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Phase modulation of robust variable structure control for nonlinear aircraft

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Abstract—**This paper concerns phase plane description and modulation of the performance for a nonlinear Unmanned Aerial Vehicle (UAV) system based on Variable Structure Control (VSC) by reaching sliding mode. The novelty lies in the application of a phase analysis approach to achieve a robust controller for a complex nonlinear system. The aircraft dynamics are introduced and approximately linearized and decoupled on-line using feedback linearization theory. Then the sliding mode control (SMC) scheme is accomplished for the decoupled sub-channels. The phase modulation method is applied for theoretically ensuring further robustness. The simulation results demonstrate the efficiency and effectiveness of the proposed strategy.**

Keywords-nonlinear aircraft; feedback linearization; sliding mode; phase modulation; robust controller

I. INTRODUCTION

Flight control systems have strict real application requirements to achieve high reliability against model uncertainties. The model-based approach to sustainable control for dynamic systems (based on analytical redundancy instead of hardware redundancy) has long been emphasized for achieving robustness and minimizing the effects of modelling uncertainty to the system [1-4]. Some approaches are based on robust control or passive fault tolerant control (FTC) as an alternative to achieve system reconfiguration [5-7]. Here we consider the robust approach for passive flight FTC system.

Advanced high-performance aircraft, not only have the characteristics of high nonlinearity and are Multiple Input and Multiple Output (MIMO) from a control standpoint, but also require high manoeuvrability with static instability [8]. For the purpose of efficiency and simplification, the feedback linearization technique is well-proven and has been developed to be one of the feasible control strategies in the study of nonlinear system, especially for aircraft [9], [10]. Feedback linearization can remove nonlinear features from the system and provide a linearized and decoupled closed-loop form. In addition to these features, dynamic linearization has advantages such as insensitivity to parameter changes and disturbances, and simplicity in physical realization [11-13].

For the linearized and decoupled aircraft system, a further robust controller is required to achieve tracking accuracy and passive FTC performance. As a main mode of VSC, the SMC technique turns out to be characterized by high simplicity and robustness [14], [15]. The main idea at the basis of the SMC strategy is the design of a particular control surface to coerce the controlled system trajectories into the sliding manifold to achieve expected performance via rapid switching between positive and negative control gains, resulting in variable structure of the control law. An advantage of sliding behavior is its insensitivity similar as the on-line feedback linearization strategy. The undesired phenomenon of so-called "chattering" of real-time SMC system, which is due to the finite switching frequency, could be avoided by using an approximated saturation function instead of the sign function during SMC system design [16-18]. In this study, the simultaneously worked feedback linearization controller and sliding mode controller optimize the system response against most of disturbances and even chattering.

As another aspect to this work, the literature of the development of SMC within phase modulation is well summarized in the work of [19] and [20]. By just using a real-time system response signal, a simple and effective phase modulation strategy can be used to purposely rectify the controller structure and parameters. This method also facilitates an approach to theoretical analysis for ensuring robustness of the SMC system, before adding external disturbances for certification. The approach, also presents a unique and efficient criterion for studying system characterization, especially when using linearised systems approaches for which the most suitable mathematical description is difficult to conclude because of unpredictable error during linearization and decoupling.

The contribution of this paper lies in the application of the combined SMC strategy based on feedback linearization with phase modulation theory to a nonlinear aircraft system. The aircraft example includes full force and moment longitudinal and lateral dynamics, which are deliberately linearised and decoupled into three second order sub-systems. The phase diagrams are then feasibly obtained and successfully used as criteria for achieving system robustness. The designed SMC systems illustrate the efficiency for real-time application.

Section II introduces the theoretical foundation of the control strategies. The mathematical model and linearization processing for a nonlinear UAV, the Machan, are introduced in Section III. The modulation analysis and simulation results are

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given in Section IV to illustrate the proposed approach. The concluding discussion is given in Section V.

II. CONTROL SYSTEM SCHEME

A. Nonlinear Feedback Linearization

The concept of feedback linearization makes use of the principle of transforming a smooth non-linear dynamical system into linear input-output form [21].

For the MIMO affine nonlinear system:

$$\left.\begin{aligned} x &= f(x) + G(x)u \\ y &= H(x) \end{aligned}\right\} \tag{1}$$

where $x = (x_1, \dots, x_n)^T$ is an n-D (dimensional) states vector of the system, $u \epsilon R^m$ and $y \epsilon R^m$ are the input and output vectors of the system, $f \epsilon R^n$ is a sufficiently smooth vector field. $G(x) = [g_1(x)\ g_2(x) \dots g_m(x)]^T$, $g_i\ (i = 1,2, \dots, m)$ is an m-D sufficiently smooth vector field , $H(x) = [h_1(x)\ h_2(x) \dots h_m(x)]^T$, $h_i\ (i = 1,2, \dots, m)$ is a sufficiently smooth scalar function.

