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Robust adaptive actuator failure compensation controller for systems with unknown time-varying state delays

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Abstract—**An output feedback model reference adaptive controller is developed for a class of linear systems with multiple unknown time-varying state delays and in the presence of actuator failures. The adaptive controller is designed based on SPR-Lyapunov approach and is robust with respect to multiple unknown time-varying plant delays and to an external disturbance with unknown bounds. Closed-loop system stability and asymptotic output tracking are proved using suitable Lyapunov-Krasovskii functional and Simulation results are provided to demonstrate the effectiveness of the proposed controller.**

Keywords- Robust adaptive control, Output feedback, Actuator failure, State delay systems.

I. INTRODUCTION

Component failures occur in many practical systems and may cause performance deterioration and catastrophic accidents. There have been many studies in the literature on control of systems with component failures [1]-[11]. In these papers, different design methods including multiple model, switching and tuning designs, fault detection and diagnosis designs, robust control designs and adaptive designs are used. Compared to other approaches, the direct adaptive control approach has the key advantage that it can provide theoretically provable asymptotic tracking in addition to stability, in the presence of large parameter variation and uncertainties. Important results for direct adaptive control of systems with actuator failures exist in [8]-[11].

Delay phenomena are frequently encountered in mechanics, physics, applied mathematics, biology, economics and engineering systems. In the presence of time delay, the design of fault tolerant controller becomes more complex. Therefore, the problem of fault tolerant adaptive control of delay systems has received little attention. For example, in [12] a fault detection and accommodation method is considered for nonlinear state delay systems, based on an iterative design of an observer which monitors the variations of the system dynamics and the control signal is formed by treating component failures as uncertainties. In [13] and [14], state feedback controllers are developed within the framework of Linear Matrix Inequalities for a class of linear systems with time delay in control inputs and constant actuator failures of stuck-type. A direct state feedback adaptive control scheme is introduced in [15] for linear state delay systems with unknown plant dynamics and unknown constant stuck failures in actuators. The same problem is solved for decentralized systems in [16]. Based on a linear matrix inequality technique and an adaptive method, [17] suggests adaptive reliable controllers against loss of effectiveness unknown actuator failures, but with the assumption that the system parameters are known.

In a recent work [18], an output feedback adaptive controller is designed for state delay systems with unknown parameters and unknown constant failures of stuck-type. To the best knowledge of authors, it is the first output-feedback model reference adaptive controller for compensating actuator failures in time delay systems. In this study, a new controller structure is developed to have robustness with respect to an external bounded disturbance with unknown bounds in addition to multiple unknown time-varying plant delays.

II. PROBLEM FORMULATION

In this section, the control problem is formulated. Consider a linear state delay plant described by

$$\begin{aligned}&\dot{x}(t) = Ax(t) + Bu(t) + \sum_{l=1}^{M} A_{dl}x(t - d_l(t)) + b_f f(t),\\ &x(\theta) = x_0,\ \ \theta \in [-d_{\max}, 0]\\ &y(t) = cx(t)\end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re$ is the plant output and $u(t) = [u_1, ..., u_m]^T \in \Re^m$ is the input vector whose elements may fail during system operation. $f(t) \in \Re$ is the external disturbance with $|f(t)| \le f^*$. The constant matrices $A \in \Re^{n\times n}$, $A_{dl} \in \Re^{n\times n}$, $l = 1,..., M$ and $B = [b_1,..., b_m] \in \Re^{n\times m}$ and the vectors $c \in \Re^{1\times n}$ and $b_f \in \Re^{n\times 1}$ are unknown. The time delays $d_l(t)$ are nonnegative differentiable functions, satisfying

$$0 \le d_l(t) \le d_{\max}, \qquad \dot{d}_l(t) \le \bar{d} < 1, \quad l = 1,...,M \tag{2}$$

where $d_{\max}$ and $\bar{d}$ are some unknown positive constants.

