UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

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Invariant Control of Non-Linear Elements in a Stacked High Redundancy Actuator

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***Abstract*—The High Redundancy Actuator (HRA) concept aims to provide a single actuator comprising many cooperating actuation elements. The potential benefits of this include improved overall reliability, availability, and reduced need for oversizing of actuators in safety critical applications. This paper deals with the question of distributing the load evenly between a stack of elements despite non-linear characteristics.**

The approach is to separate the state space into a high dimensional internal and a low dimensional invariant (or external) subspace. If the internal states can be decoupled and damped, the input-output behaviour only depends on the few states of the invariant subspace. In other words: the high redundancy actuator with many redundant elements behaves just like a conventional single actuator, and classical control strategies can then be applied.

This approach is demonstrated here for an HRA constructed from simple spring-damper-actuator elements. The non-linear behaviour is required to simulate the element behaviour at the end of the available travel. Without equalisation, excessive accelerations are caused by individual elements hitting the end stop, and this can be avoided by applying the proposed element tuning.

***Index Terms*—electromagnetic actuation, fault tolerance, multi-variable control, non-linear control**

I. High Redundancy Actuation

High Redundancy Actuation (HRA) is a novel approach for designing a fault tolerant actuator that comprises a relatively large number of actuation elements (see Figure 1). As a result, faults in the individual elements can be inherently accommodated without resulting in a failure of the complete actuator system.[1]

The concept of the High Redundancy Actuation (HRA) is inspired by the human musculature. A muscle is composed of many individual muscle cells, each of which provides only a minute contribution to the force and the travel of the muscle. These properties allow the muscle as a whole to be highly resilient to damage of individual cells. The aim of this project is not to replicate muscles, but to use the same principle of co-operation with existing technology to provide intrinsic fault tolerance.

[1]This project is a cooperation of the Control Systems group at Loughborough University, the Systems Engineering and Innovation Centre (SEIC), and the actuator supplier SMAC Europe limited. The project was funded by the Engineering and Physical Sciences Research Council (EPSRC) of the UK under reference EP/D078350/1.

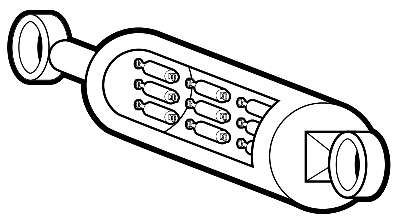

Figure 1. High Redundancy Actuator

An important feature of the High Redundancy Actuator is that the elements are connected both in parallel and in series. While the parallel arrangement is commonly used, the serial configuration is rarely employed, because it is perceived to be less efficient. However, the use of elements in series is the only configuration that can deal with the lock-up of an element. In a parallel configuration, this would immediately render all elements useless, but in the series configuration it only leads to a slight reduction of available travel (see Steffen et al. 2007, 2008a for details).

The paper is organised as follows: it starts with the motivation and background in Section 2, followed by the non-linear model of an actuator stack consisting of elements with a non-linear characteristic in Section 3. The control goal is presented in Section 4, followed by the solution in Section 5. Sections 6 presents a simulation example, and Section 7 proposes an extension for more complex HRA configurations.

II. Motivation

While the parallel configuration of actuation elements is well established and researched, the dynamic behaviour of elements in series is very different. The

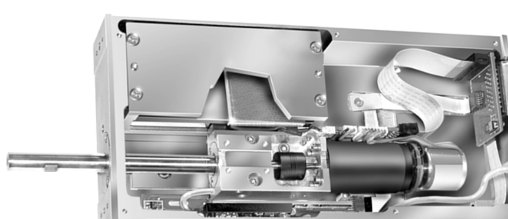

Figure 2. Electromechanical actuator

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reason is that between each element in series, there is a moving mass. This creates a high number of mechanical degrees of freedom, in turn leading to a high order dynamic model. The model needs to describe the position and speed of each mass separately, and there may also be further states internal to each element.

For the envisioned number of elements (10x10 or more), this may lead to a model with hundreds of states. Dealing with this complexity constructively is crucial for the success of the high redundancy actuator, because the standard approach of using sophisticated instrumentation may not be suitable. The goal of this paper is to reduce the model and instrumentation complexity to a level comparable to a conventional actuator.