For the affine nonlinear system (1), define γ_i to be the smallest integer such that at least one of the inputs appears in $y_i^{(\gamma_i)}$ using Lie derivatives as:

$$y_i^{(\gamma_i)} = L_f^{\gamma_i} h_i + \textstyle\sum_{j=1}^{m} (L_{g_j} L_f^{\gamma_i - 1} h_i u_j) \tag{2}$$

with at least one of the $L_{g_j} L_f^{\gamma_i - 1} h_i \neq 0 \forall x$, and u_j is the jth row of u. The input-output relation can then be defined as:

$$\begin{bmatrix} y_1^{(\gamma_1)} & y_2^{(\gamma_2)} & \dots & y_m^{(\gamma_m)} \end{bmatrix}^T = A(x) + B(x)u \tag{3}$$

$$A(x) = \begin{bmatrix} L_f^{\gamma_1} h_1(x) & L_f^{\gamma_2} h_2(x) & \dots & L_f^{\gamma_m} h_m(x) \end{bmatrix}^T \tag{4}$$

$$B(x) = \begin{bmatrix} L_{g_1} L_f^{\gamma_1 - 1} h_1 & \cdots & L_{g_m} L_f^{\gamma_1 - 1} h_1 \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{\gamma_m - 1} h_m & \cdots & L_{g_m} L_f^{\gamma_m - 1} h_m \end{bmatrix} \tag{5}$$

If the matrix $A(x) \epsilon R^{m \times m}$ is invertible, then the system can be linearized by decoupling the non-linear terms in (3) by choosing u as follows:

$$u = B^{-1}(x)[-A(x) + v] \tag{6}$$

which leads to the closed-loop decoupled, linear system:

$$\begin{bmatrix} y_1^{(\gamma_1)} & y_2^{(\gamma_2)} & \dots & y_m^{(\gamma_m)} \end{bmatrix}^T = \begin{bmatrix} v_1 & v_2 & \cdots & v_m \end{bmatrix}^T \tag{7}$$

Once linearization has been achieved, any further control objectives may be easily met [22], [23].

Furthermore, if the system has the total relative degree $\gamma = \gamma_1 + \gamma_2 + \dots + \gamma_m < n$, the standard MIMO form for system states can be reformulated as:

$$\left.\begin{aligned} z_1^1 = h_1(x), z_2^1 = L_f h_1(x), \dots, z_{\gamma_1}^1 = L_f^{\gamma_1 - 1} h_1(x) \\ z_1^2 = h_2(x), z_2^2 = L_f h_2(x), \dots, z_{\gamma_2}^2 = L_f^{\gamma_2 - 1} h_2(x) \\ \vdots \\ z_1^m = h_m(x), z_2^m = L_f h_m(x), \dots, z_{\gamma_m}^m = L_f^{\gamma_m - 1} h_m(x) \end{aligned}\right\} \tag{8}$$

where z are the chosen transformed states for linearization and decoupling. The system in (8) can be converted into a pseudo-linearized system as:

$$\left.\begin{aligned} \dot{z}_1^1 &= z_2^1 \\ &\vdots \\ \dot{z}_{\gamma_1}^1 &= a_1(z, v) + \textstyle\sum_{j=1}^{m} b_j^1(z, v) u_j \\ &\vdots \\ \dot{z}_1^m &= z_2^m \\ &\vdots \\ \dot{z}_{\gamma_m}^m &= a_m(z, v) + \textstyle\sum_{j=1}^{m} b_j^m(z, v) u_j \\ v &= P(z, v) + Q(z, v) u \\ y_1 &= z_1^1 \\ &\vdots \\ y_m &= z_1^m \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} a_i(z, v) &= L_f^{\gamma_i} h_i \cdot \Phi^{-1}(z, v) \\ b_j^i(z, v) &= L_{g_i} L_f^{\gamma_i - 1} h_i \cdot \Phi^{-1}(z, v) \\ p_i(z, v) &= L_f \eta_i \cdot \Phi^{-1}(z, v) \\ q_{ij}(z, v) &= L_{g_i} \eta_i \cdot \Phi^{-1}(z, v) \end{aligned}\right\} \tag{10}$$

Note that $\in R^{n-\gamma}$, $Q \in R^{(n-\gamma) \times m}$ and the aforesaid state transform $\Phi: x \to (z, v)$ is a diffeomorphism that maps x onto standard coordinates. The transformed input vector v has the expression from (9) as:

$$v = \Phi(z, v, u) \tag{11}$$

Once the linearization has been achieved, any further control objectives may be easily met.

B. Variable Structure Control and Robustness Analysis

As mentioned above, the nonlinear terms of the system can be eliminated by selecting an appropriate set of input transformations. However, the input-output linearization only fits in with systems with accurate models. In order to ensure control system robustness in the presence of system uncertainties, such as parameter uncertainties or unmodelled dynamics, the sliding mode control based on variable structure theory is chosen and applied to the linearized system.