In this paper, one important type of actuator failure modeled as

$$u_j(t) = \bar{u}_j, \qquad j \in \{1,2,...,m\} \tag{3}$$

is considered, where the constant values $\bar{u}_j$ and the failure time instants t_j are unknown. With this type of actuator failure, the input $u(t)$ is defined as

$$u(t) = v(t) + \sigma(\bar{u} - v(t)) \tag{4}$$

where $\bar{u} = [\bar{u}_1,...,\bar{u}_m]^T$,

$$\sigma = diag\{\sigma_1, \sigma_2,...,\sigma_m\}, \quad \sigma_i = \begin{cases} 1, & u_i(t) = \bar{u}_i \\ 0, & u_i(t) \ne \bar{u}_i \end{cases}$$

and $v(t)$ is an applied control input to be designed. For this important type of actuator failure it is a basic assumption that [8]:

(A1)- If the system parameters and actuator failures (up to $m-1$ failures) are known, the remaining actuators can still achieve a desired control objective.

The adaptive control objective is to determine an output feedback $v(t)$ for the plant (1) with unknown parameters and unknown actuator failures (3) such that despite the control error $u - v = \sigma(\bar{u} - v)$, all signals of the closed-loop system remain bounded and the plant output $y(t)$ follows the output $y_m(t)$ of a stable reference model with the transfer function

$$y_m(t) = W_m(s)r(t), \qquad W_m(s) = \frac{1}{D_m(s)} \tag{5}$$

asymptotically; i.e., $\lim_{t\to\infty} e(t) = \lim_{t\to\infty}(y(t) - y_m(t)) = 0$. In the above equation, $D_m(s)$ is a monic Hurwitz polynomial and $r(t)$ is the reference input which is assumed to be uniformly bounded and piecewise continuous.

For simplicity the "equal control" design

$$v_1(t) = v_2(t) = ... = v_m(t) \equiv v_o(t) \tag{6}$$

which assumes that the control inputs to all actuators are the same, is used. It is reasonable in many practical applications.

Consider the transfer function of the system without delay as

$$y(t) = \sum_{j=1}^{m} \frac{k_{pj} N_j(s)}{D(s)} u_j(t), \quad c(sI - A^{-1})b_j = k_{pj} \frac{N_j(s)}{D(s)} \tag{7}$$

where k_{pj} is a scalar, $N_j(s)$ is a monic polynomial and $D(s)$ is a monic polynomial of degree n.

With the assumption that p (up to $m-1$) actuators fail and from (4) and (6) the closed-loop system (7) can be expressed as

$$y(t) = W_o(s)v_o(t) + \bar{y}(t) \tag{8}$$

where

$$\bar{y}(t) = \sum_{j=j_1,...,j_p} \frac{k_{pj} N_j(s)}{D(s)} \bar{u}_j(t),$$

$$W_o(s) = \frac{\sum_{j \ne j_1,...,j_p} k_{pj} N_j(s)}{D(s)} \equiv \frac{k_p N_{uf}(s)}{D(s)} = c(sI - A)^{-1} b, \tag{9}$$

$$b = \sum_{j \ne j_1,...,j_p} b_j \ ,$$

To achieve control objective, it is assumed that $W_o(s)$ satisfies the following assumptions:

(A2)- $N_{uf}(s)$ is stable for each number of failures.

(A3)- The high-frequency gain k_p has the same sign for each number of failures and $sign[K_r^*] = sign[k_p]$.

(A4)- The relative degree of the transfer function $W_o(s)$ is $n^* = 1$ for each number of failures.

III. Error Equation

In order to design the adaptive controller, a suitable error equation parameterization is obtained in this section. For this purpose, first consider the system without delay (7) and when the plant parameters and actuator failures are known. The controller structure is defined as

$$\begin{aligned} &v_o^*(t) = K_e^* y(t) + K_1^{*T} x_1(t) + K_2^{*T} x_2(t) + K_r^* r(t) + K_3^*, \\ &x_1(t) = H(s)v_o(t), \ x_2(t) = H(s)y(t), \\ &H(s) = \frac{\alpha(s)}{\Lambda(s)}, \end{aligned} \tag{10}$$

in which $\Lambda(s)$ is a monic Hurwitz polynomial of degree n, $K_1^* \in \Re^{n-1}, K_2^* \in \Re^{n-1}, K_r^* \in \Re$, $K_e^* \in \Re$ are the parameters of the standard controller structure for MRAC of systems without delay and $\alpha(s) = [1, s, \ldots s^{n-2}]^T$.

