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Stability analysis and Control design of a Class of Event Based Control Systems

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Abstract—**Event Based Control (EBC) provides a reduction of mean control rates, which is an important advantage in control systems, specially when network environment gets involved. For this reason, the study of design methodologies for EBC systems that met desired specifications regarding stability and performance issues are a valuable research field. This work presents a control design process applied to a class of EBC systems using LMIs, including stability issues on the light of Asynchronous Dynamical System theory. The application of the proposed methodology is presented by an example, showing good performance in simulation results.**

Index Terms—**Event Based Control, Linear Matrix Inequality, Asynchronous Dynamical Systems, Control Application**

I. INTRODUCTION

Event Based Control (EBC) has been considered complex and, in the past, the lack of a well established theory and design methods has limited its use to special cases. However, the interest in EBC control strategies is growing due to its advantages. In particular, its reduction on resource usage, which allows a reduction of the control rate [1], [2], [3], [4]. This characteristic has great importance in embedded systems, where life-time of devices is limited by batteries, or in networked environment, where the reduction on the usage of bandwidth is desirable. For instance, both requirements must be fulfilled in the case of wireless sensor networks.

Following the EBC approach, different alternatives have been developed. In [1], [2], improved system response with a reduction in the control rate is obtained using impulse control signals and observers. In [5], a piecewise control signal based scheme with level-triggered sampling is proposed. More recently in [4], the idea of sporadic control is introduced, where a minimum time difference between sampling times is proposed. A similar control scheme, which is experimentally implemented, presents an improvement of the system response in the presence of delay, jitter and noise, [6]. Moreover, PID controllers have been studied using event based sampling, [7]. The work in [8] presents a modified PID control structure that minimizes different problems related to event based sampling approach.

In addition, different schemes have been analyzed using state-space approach, showing stability properties, [9], and the existence of a lower bound for inter-events time, [10], for instance. This methodology has been extended to NCS systems in [11], including quantization effects. In [12], experimental results have been recently presented, evaluating an event based state-state approach. In [13], similar considerations are obtained for an output-based event-triggered control scheme using LMI techniques.

Therefore, different EBC schemes have been developed and evaluated. The advantages obtained applying such schemes encourage to develop new alternative approaches and specific design methodologies for such systems.

The objective of this work is to describe a design methodology useful for EBC systems, which guarantees stability and some design specifications and its validation under simulation test. The design procedure facilitates the improvement of the dynamics of the resulting EBC system, which will be experimentally implementable. The control scheme switches between different controllers following an event-triggered sampling scheme which is proposed in [6]. The stability of the closed-loop system is determined using a standard LMI methodology, [14], taking into account results derived from Asynchronous Dynamical Systems (ADS) theory. This theory is an interesting resource since it is applicable to Networked Control Systems (NCS), [15], [16].

The paper is organized as follows: first, the particular structure of the EBC scheme under study is described in Section II. In the next section, the analysis of the proposed EBC system is presented, based on the use of the ADS theory. In Section IV, the proposed approach for the design of the EBC controller is described. This process includes the LMI based stability analysis. An example of the proposed methodology is presented in Section V, where its performance is analyzed by means of simulations. Finally, conclusions end the paper.

II. STATEMENT OF THE PROBLEM

Event Based Control is based on triggering control action only when a condition is met [2]. The main advantage of this control strategy is that the required number of control actions can be considerably reduced while maintaining the system under control.

Figure 1 presents the scheme of a basic EBC system, where a reference signal is explicitly included. The difference from a typical feedback control system is the event-based sampling technique used to perform the feedback.

Let us consider a linear, or linearized, system which can be described by the next equations:

$$\begin{aligned} \dot{x_p}(t) &= f(x,u) = Ax_p(t) + Bu(t) \\ y(t) &= g(x,u) = Cx_p(t) + Du(t) \end{aligned} \qquad (1)$$

Here, for the sake of simplicity, linear SISO systems with $D = 0$ are considered. Introducing a zero order hold (ZOH) in the input of the system, the control signal value u_k, which is generated by a digital controller, is maintained constant between consecutive control events. The closed-loop correction acts only when the event firing rule is met, e.g. when a selected signal level reaches a limit value defined during design stage. Hence, the continuous control signal, that is, the system's input signal, is

$$u(t) = h(t - t_k)u_k, \quad t \in [t_k, t_{k+1}), k = 0, \ldots, \infty \tag{2}$$

where u_k is the event-triggered control signal and $h(t)$ is the Heaviside step function.

