left[\frac{1}{2}I_k+\frac{1}{2}m_k l_{g2}^2\right]+m_k l_1 l_{g2}\cos\left(\varphi-\theta\right)\dot{\varphi}\dot{\theta} \\ &\quad -\left(m_r l_{g1}+m_k l_1\right)g\sin\left(\varphi\right)-m_k g l_{g2}\sin\left(\theta\right) \end{aligned} \tag{4}$$

In the next few steps partial derivations of the Lagrangian are obtained in order to get two nonlinear motion equations for the system:

$$\begin{aligned} \tau &= \ddot{\varphi}\left(I_r+m_k l_1^2\right)+\ddot{\theta}m_k l_1 l_{g2}\cos\left(\varphi-\theta\right)+m_k l_1 l_{g2}\dot{\theta}^2\sin\left(\varphi-\theta\right) \\ &\quad +\left(m_r l_{g1}+m_k l_1\right)g\cos\left(\varphi\right) \end{aligned} \tag{5}$$

$$\begin{aligned} 0 &= \ddot{\varphi}m_k l_1 l_{g2}\cos\left(\varphi-\theta\right)+\ddot{\theta}\left(I_k+m_k l_{g2}^2\right)-m_k l_1 l_{g2}\dot{\varphi}^2\sin\left(\varphi-\theta\right) \\ &\quad +m_k g l_{g2}\cos\left(\theta\right) \end{aligned} \tag{6}$$

For simplification of these equations following substitutions in equation (5) and equation (6) are introduced:

$$\begin{aligned} A_1 &= I_r+m_k l_1^2 & B_1 &= m_k l_1 l_{g2} \\ A_2 &= m_k l_1 l_{g2} & B_2 &= I_k+m_k l_{g2}^2 \\ A_3 &= g\left(m_r l_{g1}+m_k l_1\right) & B_3 &= m_k g l_{g2} \end{aligned} \tag{7}$$

The derivation of the $\ddot{\varphi}$ and $\ddot{\theta}$ are expressed from the motion equations as functions of all other variables plus friction coefficients (k_1 and k_2). Then we can obtain final nonlinear motion equations as:

$$\ddot{\varphi} = \frac{1}{A_1}\left[\tau - A_2\cos(\varphi-\theta)\left(\frac{1}{\frac{B_1A_2}{A_1}\cos(\varphi-\theta)-B_2}\left[-B_1\dot{\varphi}^2\sin(\varphi-\theta)-B_1\frac{A_2}{2A_1}\sin(2(\varphi-\theta))\dot{\theta}^2\right.\right.\right.$$
$$\left.\left.\left.-B_1\frac{A_3}{A_1}\cos(\varphi)\cos(\varphi-\theta)+B_3\cos(\theta)+\frac{B_1}{A_1}\cos(\varphi-\theta)\tau\right]\right)-A_2\dot{\theta}^2\sin(\varphi-\theta)-A_3\cos(\varphi)+k_1\dot{\varphi}\right] \tag{8}$$

$$\ddot{\theta} = \frac{1}{B_2}\left[-B_1\cos(\varphi-\theta)\left(\frac{1}{\frac{B_1A_2}{B_2}\cos(\varphi-\theta)-A_1}\left[\frac{A_2B_1}{2B_2}\sin(2(\varphi-\theta))\dot{\varphi}^2+A_2\sin(\varphi-\theta)\dot{\theta}^2\right.\right.\right.$$
$$\left.\left.\left.-B_1\frac{A_3}{A_1}\cos(\varphi)\cos(\varphi-\theta)+A_3\cos(\varphi)-\tau\right]\right)+B_1\dot{\varphi}^2\sin(\varphi-\theta)-B_3\cos(\theta)+k_2\dot{\theta}\right] \tag{9}$$

Although the dynamic behaviour of most physical systems is nonlinear, many of these systems behave "almost linearly" at and near nominal operating points or along nominal trajectories. In our case we have done the linearization in the upper position of both arms. Defining the space-state vector as $x=\begin{bmatrix}\varphi & \theta & \dot{\varphi} & \dot{\theta}\end{bmatrix}^T$, then the linearized state space model can be written in the following common matrix form:

$$\dot{x} = Ax + Bu \tag{10}$$

Due to the complexity of the functions equation (8) and equation (9) all necessary calculations have been done in Matlab/Simulink using the Symbolic Math toolbox. Consequently the resulting state-space matrices are in the form:

$$A=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 10{,}7135 & -7{,}1616 & -2{,}424 & 0\\ -15{,}7554 & 43{,}3201 & 0 & -0{,}027\end{bmatrix} \quad B=\begin{bmatrix}0\\ 0\\ 40{,}0328\\ -58{,}8731\end{bmatrix} \tag{11}$$

3 LQ CONTROL

The linear quadratic controller can be synthesised from the state space model (10), with matrices (11). The state feedback gains K is calculated by minimizing the criterion equation (12):

$$J = \int (x'Qx + u'Ru)dt \tag{12}$$

This is done by solving the Algebraic Riccati Equation (13):

$$A^T P + PA + Q = PBR^{-1}B^T P \tag{13}$$

Where K is given by equation (14):

$$K = R^{-1}B^T P \tag{14}$$

In Matlab environment this is solved using function lqrd. For real time experiments the sample time was set to 0,01s. The weight matrix Q and R have been determined using brute force search method in a limited range of values in the matrices. The search criterion was to obtain the controller eigenvalues without the complex part. The final settings used for simulation was following:

$$Q = \begin{bmatrix} 19 & 0 & 0 & 0 \\ 0 & 26 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$R = [3{,}5] \tag{16}$$

with the corresponding LQ control gain as:

$$K = [-1{,}54 \quad -7{,}11 \quad -0{,}69 \quad -1{,}22] \tag{17}$$

4 RESULTS

This section presents some results with the LQ control of our physical pendubot model in real time, using the Matlab/Simulink xPC Target configuration. The target PC is equipped with a I/O board, Humusoft MF-624, that is connected to a control unit Mitsubishi MR-J2S-40A. The control unit directly controls the motor in torque control mode and also reads the position the motor shaft (pendubot arm). The position of pendulum is measured by IRC sensor that is connected directly to the PC I/O board. This configuration is shown on figure 3.