In order to drive the error equation, $y(t)$ from (8) is substituted in (10) and the closed-loop system is obtained as

$$y(t) = W_o(s)(1 - K_1^{*T} H(s) - K_2^{*T} H(s) W_o(s) - K_e^* W_o(s))^{-1} \times [K_2^{*T} H(s)\bar{y} + K_e^* \bar{y} + K_r^* r + K_3^*] + \bar{y}(t). \quad (11)$$

Now suppose that there exist constant matrices a_{dl}^* and F of appropriate dimensions such that

$$A_d = b a_{dl}^{*T}, \quad b_f = bF.$$

Using directly the analysis in [18] and [19], the tracking error $e(t) = y(t) - y_m(t)$ can be written as

$$\begin{aligned} e(t) = \frac{k_p}{D_m(s)}[&v_0(t) - K_1^{*T} x_1(t) - K_2^{*T} x_2(t) \\ &- K_e^* y(t) - K_3^* + (1 - K_1^{*T} H(s)) F f(t) \\ &+ \sum_{l=1}^{M} a_{dl}^{*T} x(t - d_l) - K_1^{*T} H(s) \sum_{l=1}^{M} a_{dl}^{*T} x(t - d_l)]. \end{aligned} \quad (12)$$

To find a suitable error equation parameterization the dynamic system

$$z(t) = \sum_{l=1}^{M} K_1^{*T} H(s)[a_{dl}^{*T} x(t - d_l)] = \sum_{l=1}^{M} K_{zl}^{*T} z_{xl}(t) \quad (13)$$

is defined in which,

$$\begin{aligned} K_{zl}^{*T} &= [K_1^{*1T} a_{dl}^{*T}, K_1^{*2T} a_{dl}^{*T}, \ldots, K_1^{*(n-1)T} a_{dl}^{*T}], \\ z_{xl}(t) &= H_n(s)[x(t - d_l)], \\ H_n(s) &= \frac{[I_{n\times n} s^{n-2}, \ldots, I_{n\times n} s, I_{n\times n}]}{\Lambda(s)} \in \Re^{n(n-1)\times n}. \end{aligned}$$

By decomposing $z_{xl}(t)$ into two components we have

$$\begin{aligned} z_{xl}(t) &= z_{el}(t) + z_{ml}(t), \\ z_{ml}(t) &= H_n(s)[x_{ml}(t - d_l)], \\ z_{el}(t) &= H_n(s)[e_x(t - d_l)], \\ e_x(t - d_l) &= x(t - d_l) - x_m(t - d_l), \end{aligned} \quad (14)$$

Using (13) and (14), the error equation (12) can be rewritten as follows

$$\begin{aligned} e(t) = &\frac{k_p}{D_m(s)}[v_0(t) - K^{*T} w(t)] - \frac{k_p}{D_m(s)}[K_m^{*T} w_m(t) \\ &+ \sum_{l=1}^{M} K_{dl}^{*T} e_x(t - d_l(t)) + \sum_{l=1}^{M} K_{zl}^{*T} z_{el}(t) \\ &+ (1 - K_1^{*T} H(s)) F f(t)] \end{aligned} \quad (15)$$

where $K_{dl}^* = -a_{dl}^*$ and

$$\begin{aligned} K^* &= [K_e^*, K_1^{*T}, K_2^{*T}, K_r^*, K_3^*]^T, \\ w(t) &= [e, x_1^T, x_2^T, r, 1]^T, \\ K_m^* &= [K_{x_m}^{*T}, K_d^{*T}, K_z^{*T}]^T, \ K_{x_m}^* = c_m^T K_e^{*T} \\ w_m(t) &= [x_m^T(t), x_m^T(t - d(t)), z_m^T(t)]^T. \end{aligned}$$

