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A novel decoupling control method for multivariable systems with disturbances

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***Abstract*—This paper presents a novel decoupling method for multivariable systems with disturbances. In this method, the undesirable coupling parts in each loop are treated as the output disturbances. These disturbances, as well as the external disturbances, can be actively rejected by the equivalent-input-disturbance (EID) approach. The parameters of the controller in each loop can be designed independent of each other. A typical example demonstrates the simplicity in parameters design and good performance in decoupling control and disturbance rejection.**

***Index Terms*—Decoupling control, linear systems, disturbance rejection, state observer, equivalent-input-disturbance, pole assignment**

I. Introduction

Multiple-inputs and multiple-outputs (MIMO) processes are often encountered in industry, such as chemical reactor, continuous stirred tank reactor, *et. al*. Because of the interaction between each control loop, the controller design for MIMO system becomes a complex problem and the common single-input single-output (SISO) control theories are not effective for MIMO systems. However, the practical control engineers are always accustomed to deal with the MIMO system loop by loop, which requires the common SISO control methods, such as PID control, optimal control. To solve such a problem, one of the control strategies is trying to eliminate the interactions between control loops, which is known as decoupling control.

The decoupling control theory has been well developed and established for several decades. To measure the interaction of the MIMO systems, Bristol [1] introduced relative gain array method for the input-output pairings and controller design. For linear systems, an effective approach to the internal model design [2] is proposed for decoupling stable square multivariable process with delays. A transfer function matrix decoupling approach is presented for the MIMO Smith Scheme [3]. Although many significant results have been proposed, the decoupling performance is still desired to be improved and some key problems, such as robustness, disturbance rejection, and other practical concerns [4] continue to pose serious challenges.

It is well known that the disturbance is of a primary concern for control system design, and is even the ultimate objective. This problem, in conjunction with decoupling control, has attracted many researchers. Feedback control [5] may be effective for reducing the disturbance and making disturbance rejection tuning independent of the controller design, if the disturbance is measurable. Due to the difficulties in obtaining exact model or disturbance, decoupling control schemes are often constructed to estimate the cross-couplings and disturbance at the same time; and a number of observer-based methods are making progress. Such as the disturbance observer based method (DOB) [6], the perturbation observer approach (POB) [7], disturbance decoupling control [8] based on the active disturbance rejection control (ADRC) [9] and so on. However, DOB requires the inverse dynamics model and POB needs the accurate model of the plant. The ADRC based decoupling method overcomes some drawbacks of the existing methods. However, the parameters in the resulting controller are usually very complicated.

In an MIMO system, the output of one loop is always the sum of all inputs actions. The actions of other loops are the resource of the interactions and bring bad effect on the stability and performance, which is just like output disturbances occurring in this loop. Intuitionally, we may think that if such "output disturbances" can be fully overcame or compensated, the output of one loop is only determined by the relevant input, and then the MIMO system can be well decoupled. One of the novelties of this paper is that, we treat the undesirable coupling parts of each loop as its output disturbances, and compensate them together with the external disturbance. The equivalent-input-disturbance (EID) approach [10], [11] is a relatively new disturbance rejection method. It can reject more than one disturbance simultaneously and compensate any kind of disturbance effectively without knowing prior information. These important features motivated us to design the decoupling controller using the EID-based method.

This paper first constructs a configuration of the decoupling control based on the EID approach. Then, the controller parameters for each loop are designed independently. Finally, simulation results are shown for validating the proposed method.

II. Configuration of the Dynamic Decoupling Control

Consider a linear time-invariant MIMO system. Let r_i and u_i, $i = 1, 2, \cdots, n$ denote the reference inputs and the control inputs, respectively. Suppose that the open loop transfer function matrix of the process is described by

$$G(s) = [g_{ij}(s)]_{n\times n} \tag{1}$$

Then for the *i*-th loop, the output is given by

$$y_i = \sum_{j=1}^{n} g_{ij}(s)u_j, \quad i = 1, 2, \cdots, n \tag{2}$$

We divide it into two parts:

$$y_i = g_{ii}(s)u_i + \sum_{j=1, j\neq i}^{n} g_{ij}(s)u_j \tag{3}$$

where $g_{ii}(s)$ is regarded as the transfer function of the plant in the *i*-th loop, the second term in the right hand side is treated as a disturbance signal. So that it becomes an SISO system with disturbances for each loop.