In this work, the event-based sampling of a continuous signal $z(t)$ is defined by the next expressions, with $k \geq 1$

$$\begin{aligned} z_k &= z(t_k) \\ d^{(k)}(t^*) &= z(t_{k-1} + t^*) - z(t_{k-1}) \\ t_k &= t_{k-1} + T_k \\ T_k &= \begin{cases} T_{min} & \text{if } \delta^{(k)}(t^*) \geq \delta_{limit} \\ & \quad \text{and } t^* \leq T_{min} \\ t^* & \text{if } \delta^{(k)}(t^*) \geq \delta_{limit} \\ & \quad \text{and } t^* \in (T_{min}, T_{max}) \\ T_{max} & \text{if } \delta^{(k)}(t^*) < \delta_{limit} \\ & \quad \text{and } t^* = T_{max} \end{cases} \end{aligned} \tag{3}$$

where t^* is the continuous time which is reset at every sampling time t_k, $t^* \in [0, T_k]$. The difference between the actual value of the signal and the last sampled value, $d^{(k)}(t^*)$, is a switched continuous function depending on the $k-th$ event. The parameter $\delta^{(k)}$ is the value which fires control action, continuously updated according to the following rule:

$$\delta^{(k)}(t^*) \quad = \quad K_p^s \left| d^{(k)}(t^*) \right| + K_i^s \int_0^{t^*} \left| d^{(k)}(t) \right| dt \tag{4}$$

This expression is reset at event time, i.e. $\delta^{(k)}(0) = 0$. Thus, the new sampling is triggered at $t_k = t_{k-1} + T_k$ time, when $\delta^{(k)}$ reaches a predefined limit δ_{limit} or when t^* reaches the limit T_{max}. Equation 4 leads to a proportional-integral sampling scheme, where a minimum difference between sampling times T_{min} (sporadic control approach [4]) and a maximum difference between sampling times T_{max} are defined. Hence, $T_k \in [T_{min}, T_{max}]$.

In the same way, an integral action is added to the proportional sampling (or deadband sampling, [1]) to reduce the sticking phenomenon, [17]. Initially, when the reference varies, the proportional sampling is enough to guide the system towards the desired state. However, when it is close to the reference signal, no action will be triggered as long as $\delta^{(k)}(t^*)$ is maintained inside its deadband, so a non null error stands. In such case, the integral part of the event sampling rule will fire the control action after a time interval. This interval depends on the parameter K_i^s, defined in the event sampling policy. The choice of values of the parameters δ_{limit}, K_i^s, and K_p^s will have a direct impact on the performance of the system.

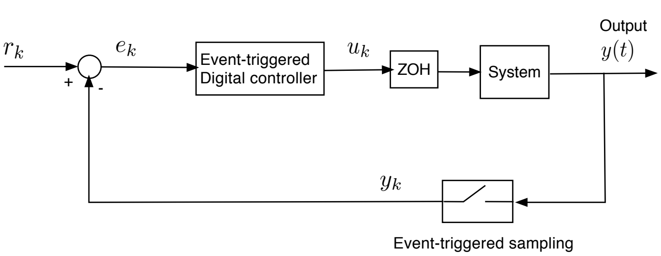

Figure 1. Scheme of a basic EBC system

Considerations about T_{max}. The existence of a maximum inter-sampling time can be considered that leads to unnecessary control actions. However, the definition of T_{max} helps to minimize the sticking problem, [6]. Note that, in practice, this definition affects only in cases when the open-loop system is stable and $\delta^{(k)}$ is strictly zero, depending on the value of δ_{limit} and K_i^s. It is introduced to prioritize the minimization of the sticking phenomenon, with a possible penalty in the resource optimization which can be adjusted depending on the value of T_{max}. In addition, it facilitates the mathematical description proposed below.