IV. ADAPTIVE CONTROLLER DESIGN

For system with state delays and unknown parameters and actuator failures, the adaptive controller is designed in this section. Using the error equation (17) the controller structure

$$v_0(t) = K^T(t) w(t) - K_I \operatorname{sgn}(e(t)) \int_0^t |e(t)| dt, \quad (16)$$

is suggested, in which $K(t) = [K_e, K_1^T, K_2^T, K_r, K_3]^T$ is the estimate of the unknown parameter vector K^* and K_I is a positive constant scalar. This control law is composed of two terms. The first component $K^T(t) w(t)$ has the standard structure for output feedback control of systems with actuator failures [9]. The integral term $K_I \operatorname{sgn}(e(t)) \int_0^t |e(t)| dt$ is used to achieve robustness with respect to unknown plant delays and external disturbance [20].

Introducing the parameter error $\tilde{K}(t) = K(t) - K^*$ and using (16), the tracking error (15) can be expressed as

$$\begin{aligned} e(t) = &\frac{k_p}{D_m(s)}[\tilde{K}^T w(t) - K_I \operatorname{sgn}(e(t)) \int_0^t |e(t)| dt] \\ &- \frac{k_p}{D_m(s)}[K_m^{*T} w_m(t) + \sum_{l=1}^{M} K_{dl}^{*T} e_x(t - d_l(t)) \\ &+ \sum_{l=1}^{M} K_{zl}^{*T} z_{el}(t) + (1 - K_1^{*T} H(s)) F f(t)] \end{aligned} \quad (17)$$

As the usual design method of MRAC for systems without delay, the augmented state vector $\hat{x}=[x^T, x_1^T, x_2^T]^T$ is defined. Let $\hat{e}=\hat{x}-\hat{x}_m$ where $\hat{x}_m(t)$ is the state of a nonminimal realization $\hat{c}(sI-\hat{A})^{-1}\hat{b}K_r^*$ of $W_m(s)$. Then the state space representation

$$
\begin{aligned}
\dot{\hat{e}}(t) &= \hat{A}\hat{e}(t)+\hat{b}[\widetilde{K}^T(t)w(t)-K_I \operatorname{sgn}(e(t))\int_0^t |e(t)|dt] \\
&\quad -\hat{b}[K_m^{*T} w_m(t)+\sum_{l=1}^{M} K_{dl}^{*T} L^T \hat{e}(t-d_l(t)) \\
&\quad +\sum_{l=1}^{M} K_{zl}^{*T} C_e \hat{z}_{el}(t)+(1-K_1^{*T}H(s))Ff(t)], \\
\dot{\hat{z}}_{el}(t) &= A_e\hat{z}_{el}(t)+B_e L^T \hat{e}(t-d_l(t)), \\
z_{el}(t) &= C_e\hat{z}_{el}(t), \\
e(t) &= \hat{c}\hat{e}(t),
\end{aligned}
\tag{18}
$$

is obtained for (17), where $L=[I_{n\times n}, 0_{n\times(n-1)}, 0_{n\times(n-1)}]^T$ and the triple (A_e, B_e, C_e) is a minimal state space realization for the stable transfer matrix $H_n(s)$.

Because the relative degree is assumed to be one, $W_m(s)=\hat{c}(sI-\hat{A})^{-1}\hat{b}K_r^*$ is SPR. Therefore, according to MKY lemma [21], for any symmetric positive definite matrix $L_c = L_c^T > 0$, there exist a symmetric positive definite matrix $P=P^T>0$, a positive scalar $\upsilon>0$ and a vector q such that

$$
\begin{aligned}
\hat{A}^T P+P\hat{A} &= -qq^T-\upsilon L_c, \\
P\hat{b}K_r^* &= \hat{c}^T.
\end{aligned}
\tag{19}
$$

Since A_e is stable, there exist symmetric positive definite matrices $P_{zl}=P_{zl}^T>0$ and $Q_{zl}=Q_{zl}^T>0$ that satisfy

$$
A_e^T P_{zl}+P_{zl}A_e=-Q_{zl}, \quad l=1,\dots,M \tag{20}
$$

Now we are ready to state the following theorem.