For clarity, we take a two-input two-output (TITO) system as an example, as shown in Fig.1. For the plant g_{11}, we only need to design a controller c_1 to reject the signal through g_{12}, as well as the disturbance d_1. Similarly, one designs controller c_2. Due to this process, we can also notice that only the diagonal elements of the transfer function are needed in the compensation instead of the whole information.

For convenience, the processes are formulated in state-space forms. Then the plant in the *i*-th loop is given by

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D_i d_i(t) \\ y_i(t) = C_i x_i(t) + \Phi_i \varphi_i(t) \end{cases} \tag{4}$$

where A_i, B_i and C_i are matrices obtained by the transfer function $g_{ii}(s)$, $i = 1, 2, \cdots, n$, $\Phi_i \varphi_i(t)$ denotes the signal $\sum\limits_{j=1, j\neq i}^{n} g_{ij}(s)u_j$, which need not to be known exactly, and $D_i d_i(t)$ is the external disturbance.

Now, a transformation is required. For the output $y_i(t)$, there must exist a matrix Ψ_i such that

$$y_i(t) = C_i[x_i(t) + \Psi_i \varphi_i(t)] \tag{5}$$

Let

$$\tilde{x}_i(t) = x_i(t) + \Psi_i \varphi_i(t) \tag{6}$$

Combing (5) and (6), the state equation (4) is derived as

$$\begin{cases} \dot{\tilde{x}}_i(t) = A_i \tilde{x}_i(t) + B_i u_i(t) + [D_i d_i(t) - A_i \Psi_i \varphi_i(t)] \\ y_i(t) = C_i \tilde{x}_i(t) \end{cases} \tag{7}$$

where $D_i d_i(t) - A_i \Psi_i \varphi_i(t)$ is regarded as whole disturbances imposed on this loop.

An EID of the plant is defined to be a signal on the control input channel that produces the same effect on the output as disturbances do for all $t \geq 0$. The EID of the disturbance must exist under the condition that the plant does not have any zeros

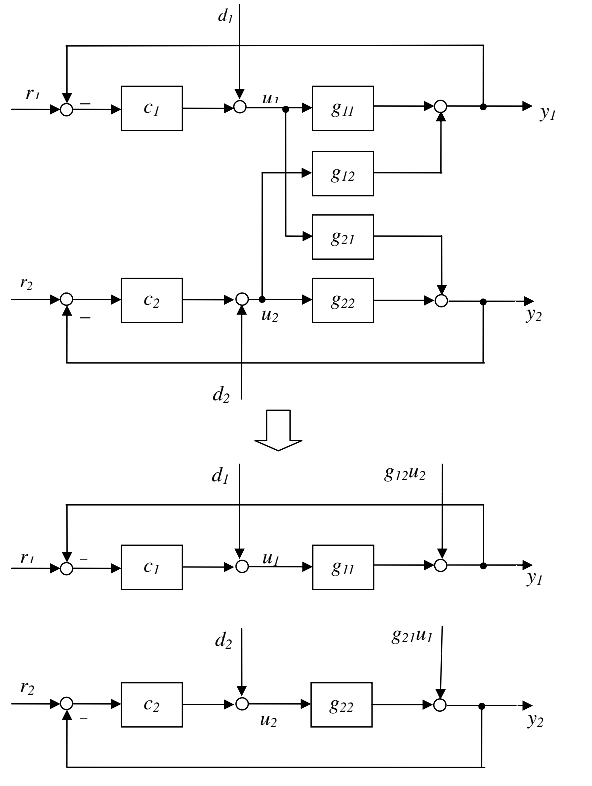

Fig. 1. Configuration of TITO decoupling control system

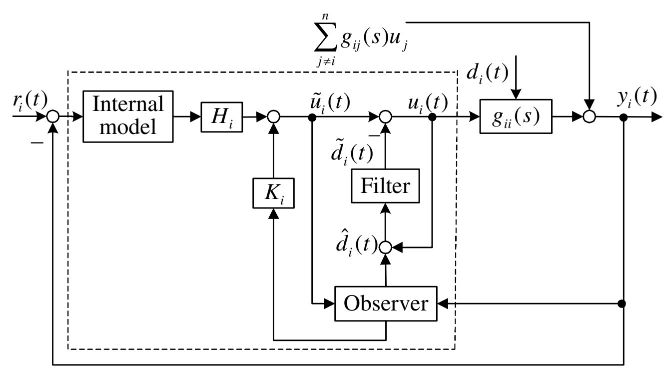

Fig. 2. Configuration of the EID method

on the imaginary axis [10]. Suppose that a disturbance $de_i(t)$ is imposed on the control input channel of the *i*-th plant, then (7) can be described as the following equivalent model