For every event at t_k derived from equations (3), the open-loop linear system (1) can be discretized, leading to the next representation,

$$\begin{aligned} x_{k+1}^p &= \phi_k x_k^p + \Gamma_k u_k \\ y_k &= C x_k^p \\ \phi_k &= \phi(T_k) = e^{AT_k} \end{aligned} \tag{5}$$

$$\Gamma_k = \Gamma(\quad T_k) = \int_0^{Tk} e^{As} ds B \tag{6}$$

Due to the event-based sampling policy (3), T_k can take any value in the interval $[T_{min}, T_{max}]$. Therefore, $\phi_k \in \mathcal{C}([T_{min}, T_{max}])$, that is, it is a continuous function with respect to variable T_k in the interval $[T_{min}, T_{max}]$, that depends on matrix A. However, considering the nature of t_k, ϕ_k is a jump or switched function respect to the index k.

Let the structure of the controller be given by the next equations:

$$\begin{aligned} x_{k+1}^c &= A_k^c x_k^c + B_k^c e_k \\ u_k &= C_k^c x_k^c + D_k^c e_k \end{aligned} \tag{7}$$

where $e_k = r_k - y_k$. Defining the extended state-vector $\bar{x}_k = [x_k^{pT}, x_k^{cT}]^T$, the closed-loop system can be represented using the following equations:

$$\begin{aligned} \bar{x}_{k+1} &= \begin{bmatrix} \phi_k - \Gamma_k D_k^c C & \Gamma_k C_k^c \\ -B_k^c C & A_k^c \end{bmatrix} \bar{x}_k + \begin{bmatrix} \Gamma_k D_k^c \\ B_k^c \end{bmatrix} r_k \\ &= \bar{A}_k \bar{x}_k + \bar{B}_k r_k \end{aligned} \tag{8}$$

An EBC system following the sampling rule (3) presents the usual advantages shown in the literature for EBC systems, but with an improved behavior respect to the sticking problem, [6]. However, analysis and design techniques for this class of systems are needed, since the control structure could vary at each event.

In the next sections, there is presented an approach valid for determining the stability of the EBC systems using ADS and LMI theory and a design methodology adequate for fulfilling some specifications.

III. Stability of EBC system under study

Some results from the ADS theory can be applied to the proposed EBC system, under several conditions. Relevant results from [15] are summarized to clarify the discussion. First, the definition of ADS systems:

Definition 1. An asynchronous dynamical system (ADS) with rate constraints on events is a tuple

$$\mathcal{A} = (\mathbb{R}_+, \{1, ..., N\}, \mathbb{R}^n, E, R, \mathcal{I}, F) \tag{9}$$

where $\mathbb{R}_+$ is time, $\{1, ..., N\}$ is the discrete state-space, $\mathbb{R}^n$ is the continuous state-space, E is the set of events, $R = \{r_1, \ldots, r_M\}$ is the set of event rates, $\mathcal{I} : 1, ..., N \to 2^E$ is the discrete state-event function, and F is the set of continuous dynamical system functions. By definition, $\mathcal{I}(i) = \left\{E_{i_1}, E_{i_2}, \ldots, E_{i_{M_i}}\right\}$ is the *i*th discrete state-event set, where $e_j^{(i)} \in E$ for $j = 1, ..., M_i$. An ADS has associated a discrete state $s(t)$ and a continuos state $x(t)$. $s(t) = i$ if and only if the events in $\mathcal{I}(i)$ have occurred and $\dot{x} = f_i(x)$. If the evolution of x is given by a difference equation, then, the last equations is substituted by $x_{k+1} = f_i(x_k)$.

Now, the definition of the stability of such systems:

Definition 2. An ADS with continuous state dynamics $x(t)$ is exponentially stable if

$$\lim_{t\to\infty} e^{\alpha t} \|x(t)\| = 0,$$

for some $\alpha > 0$. If the dynamics is given by a discrete state x_k, the condition is

$$\lim_{k\to\infty} \alpha^k \|x(t)\| = 0$$

Focussing the discussion on the discrete case, the next theorem summarizes the main result from [15]: Consider an ADS system $\mathcal{A}$ with discrete state dynamics x_k ($x_{k+1} = f_i(x_k)$), the ith discrete state-event set $\mathcal{I}(i) = \left\{E_{i_1}, E_{i_2}, \ldots, E_{i_{M_i}}\right\}$ and a set of event rates $R = \{r_1, .., r_M\}$ in which r_i satisfying $0 \le r_i \le 1$ is the rate of occurrence of event $E_i \in E$ over time. So, over any time period $[t, t+T]$ for large enough T, $r_i T$ is the total amount of time that E_i has occurred.