Taking into account the presence of uncertainties in the nonlinear system (1), and (7) becomes:

$$\begin{bmatrix} y_1^{(\gamma_1)} & y_2^{(\gamma_2)} & \dots & y_m^{(\gamma_m)} \end{bmatrix}^T = (A + \Delta A) + (B + \Delta B)u \tag{12}$$

where the uncertainties represented by $\|\Delta B\|$ and $\|\Delta A\|$ are bounded. The switching surface is chosen as [24]:

$$s_i = \left(\frac{d}{dt} + \lambda_i\right)^{\gamma_i} (y_i - y_{iref}) \tag{13}$$

where λ_i is a positive constant and y_{iref} the reference command signal. Differentiating (13) with respect to time t leads to:

$$\dot{s}_i = y_i^{\gamma_i} + \textstyle\sum_{j=0}^{\gamma_i - 1} k_{ij} \left(y_i^j - y_{iref}^j\right) - y_{iref}^{\gamma_i} \tag{14}$$

In order to design a robust controller, the exponential approaching law is selected for SMC. The control input can then be expressed in the following form [25]:

$$u = B^{-1}(x)\{Y_{ref} - A - KY - \varepsilon \text{sgn}(s)\} \tag{15}$$

$$KY = \begin{bmatrix} \sum_{j=0}^{\gamma_1-1}\left[k_{1j}\left(y_1^j - y_{1ref}^j\right)\right] \\ \sum_{j=0}^{\gamma_2-1}\left[k_{2j}\left(y_2^j - y_{2ref}^j\right)\right] \\ \vdots \\ \sum_{j=0}^{\gamma_m-1}\left[k_{mj}\left(y_m^j - y_{mref}^j\right)\right] \end{bmatrix} \quad (16)$$

Substituting u and (12) into (14) yields to:

$$\dot{s} = \Delta A - \varepsilon sgn(s) - \Delta B(x)B^{-1}(x)\left[KY + \varepsilon sgn(s) + A - Y_{ref}\right] \quad (17)$$

where $Y_{ref} = \left[y_{1ref}^{\gamma_1} \quad y_{2ref}^{\gamma_2} \quad \cdots \quad y_{mref}^{\gamma_m}\right]^T$.

In order to guarantee asymptotic stability of the control system, $s^T\dot{s} < 0$ must be ensured yields to:

$$s^T\dot{s} \le \|s\|[\|\Delta A\| - \varepsilon + \|\Delta BB^{-1}KY\| + \varepsilon\|\Delta BB^{-1}\| + \|\Delta BB^{-1}A\| + \|\Delta BB^{-1}Y_d\|] \quad (18)$$

If $\|\Delta B\| < \|B\|$, and satisfy $\varepsilon > (\|\Delta A\| + \|\Delta BB^{-1}KY\| + \|\Delta BB^{-1}A\| + \left\|\Delta BB^{-1}Y_{ref}\right\|)/(1 - \|\Delta BB^{-1}\|)$, then $s^T\dot{s} < 0$ can be guaranteed. This means that the reaching condition of the sliding mode is tenable and the desired sliding motion is reachable by means of a suitable control law u [26], [27].

C. Phase Modulation Analysis

A proper selection of parameters that satisfy the above-cited conditions can ensure stability and robustness of the control system, although this is a sufficient but not necessary condition for reachability of the sliding mode. Moreover, the system uncertainties come from the unmodelled information and linearization error make the transformed system, even a simple one, more complicated to obtain desirable results.

Phase plane portraits are traditionally used to graphically show the SMC working performance [28], [29]. Recent research considers it as a modulation strategy to instructively adjust system structure and parameters for ensuring the system robustness [19], [20]. The basic principle of phase modulation is introduced in Fig. 1. The system real-time response error e and its derivative form $\dot{e}$ are used to establishing the coordinates. The switching line $s = 0$ defined by VSC theory divides the phase plane into four regions dominated by negative and positive feedback, respectively. Once the control system reach the designed sliding surface, the phase trajectory will frequently cross the switching line to repeatedly enter the different regions for compensating system uncertainties through switching between negative and positive feedback with high-frequency [30].

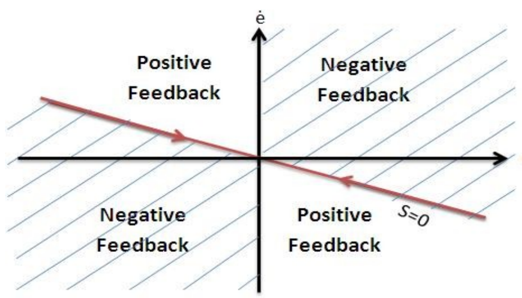

Figure 1. Phase plane with switching line

III. NONLINEAR MACHAN AIRCRAFT

The aircraft chosen is a UAV (or remotely piloted vehicle), the Machan, used as a development vehicle by Marconi Avionics, RAE Farnbrough and NASA Dryden for research on high incidence flight and non-linear control laws [31]. The methodology has wider application for FTC for systems with known nonlinear dynamic structure.