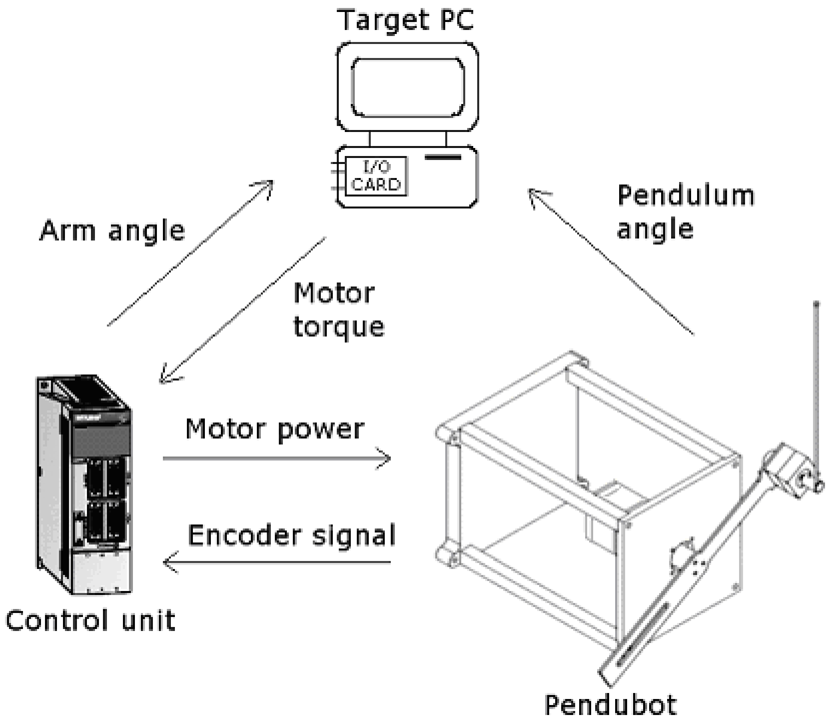

Figure 3: Pendubot hardware connection

The obtained behaviour of pendubot is illustrated on figure 4. From the figure can be seen that the system is slightly oscillating around the chosen equilibrium position. Approximately at the time instants 23 s and 24,5 s there have been introduced two disturbances, which were successfully handled by the controller.

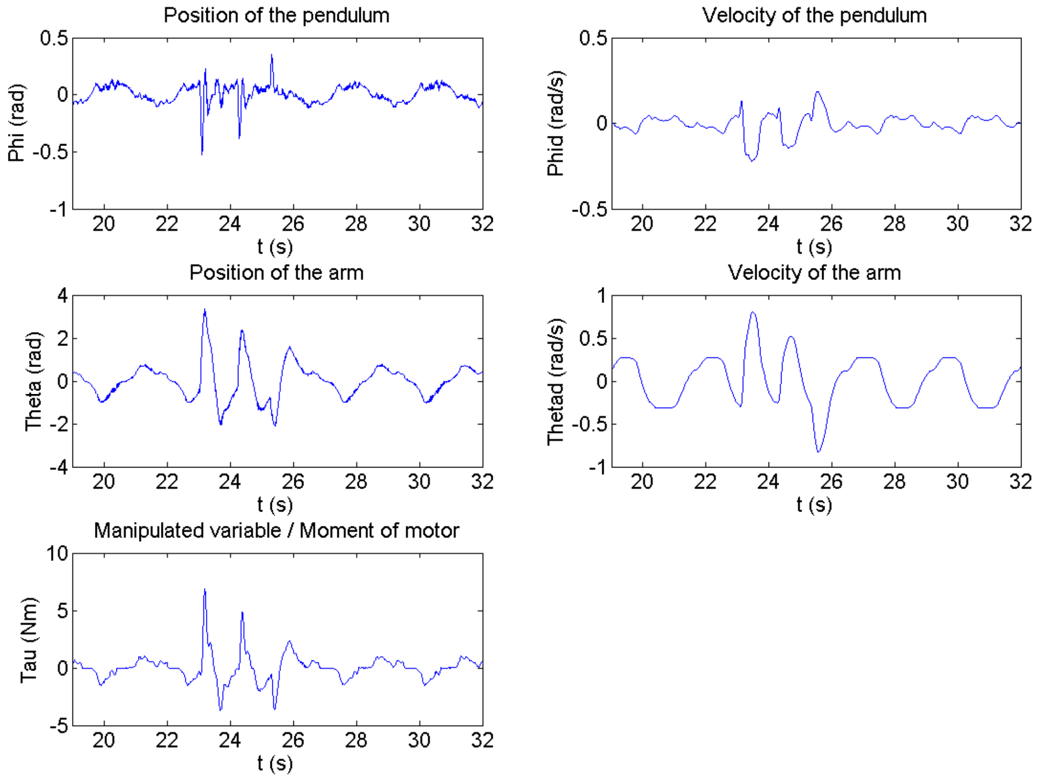

Figure 4: Experimental results

(in the figure Phid and Thetad are derivations of the angles Phi and Theta)

5 CONCLUSION

The reported outcomes are only results of the initial research done on this physical pendubot system. Further research will be directed on more precise non-linear pendubot model and MPC strategies.

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