Theorem 1: Consider the system (1) with actuator failures (3) and the reference model (5). Suppose that assumptions (A1) to (A4) hold. Then for any positive number γ and positive definite matrix $\Gamma=\Gamma^T>0$, the adaptive control (16) with coefficients

$$
\begin{aligned}
\dot{K}(t) &= -\operatorname{sgn}(\rho^*)\Gamma e(t)w(t), \\
K_I &= \gamma \operatorname{sgn}(\rho^*),
\end{aligned}
\tag{21}
$$

assures that all the closed-loop signals are bounded and the tracking error $e(t)$ converges to zero asymptotically.

Proof To prove this theorem, choose the Lyapunov-Krasovskii functional

$$
\begin{aligned}
V(t) &= \hat{e}^T(t)P\hat{e}(t)+\sum_{l=1}^{M} \hat{z}_{el}^T(t)P_{zl}\hat{z}_{el}(t) \\
&\quad +\sum_{l=1}^{M}\int_{t-d_l(t)}^{t}\hat{e}^T(s)\upsilon L_c\hat{e}(s)ds+\frac{|\rho^*|}{\gamma}(-\gamma\int_0^t|e(t)|dt+\eta^*)^2 \\
&\quad +(\widetilde{K}(t)-\hat{K}_1)^T\Gamma^{-1}(\widetilde{K}(t)-\hat{K}_1)|\rho^*|
\end{aligned}
\tag{22}
$$

in which $\gamma>0$ is a constant scalar and $\rho^*=K_r^{*-1}$. The vector $\hat{K}_1$ is defined as $\hat{K}_1=-\frac{r}{2}[\rho^*,0,\dots,0]^T$. The parameters $r>0$ and $\eta^*>0$ with arbitrary values will be defined later.

According to the update law (21) and using (19) and (20), the time derivative of $V(t)$ along (18) is

$$
\begin{aligned}
\dot{V}(t) &= -\hat{e}^T(t)qq^T\hat{e}(t)-r\hat{e}^T P\hat{b}\hat{b}^T P\hat{e} \\
&\quad -\sum_{l=1}^{M}\nu(1-\dot{d}_l(t))\hat{e}^T(t-d_l(t))L_c\hat{e}(t-d_l(t)) \\
&\quad -\sum_{l=1}^{M}\hat{z}_{el}^T(t)Q_{zl}\hat{z}_{el}(t)-2\sum_{l=1}^{M}\hat{e}^T P\hat{b}K_{dl}^{*T}L^T\hat{e}(t-d_l(t)) \\
&\quad -2\sum_{l=1}^{M}\hat{e}^T P\hat{b}K_{zl}^{*T}c_e\hat{z}_{el}+2\sum_{l=1}^{M}\hat{z}_{el}^T P_{zl}B_e L^T\hat{e}(t-d_l(t)) \\
&\quad -2\hat{e}^T P\hat{b}K_m^{*T}w_m-2\hat{e}^T P\hat{b}K_I \operatorname{sgn}(e(t))\int_0^t|e(t)|dt \\
&\quad -2|\rho^*||e(t)|(-\gamma\int_0^t|e(t)|dt+\eta^*)+2\hat{e}^T P\hat{b}(1-K_1^{*T}H(s))Ff(t)
\end{aligned}
\tag{23}
$$

for $t\in(T_i, T_{i+1})$, $i=0,1,\dots,m_0$.

Because $D_m(s)$ and $H_n(s)$ are stable and the reference input $r(t)$ is bounded, the reference signals $x_m(t)$, $x_m(t-d)$ and $z_m(t)$ are bounded. Therefore there exists a constant w_m^* such that $\|w_m(t)\|\le w_m^*$ and the following inequality can be written for the eighth term of (23).

$$
\begin{aligned}
&-2\hat{e}^T P\hat{b}K_m^{*T}w_m \le 2|\hat{e}^T P\hat{b}K_r^*||\rho^*|\|K_m^{*T}\|\|w_m\| \\
&\le 2|e(t)||\rho^*|\|K_m^{*T}\|w_m^*
\end{aligned}
\tag{24}
$$