The Machan Euler equations relate the forces X, Y, Z and moments L, M, N in the aircraft body axes to the angular and linear velocities in the inertial axes are shown as:

$$\left.\begin{aligned} m(\dot{u} + qw - rv) &= X \\ m(\dot{v} + ur - pw) &= Y \\ m(\dot{w} + vp - qu) &= Z \\ I_x\dot{p} + (I_z - I_y)rq &= L \\ I_y\dot{q} + (I_x - I_z)pr &= M \\ I_z\dot{r} + (I_y - I_z)pq &= N \end{aligned}\right\} \quad (19)$$

where, m is the mass of the aircraft; I_x, I_y, I_z are the moments of inertia about the axes through the centre of gravity parallel to the aircraft body axes; u, v and w are the forward, side and vertical velocity of the aircraft respectively; p, q and r are the roll, pitch and yaw rates, respectively.

The aerodynamic force and moment equations are:

$$\left.\begin{aligned} X &= X_E - Dcos\alpha + (L_w + L_T)sin\alpha - mgsin\theta \\ Y &= Y_a + mgcos\theta sin\phi \\ Z &= -(L_W + L_T)cos\alpha - Dsin\alpha + mgcos\theta cos\phi \\ L &= L_a + L_E \\ M &= M_a + L_w(cg - 0.25)\bar{c} - L_q(l_t + 0.25 - cg) \\ N &= N_a \end{aligned}\right\} \quad (20)$$

where α (degrees) is the angle of attack; θ and ϕ (degree) are the pitch and roll angles, respectively; Y_a (N) is the side force; cg (m) is the position of the aircraft centre of gravity; X_E (N) is the thrust force due to the engine; l_t (N·m^{-1}) is the tail moment; D (N) is the force acting on the airframe; L_w, L_T and L_q (N) represent the wing, total tail and tail lift due to the pitch rate respectively; M_a, N_a and L_a (N·m^{-1}) are the pitching, yawing and rolling moment components respectively; $\bar{c}$ (m) is the mean aerodynamic chord and L_E (N·m^{-1}) is the rolling moment due to the engine.

The first order non-linear engine dynamic is given as:

$$\dot{X}_E = (P_{max}T_H\delta_p - X_EU_2)/K_e \quad (21)$$

where, P_{max}, T_H, δ_p, K_e and U_2 represent the maximum engine power, the throttle demand, the propeller efficiency, the engine rise rate and the air flow rate, respectively. The parameter details are given in Aslin 1985. The open-loop Machan UAV is unstable, thus a closed-loop "base-line" control system must be configured for stability before the further robust or FTC system can be developed.

To simplify the system, this paper only considers the angle states and their rates. Thus the system state vector x, and the output state vector y for the nonlinear aircraft model are chosen as:

$$x = [\phi \quad \theta \quad \psi \quad p \quad q \quad r]^T$$
$$y = [\phi \quad \theta \quad \psi]^T \tag{22}$$

where ψ is yaw angle.

A. State-space Description

The states p, q and r in (19) can be expressed as:

$$\left.\begin{aligned} \dot{p} &= \frac{(I_y - I_z)}{I_x} rq + \frac{L}{I_x} \\ \dot{q} &= \frac{(I_z - I_x)}{I_y} pr + \frac{M}{I_y} \\ \dot{r} &= \frac{(I_x - I_y)}{I_z} pq + \frac{N}{I_z} \end{aligned}\right\} \tag{23}$$

Additionally, the roll, pitch and yaw angles ϕ, $\theta$and ψ can be expressed in the terms of p, q and r as:

$$\left.\begin{aligned} \dot{\phi} &= p + qsin\phi tan\theta + r\, cos\phi tan\theta \\ \dot{\theta} &= qcos\phi - r\, sin\phi \\ \dot{\psi} &= qsin\phi sec\theta + rcos\phi sec\theta \end{aligned}\right\} \tag{24}$$

The real input vectors for the aircraft system are as:

$$u_{real} = [\delta_a \quad \delta_e \quad \delta_t]^T \tag{25}$$

where δ_a, δ_e, δ_t are the aileron, elevator and rudder input, respectively. By only considering the nonlinear part of system, the input vectors u of feedback linearization and decoupling control issue in (1) is chosen as described in (23):

$$u = [u_1 \quad u_2 \quad u_3]^T = \begin{bmatrix} \frac{L}{I_x} & \frac{M}{I_y} & \frac{N}{I_z} \end{bmatrix}^T \tag{26}$$

Then u_{real} can be easily calculated in the terms of u since they are linearly related from the form of the forces and moments equations [31], which can be shown as:

$$u_{real} = [l_{\delta_a} \quad l_{\delta_e} \quad l_{\delta_r}]^T u \tag{27}$$

where l_{δ_a}, l_{δ_e}, $l_{\delta_r}$are all the linear parameters.