$$\begin{cases} \dot{\tilde{x}}_i(t) = A_i \tilde{x}_i(t) + B_i[u_i(t) + de_i(t)] \\ y_i(t) = C_i \tilde{x}_i(t) \end{cases} \tag{8}$$

Note that the same variable $\tilde{x}_i$ is used for both the state of plant (7) and (8). This should not make confusion.

The configuration of the EID-based control system is shown in Fig. 2. The controller c_i in Fig. 1 is just designed using the parts within the dashed line in Fig. 2. In this figure, the internal model

$$\dot{\bar{x}}_i(t) = \bar{A}_i \bar{x}_i(t) + \bar{B}_i[r_i(t) - y_i(t)] \tag{9}$$

is employed to improve the tracking precision, in which the parameters are determined by the reference input $r_i(t)$. The Luenberger observer

$$\begin{cases} \dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i\tilde{u}_i(t) + L_i(y_i(t) - \hat{y}_i(t)) \\ \hat{y}_i(t) = C_i\hat{x}_i(t) \end{cases} \quad (10)$$

plays a key role in the disturbance estimation.

Let

$$\Delta x_i(t) = \tilde{x}_i(t) - \hat{x}_i(t). \quad (11)$$

[10] gives us the expression of the estimation value

$$\hat{d}_i(t) = B_i^+ L_i C_i \Delta x_i(t) + u_i(t) - \tilde{u}_i(t) \quad (12)$$

where $B_i^+ = (B_i^T B_i)^{-1} B_i^T$.

Then, a low-pass filter is used to select the frequency band of the signal, *e.i.*,

$$\mathscr{L}[\tilde{d}_i(t)] = F(s)\mathscr{L}[\hat{d}_i(t)] \quad (13)$$

where $\mathscr{L}(\cdot)$ is the Laplace transform and $F_i(s)$ satisfies

$$|F_i(j\omega)| \approx 1 \quad (14)$$

for all $\omega \in \Omega$, where Ω is the chosen frequency band. Note that each loop becomes an SISO system in the design, a first-order filter

$$F_i(s) = 1/(T_i s + 1) \quad (15)$$

can work well.

Thus, $\tilde{d}_i(t)$ is just the estimated compensation of the whole disturbances in the *i*-th loop. So that the control law is given by

$$u_i(t) = \tilde{u}_i(t) - \tilde{d}_i(t) \quad (16)$$

where the state feedback control law is

$$\tilde{u}_i(t) = H_i\bar{x}_i(t) + K_i\hat{x}_i(t) \quad (17)$$

III. Stability analysis and parameters design

Since the plant we considered in each loop may be influenced by any of the reference input, we set

$$r_i(t) = 0, i = 1, 2, \cdots, n,\ d_i(t) = 0 \quad (18)$$

for the *i*-th loop to guarantee that all the external inputs be zero.

Then combining (4), (10), (11) and (18) yields

$$\Delta\dot{x}_i(t) = -(A_i - L_iC_i)\Delta x_i(t) + B_i\tilde{d}_i(t) \quad (19)$$

and

$$\hat{d}_i(t) = B_i^+ L_i C_i \Delta x_i(t) + \tilde{d}_i(t) \quad (20)$$

So the transfer function from $\tilde{d}_i(t)$ to $\hat{d}_i(t)$ can be derived as

$$G_i(s) = B_i^+(sI - A_i)[sI - (A_i - L_iC_i)]^{-1}B_i \quad (21)$$

In the EID-based control system, the state feedback design does not influence the stability of the whole system. Therefore, we have the following theorem.

Theorem 1: [10] For suitably designed H_i and K_i, the control law (16) guarantees the stability of the *i*-th loop, if $A_i - L_iC_i$ is stable and

$$\|G_i(s)F_i(s)\|_\infty < 1 \quad (22)$$

where $||\cdot||_\infty$ denotes the upper bound of the maximum singular value of the function.