A. Approach

The basic idea is to split the travel equally between all actuation elements. If this is achieved, the states of the elements are no longer individual variables, and they can all be reduced into a single simple model. In other words: because the whole system behaves like a single conventional actuator, a simple conventional actuator model is sufficient to describe it.

This approach is not trivial, because the elements experience different effective loads. The element at the bottom of the assembly for example experiences a higher load, because it needs to move all the other elements in addition to the load. This paper will present a number of ways to address this problem using active and passive, feedforward and feedback approaches. The relative advantages and disadvantages are discussed, and two approaches are considered in more detail. Based on this set of options, a specific solution or combination of solutions can be selected as appropriate according to the practical requirements.

B. Literature Overview

High Redundancy Actuation is a novel approach to fault tolerance, and consequently the specific problem formulated in this paper has not been previously considered.

Previous work on High Redundancy Actuation did look into the possibility of aligning dynamics using different methods (see Steffen et al., 2008b, 2010), but without using the geometric approach. This leads to results that are much less general than the approach presented here.

A similar approach is used in rotary actuation with torque summing and velocity summing gears, see for example Ting et al. [1994], Bennett et al. [2004]. The approach has a number of characteristic differences to the systems studied in this paper: it only works for rotary motion, and the problem of complexity is much less prominent because, due to the symmetrical structure, all elements act and behave in exactly the same way. Thus the parameter tuning described in this paper is not applicable.

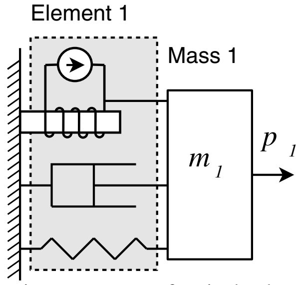

Figure 3. Dynamic components of a single element

A related problem has been studied in the dynamic behaviour of axial stacks of piezoelectric actuators by Jalili [2009]. The author models the stack as a distributed system with partial differential equations (compared to the lumped elements in this paper), which leads to similar results concerning the internal mode. However, the author makes no apttempt to control or decouple these modes internally.

The basis for decoupling internal modes is the geometric approach, because it creates a connection between the dynamics of the system and constraints formulated in terms of the states of the system. This approach was introduced by Wonham [1985] and later extended by Basile and Marro [1992]. It provides the standard solution for the disturbance decoupling problem (see Commault et al., 1997), which is the class of control problems at the heart of this paper. The geometric approach has previously been used for adaptive control of a High Redundancy Actuator in Steffen et al. [2009].

C. Symbols

diag{}	diagonal matrix
f_j	force produced by element j
g_j	strength factor for element j
F_i	force total for mass m_i
f	characteristic element function (non-linear model)
$k_d\, k_r$	damping and force constant (linear model)
m_i	mass of moving mass number i
$n_i\ n_j$	number of masses and elements
$\mathbf{Q}$	the connection matrix $\in \{-1, 0, -1\}^{j \times i}$
$\mathbb{R}$	set of real numbers
t	time
u_j	input of element j
$x_i\ \dot{x}_i\ \ddot{x}_i$	position, speed and acceleration of mass m_i

III. System Model

The basic components of an electromechanical actuation element are shown in Figure 3. From a modelling perspective, it is a typical actuated spring-mass-damper system, which can be described by NEWTONian mechanics. Three forces act upon the mass: the electromagnetic force F_{el}, the damping force F_d, and the

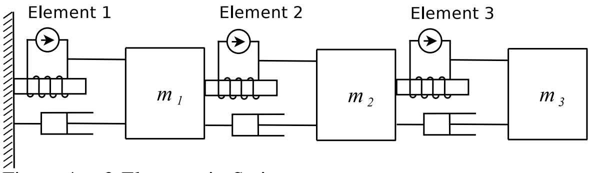

Figure 4. 3 Elements in Series

spring force $F_s = rx$ (see Davies et al. 2008 for more details). Damping and spring for are often assumed to be linear, but in reality that is rarely the case, and there are always limits to the linear region which may be relevant. Therefore, a nonlinear model is used here, using a characteristic function $f(x,\dot{x})$ to describe the behaviour of the spring and the damper:

$$m\ddot{x} = f(x,\dot{x},u) \quad .$$

Choosing x and $\dot{x}$ as states leads to a full state space model:

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x},\mathbf{u})$$

with

$$\mathbf{x} = \begin{pmatrix} \dot{x} \\ x \end{pmatrix}$$

$$\mathbf{f}(\mathbf{x},\mathbf{u}) = \begin{pmatrix} f(x,\dot{x},u) \\ \dot{x} \end{pmatrix}$$

$$\mathbf{u} = \begin{pmatrix} u \end{pmatrix} \quad .$$

Once several actuator elements are stacked, it is worth distinguishing between the mass (inertia) and the connection part of an element (generating the there forces). The connection part is attached to the element (or ground) below, so each element is subject to forces from both sides (except for the top one, as shown in Figure 4). The resulting model for actuation elements in series is:

$$\text{diag}\{\mathbf{m}\}\ddot{\mathbf{x}} \;= -\mathbf{Q}f(\mathbf{Q}^T\mathbf{x},\mathbf{Q}^T\dot{\mathbf{x}},\mathbf{u}) \qquad (1)$$

where

$$\mathbf{Q} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

is the connection matrix defining the connections between the actuators and the masses. The notation $f(\mathbf{Q}^T\mathbf{x},\mathbf{Q}^T\dot{\mathbf{x}})$ indicates that the same function f is applied element wise to the elements of $\mathbf{Q}^T\mathbf{x}$ and the derivative. More complicate configuration including parallel configurations can be modelled by supplying the corresponding connection matrix $\mathbf{Q} \in \mathbb{R}^{n_j \times n_i}$, but only stacks will be considered here with $n_i = n_j = n$, and $\mathbf{Q}$ a band diagonal matrix of the same structure as above. The time variables $\mathbf{u}$, $\mathbf{x}$, and $\mathbf{m}$ are vectors describing the input and the position of the elements, while the parameter vector $\mathbf{m}$ specifies the mass of the elements. This model is in state space form, and the state variable consists of the velocity and the position components

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{pmatrix} \in \mathbb{R}^{2n} \quad .$$

Internal states of the elements (e.g. current in the coil) can be modelled in the same way if necessary.

The scalar equations behind this vector equation are

$$m_i\ddot{x}_i = -\mathbf{q}_i\mathbf{f}$$

and

$$f_j = f(\mathbf{q}_j^T\mathbf{x},\mathbf{q}_j^T\dot{\mathbf{x}},\mathbf{u})$$

where $\mathbf{q}_i$ is the ith row of $\mathbf{Q}$, $\mathbf{q}_j^T$ is the transpose of the jth column of $\mathbf{Q}$, and $\mathbf{f}$ is a column vector of all f_j.

In fact one further term is required to solve this problem, because using equal elements will not generally be sufficient. Because the elements close to the ground have a higher load, they need to be stronger, and this is modelling using a strength factor $\mathbf{l}$. The characteristic function is scaled with this factor:

$$f_j = g_j f(\mathbf{q}_j^T\mathbf{x},\mathbf{q}_j^T\dot{\mathbf{x}})$$

or

$$\text{diag}\{\mathbf{m}\}\ddot{\mathbf{x}} \;= -\mathbf{Q}\text{diag}\{\mathbf{g}\}f(\mathbf{Q}^T\mathbf{x},\mathbf{Q}^T\dot{\mathbf{x}},\mathbf{u}) \qquad (2)$$

IV. Alignment Goal

One of the goals of high redundancy actuation is that all elements work together to perform the actuation task. This requires that they move in a coordinated and generally synchronous motion. This is called the alignment goal.

The key obstacle that the elements are not equal, because they at different positions along the stack and therefore subject to different accelerations. The naive approach of applying the same input signal to all elements does not achieve the alignment of motion. Instead, the force created by two elements cancels out for all masses except the load mass. The top element (x_3) responds first, then the middle element (x_2) begins to move, and finally the element on the base (x_1) will respond. So the input responses ripples through the system like a longitudinal wave. This phenomenon is already known from stacks of piezoelectric actuators.