Thus the affine system in (1) for this nonlinear UAV is simplified as:

$$f(x) = \begin{bmatrix} p + qsin\phi tan\theta + r\, cos\phi tan\theta \\ qcos\phi - r\, sin\phi \\ qsin\phi sec\theta + rcos\phi sec\theta \\ \frac{(I_y - I_z)}{I_x} rq \\ \frac{(I_z - I_x)}{I_y} pr \\ \frac{(I_x - I_y)}{I_z} pq \end{bmatrix}$$
$$G(x) = \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix}$$
$$H(x) = [I_{3\times3} \quad 0_{3\times3}] \tag{28}$$

B. System Feedback Linearization and Decoupling

For achieving linearization and decoupling, the pseudo-linearized system states in (9) for transforming the nonlinear system in (28) are chosen as [32]:

$$\left.\begin{aligned} z_1^1 &= \phi \\ z_2^1 = \dot{z}_1^1 &= p + qsin\phi tan\theta + r\, cos\phi tan\theta \\ z_1^2 &= \theta \\ z_2^2 = \dot{z}_1^2 &= qcos\phi - r\, sin\phi \\ z_1^3 &= \psi \\ z_2^3 = \dot{z}_1^3 &= qsin\phi sec\theta + rcos\phi sec\theta \end{aligned}\right\} \tag{29}$$

To check whether this coordinate transformation is invertible, the Jacobi Matrix for $z = z(x)$ is organized as:

$$\nabla z = \frac{\partial z(x)}{dx} =$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ (qcos\phi - rsin\phi)tan\theta & (qsin\phi + rcos\phi)sec^2\theta & 0 & 1 & sin\phi tan\theta & cos\phi tan\theta \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -qsin\phi - rcos\phi & 0 & 0 & 0 & cos\phi & -sin\phi \\ 0 & 0 & 1 & 0 & 0 & 0 \\ (qcos\phi - rsin\phi)sec\theta & (qsin\phi + rcos\phi)sec\theta tan\theta & 0 & 0 & sin\phi sec\theta & cos\phi sec\theta \end{bmatrix} \tag{30}$$

where $z = [z_1^1 \quad z_2^1 \quad z_1^2 \quad z_2^2 \quad z_1^3 \quad z_2^3]^T$.

The rank of ∇z is $det(\nabla z) = -sec\theta \neq 0$and $z = z(x)$ is a sufficiently smooth vector field with inversion, thus $z = z(x)$ is a global diffeomorphism of the system in (28) [9].

The input transformation ν in (6) takes the form:

$$\nu = P + Qu \tag{31}$$

where $\nu = [\nu_1 \quad \nu_2 \quad \nu_3]^T$, $P = [P_1 \quad P_2 \quad P_3]^T$, $Q = [Q_1 \quad Q_2 \quad Q_3]^T$ follow the description in (9), (10) as:

$$P(\bar{z}) = \frac{\partial z(x)}{dx} f(x)_{|x=f^{-1}(z)} \tag{32}$$

$$Q(\bar{z}) = \frac{\partial z(x)}{dx} G(x)_{|x=f^{-1}(z)} \tag{33}$$

Now, by choosing $\bar{z} = [z_1^1 \quad z_1^2 \quad z_1^3]^T$ to be the three output channels for the transformed system, each of these channels can be expressed as:

$$\begin{aligned} P_1 = (qcos\phi - rsin\phi)[(p &+ qsin\phi tan\theta \\ &+ r\, cos\phi tan\theta)tan\theta \\ &+ (qsin\phi + rcos\phi)sec^2\theta] + (I_y - I_z)rq/I_x \\ &+ (I_z - I_x)prsin\phi tan\theta/I_y \\ &+ (I_x - I_y)pqcos\phi tan\theta/I_z \end{aligned}$$

$$\begin{aligned} P_2 = (-qsin\phi - rcos\phi)(p + qsin\phi tan\theta + r\, cos\phi tan\theta) \\ + (I_z - I_x)prcos\phi/I_y \\ - (I_x - I_y)pq\, sin\phi/I_z \end{aligned}$$

$$\begin{aligned} P_3 = (qcos\phi - rsin\phi)[sec\theta(p &+ 2qsin\phi tan\theta \\ &+ 2r\, cos\phi tan\theta)] \\ &+ (I_z - I_x)prsin\phi sec\theta/I_y \\ &+ (I_x - I_y)pqcos\phi sec\theta/I_z \end{aligned}$$

$$Q_1 = [1 \quad sin\phi tan\theta \quad cos\phi tan\theta]^T$$
$$Q_2 = [0 \quad cos\phi \quad -sin\phi]^T$$
$$Q_3 = [0 \quad sin\phi sec\theta \quad cos\phi sec\theta]^T \tag{34}$$