Since the plant in each loop is controllable and observable, the state feedback gains and the observer can be designed independently [14]. This brings us great convenience for parameters design.

As for the state feedback gains, the following augmented system including the original plant and the internal model is considered

$$\delta\dot{x}_i(t) = \tilde{A}_i\delta x_i(t) + \tilde{B}_i u_i(t) \quad (23)$$

where

$$\delta x_i(t) = \begin{bmatrix} \tilde{x}_i(t) \\ \bar{x}_i(t) \end{bmatrix},\ \tilde{A}_i = \begin{bmatrix} A_i & 0 \\ -\bar{B}_iC_i & \bar{A}_i \end{bmatrix},\ \tilde{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix} \quad (24)$$

H_i and K_i can be optimized by using a well-known linear quadratic regulation (LQR) method [15]. The optimal state-feedback control law is given by

$$u_i^*(t) = -R_i^{-1}\tilde{B}_i^T P_i \delta x_i(t) \quad (25)$$

where $P_i = \begin{bmatrix} K_i & H_i \end{bmatrix}$ is a solution of the Riccati equation

$$P_i\tilde{A}_i + \tilde{A}_i^T P_i + Q_i - P_i\tilde{B}_iR_i^{-1}\tilde{B}_i^T P_i = 0 \quad (26)$$

$Q_i > 0$ and $R_i > 0$ are diagonal matrices to be determined by the coefficient matrices of the augmented system.

The Luenberger observer has an important function that enables the coupling parts and disturbances to be compensated for. It will achieve good performance and be easily operated for a well designed gain L_i. We use the well-known pole-placement theory in this study instead of the perfect regulation method in [10], no matter if the plant is stable. First, we need to select a time parameter T_i that satisfies (14) for the low-pass filter. Then L_i can be obtained by the following procedure.

Observer gain design algorithm

Step 1: [14] For a prescribed ζ, $0 \le \zeta \le 1$, the expected poles for the observer is chosen to be

$$\lambda_{1,2} = -\zeta\sigma \pm j\sqrt{1-\zeta^2}\sigma,$$
$$\lambda_k = -a_k\zeta\sigma,\ a_k \ge 5,\ k = 3, \cdots, m \quad (27)$$

Step 2: Choose σ in (27) and calculate the transfer function $G_i(s)$ in (21), such that (22) holds.

Step 3: Calculate the observer gain L_i such that

$$\lambda_k(A_i - L_iC_i) = \lambda_k,\ k = 1, \cdots, m \quad (28)$$

Remark 1: Only if the plant is observable, the expected poles of the observer can be arbitrarily placed by the feedback matrix L_i. It is well known that the performance of the observer is determined by the dominant poles $\lambda_{1,2}$. In addition, the following part proves that there must exist a large enough σ such that (22) holds.

It follows from (21) that

$$1/\|G_i(s)\|_\infty = \left\|\tilde{G}_i(s)\right\|_\infty \quad (29)$$

where

$$\tilde{G}_i(s) = B_i^+[sI - (A_i - L_iC_i)](sI - A_i)^{-1}B_i$$

Since $\lim_{\sigma\to\infty} |\lambda_i| = \infty$ $(i = 1, \cdots, m)$, from (28), we have $\lim_{\sigma\to\infty} \|A_i - L_iC_i\|_\infty = \infty$. For any fixed ω, the matrices $j\omega I$ and $(j\omega I - A_i)^{-1}$ are constant matrices with finite norms, respectively. So,

$$\lim_{\sigma\to\infty} \left\|\tilde{G}_i\right\|_\infty = \infty. \quad (30)$$

Then, for a large enough σ, (22) holds. In view of the modeling uncertainties, σ can be chosen a little large to handle the uncertainties and guarantee the stability of the system.

Remark 2: In the design of the proposed method, the coupling parts in each loop are treated as disturbances. In this case, even if strong multivariable interactions occur, the stability of the system will not be destroyed.

The controllers are designed loop by loop based on the analysis above. Although every loop needs a controller as discussed above, the parameters can be designed similarly among all loops and be independent of each other.