An example is shown in Figure 5 for a nominal system with $f(x,\dot{x},u) = u - 2\dot{x} - 0.5x$, $m_1 = m_2 = 0.5$ and $m_3 = 1$. A simple single input/single output (SISO) proportional controller with a phase lead compensator is used $K(s) = 2\frac{0.4s+1}{4s+1}$, and a reference step of 30 mm (10 mm per element) is used in this simulation.

This kind of wave propagation complicates the control of the actuator, because it leads to situations where the elements work against each other, and not in cooperation. It can also cause elements to run into mechanical limits, which may create further vibrations and reduce the lifetime of the system significantly.

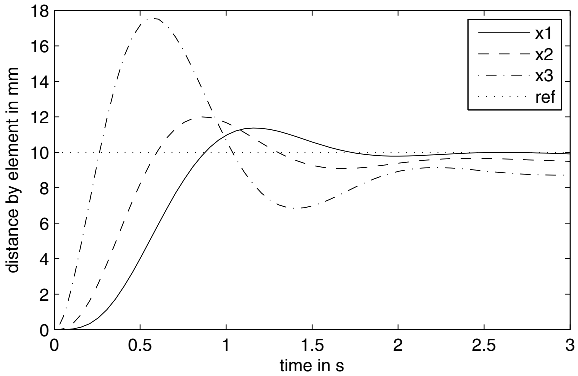

Figure 5. Delay between elements

To avoid this, the alignment goal is introduced. It states that all elements should follow a synchronous motion. Let $\Delta x_i = x_i - x_{i-1}$ denote the extensions of element number i, then the alignment goal is

$$\Delta x_1 = \Delta x_2 = \ldots = \Delta x_{n_j} \tag{3}$$

In the case of stacked elements, this can be reformulated as follows.

Definition 1. The alignment goal is to distribute the extension of the stack equally between all elements

$$\forall i : n\Delta x_i = x_n \quad . \tag{4}$$

Error signals for this goal can be defined either on the extension

$$e_i = n\Delta x_i - x_n$$

or on the position

$$e_i = x_i - \frac{i}{n} x_n \quad .$$

The following section will present a number of solutions to achieve this goal.

V. Solution

There are several approaches that can be used either individually or in combination to address the alignment goal. For an overview see [to be published], it lists a number of active and passive approaches. This paper focuses on element tuning with the goal of creating an invariant subspace. Only the strength of the elements is changed here, while the characteristics are assumed to be identical across all elements.

The feedforward approach presented here is not about controlling the deviations, but making sure that they do not occur in the first place. So the goal becomes to equalise the derivatives:

$$\Delta \ddot{x}_1 = \Delta \ddot{x}_2 = \ldots = \Delta \ddot{x}_{n_j} \tag{5}$$

assuming that all Δx_j and $\Delta \dot{x}_j$ are equal. This is the basic idea of invariant control: once the invariant condition is satisfied, invariant control can ensure that it remains valid.

In a stack, the travel of an element Δx_j can be calculated as $\mathbf{q}_j^T \mathbf{x}$. Inserting the model equation results in

$$\Delta x_j = -\mathbf{q}_j^T \operatorname{diag}^{-1}\{\mathbf{m}\}\mathbf{Q}\operatorname{diag}\{\mathbf{g}\}\mathbf{f}$$

with

$$f_j = f(\mathbf{q}_j^T \mathbf{x}, \mathbf{q}_j^T \dot{\mathbf{x}}, \mathbf{u}) \quad .$$

Because of the assumption, all f_j are equal, so the goal can be achieved if all elements of

$$\mathbf{Q}^T \operatorname{diag}^{-1}\{\mathbf{m}\}\mathbf{Q}_i\mathbf{g}$$

are equal, or

$$\mathbf{g} = \left(\mathbf{Q}^T\mathbf{Q}\right)^{-1} (1\ 1\ \cdots\ 1)^T \quad .$$

Since $\mathbf{Q}$ is invertible for the stack configuration considered here, this can be rewritten as:

$$\mathbf{g} = \mathbf{Q}^{-1}\mathbf{diag}\{\mathbf{m}\}\mathbf{Q}^{-1,T}(1\ 1\ \cdots\ 1)^T \quad .$$