After input-output feedback linearization for the system, the nonlinear aircraft dynamics have been theoretically decoupled into three 2nd order linear sub-systems with the transformed states and outputs described in (3) as:

$$\left.\begin{aligned}\dot{z} &= A_0 z + B_0 v\\ y &= C_0 z\end{aligned}\right\} \tag{35}$$

where

$$A_0 = \begin{bmatrix} A_0^1 & & \\ & A_0^2 & \\ & & A_0^3 \end{bmatrix}, A_0^i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (i = 1,2,3)$$

$$B_0 = \begin{bmatrix} B_0^1 & & \\ & B_0^2 & \\ & & B_0^3 \end{bmatrix}, B_0^i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} (i = 1,2,3)$$

$$C_0 = \begin{bmatrix} C_0^1 & & \\ & C_0^2 & \\ & & C_0^3 \end{bmatrix}, C_0^i = [1 \quad 0] (i = 1,2,3) \tag{36}$$

Each of the three sub-systems is a single-input single-output (SISO) 2nd order linear system in controllable (or phase variable) canonical form and is hence suitable for phase portrait analysis. For the 2nd order system in phase variable canonical form, the quality of the sliding mode invariance properties are satisfied, which means that once the states reach the sliding surface, the system dynamic performance is critically decided by the parameters of the designed SMC system [29].

C. *Sliding Mode Controller Design*

For the Machan system, the thrust input δ_t related to all the states is set as a very small constant value T_c to limit it's effect on the nonlinear aircraft dynamics, which could be modeled as system uncertainty. The SMC theory is used to achieve sub-controllers for each decoupled channel. The complete control system scheme is shown as Fig. 2.

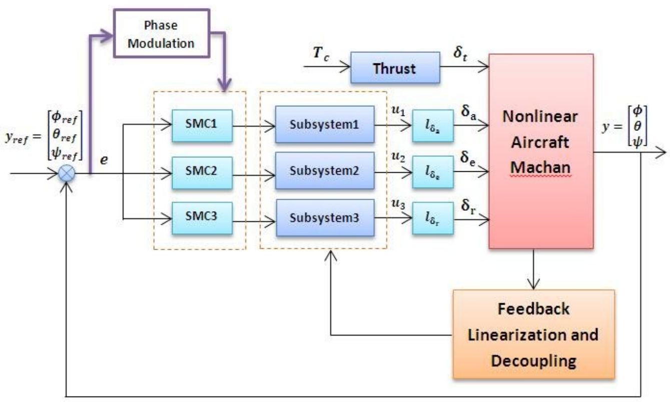

Figure 2. Control system scheme

For the 2nd order subsystems, the switching surface can be derived from (13) as:

$$s_i = k_i e_i + \dot{e}_i (i = 1,2,3) \tag{37}$$

where k_i is the adjustable parameter for SMC; $e_i = y_{ref} - y_i$ is system output error; y_{ref} and y_i are system reference input and real output, respectively.

The time derivative of (37) is then given as:

$$\dot{s}_i = k_i \dot{e}_i + \ddot{e}_i = k_i(y_{ref} - z_i^0) + (\dot{y}_{ref} - z_i^1)(i = 1,2,3) \tag{38}$$

By choosing a proper k_i for each subsystem, the sliding surface would take on desired characteristics. The SMC approaching law used in this system has the proportional form:

$$\dot{s}_i = -\varepsilon_i \text{sgn} s_i (i = 1,2,3) \tag{39}$$

where ε_i is positive constant. From (37)-(39), the subsystem control inputs $v_i (i = 1,2,3)$ derived from SMC are as:

$$v_i = k_i \dot{e}_i + \varepsilon_i \text{sgn} s_i \tag{40}$$

The above system with discontinuous control is termed a VSC since the effect of the switching surface is to alter the system feedback structure. The state trajectories on either sides of the surface $s_i = 0$ will remain in the vicinity of the sliding manifold since $s_i \dot{s}_i < 0$ on this surface. Once v_i is obtained, the system chosen input vector u and real input u_{real} could be easily calculated from (31) and (27).

IV. SIMULATION AND ANALYSIS

The SMC parameters are chosen as $K = [k_1, k_2, k_3]^T = [10,10,4]^T$, $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T = [3,3,10.6]^T$.