IV. Numerical Example

The Wood-Berry model of a pilot-scale distillation column [16] has been studied extensively. This section considers this multivariable system with delay set to zero, which is shown as

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{-18.9}{21s+1} \\ \frac{6.6}{10.9s+1} & \frac{-19.4}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \quad (31)$$

Let the reference inputs be $r_1 = 1$ and $r_2 = 1$, which were imposed at $t = 0$ s and $t = 20$ s, respectively. Then a disturbance $d_1(t) = 0.4$ was imposed on the first plant at $t = 40$ s.

The proposed method is applied to this model. Since the reference inputs are step signals, the parameters of the internal models are chosen to be

$$\bar{A}_1 = \bar{A}_2 = 0,\ \bar{B}_1 = \bar{B}_2 = 1 \quad (32)$$

As for the time constant T_1 and T_2, 0.01 will be suitable for both of the filters.

For the first loop, let

$$Q_1 = diag\left\{\begin{array}{cc} 1 & 1 \end{array}\right\},\ R_1 = 1 \quad (33)$$

Using LQR method yields

$$K_1 = -1.0,\ H_1 = 1.8233 \quad (34)$$

Set $\zeta = 1$ in (27) and choose $\sigma = 20$ in Step 2 of the observer gain design algorithm. So that they satisfy (22). Then

$$\lambda_1 = -20 \quad (35)$$

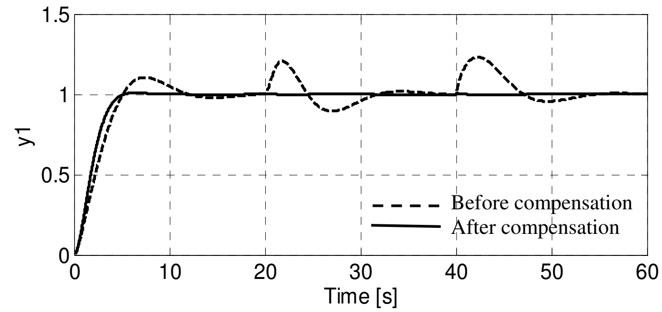

Fig. 3. The output responses for loop 1

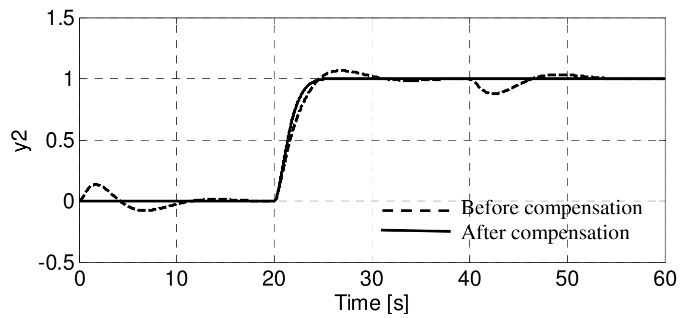

Fig. 4. The output responses for loop 2

Calculating the observer gain according to (28), yields

$$L_1 = 19.9401. \quad (36)$$

Similarly, for the second loop, choosing

$$Q_2 = diag\left\{\begin{array}{cc} 1 & 1 \end{array}\right\},\ R_2 = 1 \quad (37)$$

and

$$\lambda_2 = -20 \quad (38)$$

yields

$$K_2 = 1.0,\ H_2 = -1.5255 \quad (39)$$

and

$$L_2 = 19.9306. \quad (40)$$

The simulation results are shown in Fig.3 and Fig.4, respectively. In Fig.3, after the second step input and the disturbance being imposed, the peak to peak value (PPV) of the response in loop 1 is still less than 0.01. It occurs in loop 2 similarly; even the PPV at the beginning is only less than 0.01. It can be seen clearly that the proposed method achieves satisfactory performance in both decoupling control and disturbance rejection.

V. Conclusion

A dynamic decoupling control method has been presented for MIMO systems with disturbances. The undesirable coupling parts and external disturbances in each loop are treated as "disturbances", respectively. So that these disturbances can be effectively compensated by using the EID-based approach. The decoupling control and disturbance rejection can be carried out simultaneity without knowing the prior information of all the treated disturbances. Although every loop needs a controller, the parameters design is still very simple and can be independent of each other. Simulation results demonstrated the good performance of the proposed method.

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