With

$$\mathbf{Q}^{-1} = \begin{pmatrix} 1 & & \\ \vdots & 1 & \\ 1 & \cdots & 1 \end{pmatrix}$$

this can be rewritten as

$$g_j = \sum_{k=j}^{n} k m_k \quad .$$

In the example with three elements, this means

$$\begin{aligned} g_1 &= m_1 + 2m_2 + 3m_3 \\ g_2 &= 2m_2 + 3m_3 \\ g_3 &= 3m_3 \quad . \end{aligned}$$

The result of this tuning is shown in Figure 6. The parameters of the third element are equal to the step response in Figure **??**, and the other elements are tuned accordingly. Clearly the delay between the elements has been eliminated, and they all respond at the same time. The disturbance response (at $t = 5$) still deviates slightly between the elements, but the difference is small and not significant for most practical purposes.

VI. Simulation Results

As an example, a stack of three elements is simulated. The non-linear characteristic of each element is used to include a linear damping term, a linear spring term, and a non-linear spring term representing end stops:

$$f(\dot{x}, x, u) = u - 2\dot{x} - 0.1x - 10000h(x)$$

where

$$h(x) = \begin{cases} x & \text{if } x < 0 \\ 0 & \text{if } x \in [0, 0.35] \\ x - 0.35 & \text{if } x > 0.35 \end{cases} \quad .$$

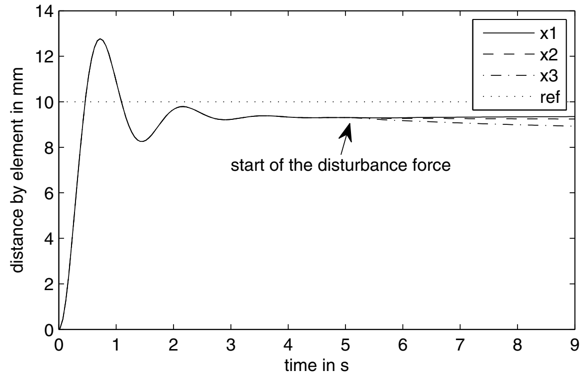

Figure 6. Step response after parameter tuning

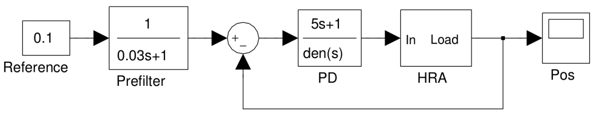

Figure 7. Simulation Control Structure

A simple PD type controller with a slight prefilter is used to control the stack as shown in Figure 7. This may not be the controller of choice for practical applications (for example because it does not provide reference tracking), but the simplicity makes the analysis of the system behaviour easier. With equal elements (without tuning), the simulation in Figure 8 shows a strong chattering effect caused by the end stops becoming active at an extension of $\Delta x = 0.35$. The state trace in Figure 9 also demonstrates that the travel is by no means equally distributed between the elements. This kind of chatter has also been observed in an experimental demonstrator of the HRA. The forces caused by the end stops and acting on the individual masses can be significant, so much so that they destroy the assembly.

Adjusting the strength of the three elements according to the methodology shown above changes the situation dramatically. The three weights are $m_1 = m_2 = 0.1\,\text{kg}$ and $m_3 = 1\,\text{kg}$, therefore the strength coefficients are

$$\begin{aligned} g_1 &= 3.3 \\ g_2 &= 3.2 \\ g_3 &= 3 \quad . \end{aligned}$$

In the implementation, these are scaled by g_3 such that g_3 becomes equal to 1. As can be seen in Figure 10, all elements of the adjusted HRA move in synchronose motion. Because the overall travel is well within the reach of the HRA, the end stops do not become relevant.