The system responses and their phase trajectories shown in Fig. 3 appear to be stable but the phase trajectories for the states ϕ and ψ are tangential to the switching line instead of crossing it, which indicates that the controllers are of dubious value without robustness. The reason for this phenomenon is that the linearized system may have conjugate poles too close to the imaginary axis, which makes the system readily unstable through the high controller gain action.

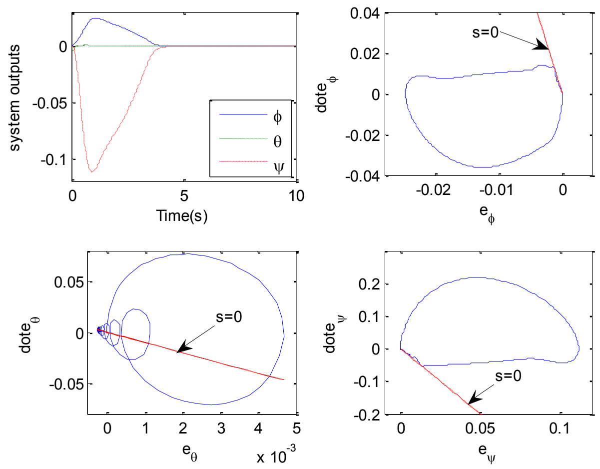

Figure 3. System responses and coordinating phase portraits

The solution is to add integrators for the corresponding states to replace system poles [30]. The new SMC parameters K are chosen as: $[k_1, k_2, k_3]^T = [8,15,5]^T$, ε is the same one as above. The developed system responses and their phase trajectories are shown in Fig. 4. The trajectories in Fig. 4 satisfy the principle of phase criterion for SMC described in Section II. For this case, the actuator faults F are chosen as step signals $F = [f_{\delta_a} \; f_{\delta_e} \; f_{\delta_r}]^T = [0.3,0.1,0.3]^T$ acting at time $t = 2s$. The system responses shown in Fig. 5 demonstrate the control scheme robustness and the efficiency of the modulation strategy.

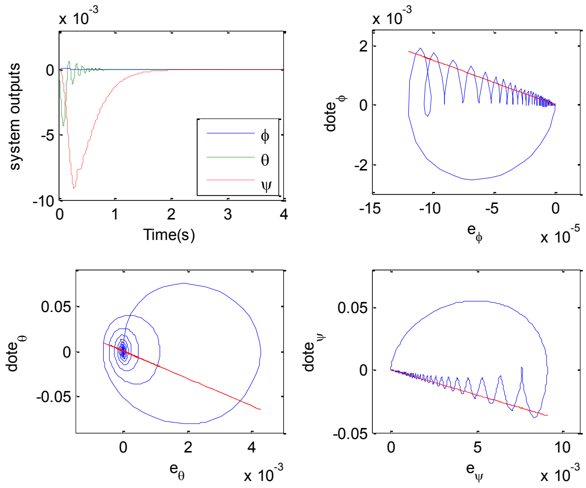

Figure 4. System responses and coordinating phase portraits

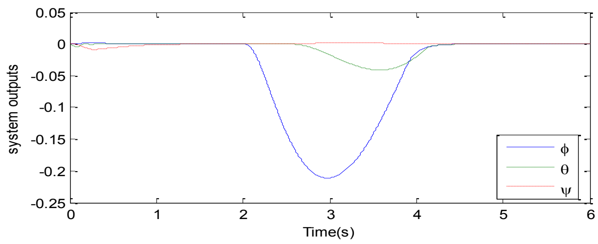

Figure 5. System responses with faults

V. CONCLUSION

The design of a passive FTC scheme for the highly nonlinear dynamics of a UAV, the Machan is achieved via VSC theory based on feedback linearization. The inner stable control loop is designed for on-line linearizing and decoupling the nonlinear system. The principle of phase plane analysis is outlined for observing the system robustness. The further SMC sub-systems are developed using the phase modulation principle to guarantee robust performance and robust stability. The designed system responses illustrate that this strategy is feasible valid and a very promising passive approach to robust FTC for flight systems.

REFERENCES

[1] R. J. Patton, “Fault-tolerant control systems: The 1997 situation”, IFAC Symp. Safeprocess’97, Hull, UK, pp.1029-1052, 1997.

[2] J. Boskovic, S. Li, and K. Raman, “Reconfigurable flight control design using multiple switching controllers and on-line estimation of damage-related parameters”, IEEE Int. Confer. on Contr. Appl., Sep 2000.

[3] Y. Wang, L. Xie, and C. E. de Souza, “Robust control of a class of uncertain nonlinear systems”, system&control letters, vol.19, August 1992, pp. 139-149.

[4] M. Bodson and J. E. Groszkiewicz, “Multivariable adaptive algotithms for reconfigureable flight control”, IEEE Trans. Contr. Syst. Technol., vol. 5, pp. 217-229, March 1997.