By choosing $\eta_1^* = \left\|K_m^{*T}\right\| w_m^*$,

$$-2\hat{e}^T p\hat{b}K_m{}^{*T} w_m \le 2\eta_1^* \left|\rho^*\right| \left|e(t)\right|. \tag{25}$$

For the last term of (23) the following inequality can be written.

$$\begin{aligned} & 2\hat{e}^T P\hat{b}(1 - K_1^{*T} H(s))Ff(t) \\ & \le \left|e(t)\right| \left\|\rho^*\right\| \left\|2(1 - K_1^{*T} H(s))F\right\| f^* = 2\eta_2^* \left|\rho^*\right| \left|e(t)\right| \end{aligned} \tag{26}$$

According to the known inequality

$$\pm 2x^T y \le x^T Sx + y^T S^{-1} y$$

that is true for any vectors x, y and any positive definite matrix S, the following expressions can be written for the fifth, sixth and seventh terms

$$\begin{aligned} & -2\sum_{l=1}^{M} \hat{e}^T p\hat{b}K_{dl}{}^{*T} L^T \hat{e}(t - d_l(t)) \\ & \le \hat{e}^T (t)P\hat{b}\Phi_1 \hat{b}^T P\hat{e}(t) + \sum_{l=1}^{M} \hat{e}^T (t - d_l(t))S\hat{e}(t - d_l(t)), \\ & -2\sum_{l=1}^{M} \hat{e}^T p\hat{b}K_{zl}{}^{*T} c_e \hat{z}_{el} \\ & \quad \le \hat{e}^T (t)P\hat{b}\Phi_2 \hat{b}^T P\hat{e}(t) + \sum_{l=1}^{M} \hat{z}_e^T S\hat{z}_e, \\ & 2\sum_{l=1}^{M} \hat{z}_{el}^T P_{zl} B_e L^T \hat{e}(t - d_l(t)) \\ & \quad \le \sum_{l=1}^{M} \hat{z}_{el}^T \Phi_3 \hat{z}_{el} + \sum_{l=1}^{M} \hat{e}^T (t - d_l(t))S\hat{e}(t - d_l(t)), \end{aligned} \tag{27}$$

where

$$\begin{aligned} \Phi_1 &= \sum_{l=1}^{M} K_{dl}{}^{*T} L^T S^{-1} LK_{dl}^* \\ \Phi_2 &= \sum_{l=1}^{M} K_{zl}{}^{*T} c_e S^{-1} c_e^T K_{zl}^* \\ \Phi_3 &= P_{zl} B_e L^T S^{-1} LB_e^T P_{zl}^T. \end{aligned}$$

Using (25), (26) and (27), choosing the coefficient K_l from (21) and defining $\eta^* = \eta_1^* + \eta_2^*$, the inequality

$$\begin{aligned} \dot{V}(t) \le & -\hat{e}^T (t)qq^T \hat{e}(t) \\ & -\sum_{l=1}^{M} \hat{e}^T (t - d_l)(\nu d^* L_c - 2S)\hat{e}(t - d_l(t)) \\ & -\hat{e}^T P\hat{b}(r - \Phi_1 - \Phi_2)\hat{b}^T P\hat{e} \\ & -\sum_{l=1}^{M} \hat{z}_{el}^T (t)(Q_{zl} - \Phi_3 - S)\hat{z}_{el}(t) \end{aligned} \tag{28}$$

with $d^* = 1 - \bar{d}$ is obtained. By choosing the arbitrary values L_c, r and Q_{zl} as

$$\begin{aligned} & \lambda_{\min}(\nu L_c) > \lambda_{\max}(2S), \\ & r > \lambda_{\max}(\Phi_1 + \Phi_2), \\ & \lambda_{\min}(Q_{zl}) > \lambda_{\max}(\Phi_3 + S) \end{aligned}$$

we have $\dot{V}(t) \le 0$ for $t \in (T_i, T_{i+1})$, $i = 0,1,...,m_0$.