VII. Generalised Configurations

This approach is not limited to stacks of actuators - it can be applied to any network of actuators. For

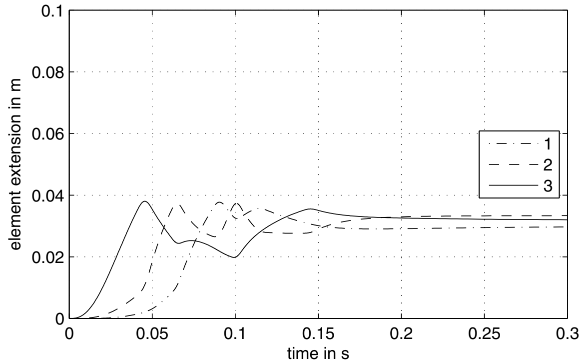

Figure 8. Simulation Results, unmatched

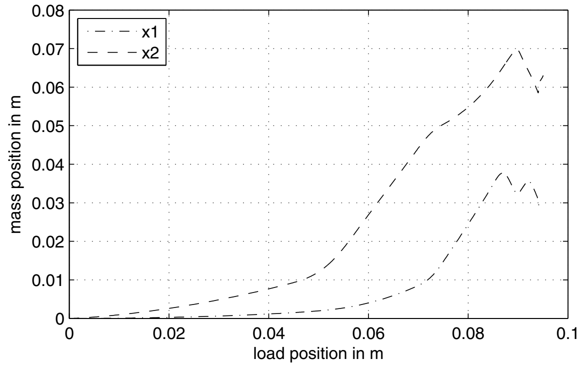

Figure 9. State trace, unmatched

example the system in Figure 12 has a connection matrix of the form

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad .$$

The equalisation can be calculated in the same way, but since $\mathbf{Q}$ is not square, the left side inverse is not unique. This agrees with the intuitive interpretation: both stacks can have the same or different strength, and the alignment goal can be achieved in either case. If the pseudo-inverse is used, they will both have the

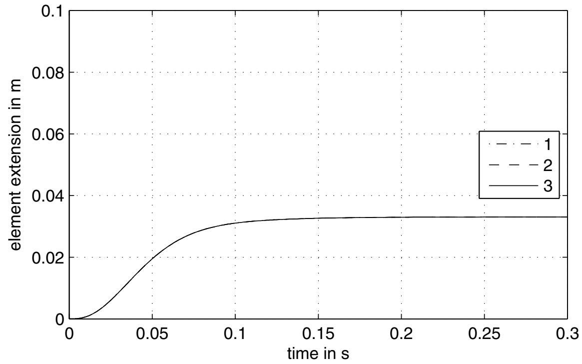

Figure 10. Simulation Results, matched

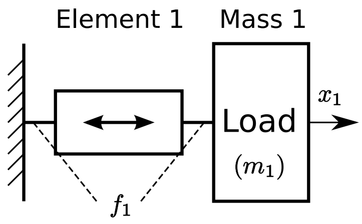

Figure 11. Abstracted Model of a Single Element

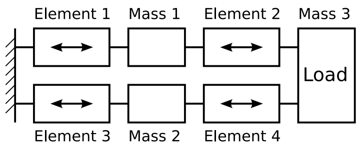

Figure 12. Example 2×2 Configuration

same strength:

$$\mathbf{Q}^* = \frac{1}{4}\begin{pmatrix} 3 & 1 & 2 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix}$$

and

$$\mathbf{g} = \lambda (m_{1,2} + m_3 \;\; m_{1,2} + m_3 \;\; m_3 \;\; m_3)^T \quad .$$

VIII. Conclusion

Starting from a model of an actuator stack, it is shown how a desired equal distribution of the speed and travel between the elements can be formulated as an invariance condition called alignment goal. The invariance can be achieved by tuning the strength of the actuation elements according to their position in the stack. While this approach is obvious within a linear framework, it is equally applicable to elements with a non-linear characteristic, as long as the characteristic is the same for all elements.

Once the alignment goal is achieved, all elements move synchronously "as one", and the stack of actuators behaves exactly the same way as a comparable single (larger) actuator. Further modes still exist in the system, but they are stable and decoupled from the inputs. Therefore it is possible to use a simple PID class controller to control an HRA.

The presented methods can be extended to more complex configurations by relaxing some of the assumptions. A simple 2x2 configuration is demonstrated, but it also covers much more extensive configuration.

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