[5] M. Blanke, “Fault-tolerant control systems”, pp.171-196, Springer-Verlag, Denmark, 1999.

[6] J. Chen, and R. J. Patton, “Robust model-based fault diagnosis for dynamic systems”, Kluwer Academic, 1999.

[7] J. Doyle, K. Zhou, K. Glover and B. Bodenheimer, “Mixed H_2-H∞ performance objectives: robust performance analysis”, IEEE Trans. on Autom. Contr., vol. 39, pp. 1564-1575, 1994.

[8] M. Donald, “Automatic flight control systems”, Prentice Hall, 1990.

[9] S. H. Lane and R. F. Stengel, “Flight control design using non-linear inverse dynamics”, Automatica, vol. 24, pp. 471-483, 1988.

[10] Y. Ochi and K. Kanai, “Design of restructurable flight control systems using feedback linearization”, J. of Guid., Contr. & Dyn.,vol. 14, 1991.

[11] S. Snell, D. Enns and W. Garrard, “Nonlinear inversion flight control for a super-manoeuvrable aircraft”, J. of Guid. Contr. & Dyn, vol. 15, 1992.

[12] Y. Wu, and Q. Zou, “Robust Inversion-based 2-DOF control design for output tracking: piezoelectric-actuator example”, IEEE Trans. Contr. Syst. Technol., vol. 17, pp. 1069– 1082, 2009.

[13] N. Hovakimyan, E. Lavretsky and C. Cao, “Dynamic inversion of multi-input non-affine systems via time-scale separation”, American Control Conference, vol. 14, 2006.

[14] D. Young, U. Ozguner and V. Utkin, “A control engineering guide to sliding mode control”, IEEE Trans. on Contr. Syst. Technol., vol. 7, pp. 328-342, 1999.

[15] X. Yan, and C. Edwards, “Nonlinear robust fault reconsturctionh and eatimation using a sliding mode observer”, Automatica, vol. 43, Sep 2007.

[16] C. Edwards and S. K. Spurgeon, “Sliding mode control: theory and application”, Taylor & Francis, London, 1998.

[17] V. I. Utkin, J. Guldner and J. Shi, “Sliding mode control in electromechanical systems”, Taylor & Francis, London, 1999.

[18] G. Bartolini, A. Ferrara and E. Usani, “Chattering avoidance by second-order sliding mode control”, IEEE Trans. on Autom. Contr., vol.43, pp. 241-246, 1998.

[19] S. Ma, A. J. Wilkinson, and K. S. Paulson, “A phase modulation-based ultrasonic communication system using variable structure control”, IEEE ICCT, pp. 857-860, Nov 2010.

[20] S. Ma, A. J. Wilkinson and K. S. Paulson, “Performance analysis of a phase modulation-based ultrasonic receiver using variable structure control”, IEEE ICCT, pp.776-779, 2011.

[21] J.-J. E. Slotine, and W. Li, “Applied nonlinear control”, Prentice Hall, USA, 1991.

[22] Y. Tang, R. J. Patton, “Active FTC for nonlinear aircraft based on feedback linearization and robust estimation”, accepted for presentation at the IFAC Symposium Safeprocess 2012, Mexico City, August 2012.

[23] Y. Tang, R. J. Patton, “Fault tolerant flight control for UAV”, accepted for presentation at the Mediterranean conference, Barcelona, July 2012.

[24] Y. B. Shtessel and J. Buffington, “Multiple time scale flight control using reconfigurable sliding mode”, AIAA J. of Guid., Contr. and Dyn., vol. 22, pp.873-883, 1999.

[25] K. D. Young, “A control engineering’s guide to sliding mode control”, IEEE International Workshop on Variable Structure Systems.pp.1-14, Tokyo, Dec 1996.

[26] M. Allen, F. Barnelli-Zazzera and R. Scattolini, “Sliding mode control of large flexible space structure”, Control Engineering Practice, vol.8, pp.861-871, 2000.

[27] B. A. White and P. M. Silson, “Reachability in variable structure control systems”, IEE Pro. D Control Theory & Appl., vol. 131, pp.85-91, 1984.

[28] B. A. White, “Range-space dynamics of scalar-variable-structure control systems”, IEE Pro. D Control Theory & Appl., vol. 133, pp.35-42, 1986.

[29] A. S. I. Zinober, “Deterministic control of uncertain systems”, Peter Peregrinus, London, 1990.

[30] Ma, "Increasing the Capacity of an Ultrasonic Communication System using Variable Structure Control," PhD thesis, Department of Engineering, The University of Hull, UK, 2011.

[31] P. Aslin, “Aircraft simulation and robust flight control system design”, DPhil thesis, Department of Electronics, University of York, UK, 1985.

[32] G. Meyer, L. R. Hunt, “Application of nonliear transfomation to automatic flight control”, Automatica, Vol. 20, pp.103-107, 1984.