Since there are only a finite number of failures in system, $V(T_{m_0})$ is finite and from

$$\dot{V}(t) \le 0, \quad t \in (T_{m_0}, \infty) \tag{29}$$

we have $V(t) \in L_\infty$ and therefore $\hat{e}(t)$, $e(t)$, $\hat{z}_e(t)$, $K(t)$, $\tilde{K}(t) \in L_\infty$. Using directly the analysis method in [21], it can be proved that all the closed loop signals are bounded and $\lim_{t\to\infty} e(t) = 0$. ■

V. Simulation Example

Consider system (1) with parameters

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix},$$

$$c = \begin{bmatrix} 1 & 1 & 0.5 \end{bmatrix}, \quad b_f = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T$$

$$A_{d1} = \begin{bmatrix} 0.1 & -0.4 & 0.4 \\ 0.1 & -0.7 & 0.3 \\ 0 & 0 & 0 \end{bmatrix}, \; d_1(t) = 4 + 0.5\sin(t)$$

$$A_{d2} = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.4 & 0.1 & -0.4 \\ 0 & 0 & 0 \end{bmatrix}, d_2(t) = 6 + 0.4\sin(t)$$

$$x(0) = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix}^T$$

and the external disturbance $f(t) = 0.2\sin(0.5t) + 0.3$ and let the transfer function of the reference model be given by

$$W_m(s) = \frac{1}{s+1}.$$

All parameters of system and the delay value are assumed to be unknown to the controller. Simulation results are obtained with the adaptive control (16) with coefficients (21), reference input $r(t) = \sin(0.2t)$ and by choosing $\Gamma = 10I$, $\gamma = .001$ and $\Lambda(s) = (s+1)^2$, for one and two actuator failure cases, respectively. The results for the case of one actuator failure: $u_1(t) = .5,\ t \geq 50$, are shown in Fig. 1. In Fig. 2, the simulation results are shown for the case when two actuators fail: $u_1(t) = 1,\ t \geq 30,\quad u_2(t) = 0,\ t \geq 70$.

VI. CONCLUSION

A robust output-feedback adaptive controller is presented for state delayed systems with unknown parameters and unknown actuator failures. The controller structure is composed of two terms. The first term is defined for compensating actuator failures. The second integral term is used to achieve robustness against unknown delay values and an external disturbance with unknown bounds. The results of this paper can be extended to higher relative degrees using normalized MRAC scheme.

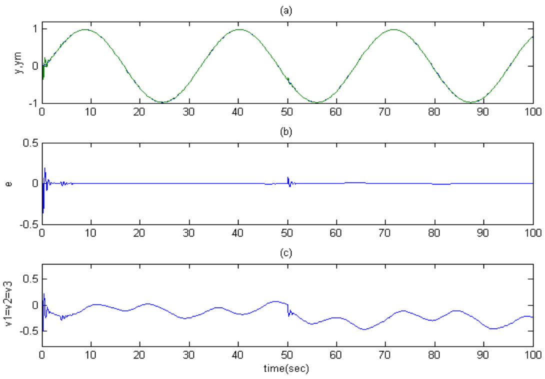

Figure 1. Simulation results for one actuator failure: (a) The plant and reference model outputs; (b) Tracking error; (c) control input.

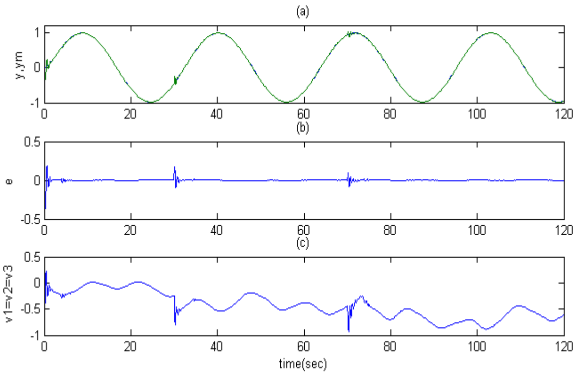

Figure 2. Simulation results for two actuator failures: (a) The plant and reference model outputs; (b) Tracking error; (c) control input.

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