In addition, suppose a Lyapunov-type function $V : \mathbb{R}^n \to \mathbb{R}_+$, which is continuously differentiable and

$$\beta_1 \|x\|^2 \le V(x) \le \beta_2 \|x\|^2 \tag{10}$$

where $\beta_{1,2} > 0$.

Theorem 3. *[15], if there exist scalars $\alpha_1, \alpha_2, ..., \alpha_M$ such that the system fulfills the condition*

$$\alpha_1^{r_1} \alpha_2^{r_2} \ldots \alpha_M^{r_M} > \alpha > 1 \tag{11}$$

and

$$V(x_{k+1}) - V(x_k) \le (\alpha_{i_1}^{-2} \alpha_{i_2}^{-2} \ldots \alpha_{i_{M_i}}^{-2} - 1)V(x_k) \tag{12}$$

for i = 1,...,N, in which i_j for $j = 1, ..., M_i$ correspond to the definition of $\mathcal{I}(i)$, the decay rate of the ADS is greater than α. Then, the ADS system is exponentially stable.

Proof: See, [15]. ∎

In order to apply those results to the EBC scheme presented in Section II, some details have to be revised.

Application of the ADS approach to event-based systems: Considering the description in Section II and the Definition 1, the EBC system described by equations (8) and event-triggered mechanism (3) is similar to an ADS system. However, in this case, the events are defined by all the possible sampling instants in a continuous interval $[T_{min}, T_{max}]$. That is, $E = \{E_1, E_2, \ldots, E_\infty\}$ is an infinite set. In addition, this fact difficulties the implementation of a real system using the sampling mechanism (3). To solve this problem, a finite set of possible sampling times is introduced:

$$\begin{aligned} \mathcal{T}_s &= [T_{min}, T_{min} + \Delta t_s, T_{min} + 2\Delta t_s, \ldots, \\ &\quad \ldots, T_{min} + n\Delta t_s = T_{max}],\ \Delta t_s = \frac{T_{max} - T_{min}}{n} \end{aligned} \tag{13}$$

with $n \in \mathbb{N}$. Now, consider the next approximation of the event-sampling approach defined in equations (3):

$$\begin{aligned} z_k &= z(t_k) \\ e^{(k)}(t^*) &= z(t_{k-1} + t^*) - z(t_{k-1}) \\ t_k &= t_{k-1} + T_k \\ T_k^{ideal} &= \begin{cases} T_{min} & \text{if } \delta^{(k)}(t^*) \ge \delta_{limit} \text{ and } t^* \le T_{min} \\ t^* & \text{if } \delta^{(k)}(t^*) \ge \delta_{limit} \text{ and } t^* \in (T_{min}, T_{max}) \\ T_{max} & \text{if } \delta^{(k)}(t^*) < \delta_{limit} \text{ and } t^* = T_{max} \end{cases} \\ T_k &= \left\lfloor \frac{T_k^{ideal}}{\Delta t_s} \right\rfloor \Delta t_s \end{aligned} \tag{14}$$

with $\delta^{(k)}$ defined by (4), $T_k \in \mathcal{T}_s$ and T_k^{ideal} represents the sampling times derived from equations (3) (which is an ideal case). Note that the equations (3) are obtained from equations (14), when $n \to \infty$. The introduction of this new event-sampling approach is motivated by the implementation which is easier than in the approach given by (3), since can be based in the use of a regular periodic sampling time Δt_s. This idea is similar to the scheme proposed in [11].

Now, considering the event-sampling scheme (14), the proposed EBC system is an ADS system with discrete dynamics $\bar{x}_k$, being $E = \{E_1, E_2, \ldots, E_n\}$. Note that the conclusions derived for this ADS system are valid for any n and, then, the results should be valid for $n \to \infty$. Therefore, the result summarized in Corollary 4 can be concluded.

Corollary 4. *The stability of the EBC system described by equations* (8) *and event-triggered mechanism* (14)*, considering the definition presented in Section II, will be guarantied if:*

- it exists a Lyapunov function $V(x_k) = x_k^T P x_k$ for some symmetric positive matrix $P > 0$ fulfilling

$$V(x_{k+1}) - V(x_k) \leq (\alpha^{-2} - 1)V(x_k)$$

for some $\alpha > 1$ when $T_k \in [T_1, T_2] \subseteq [T_{min}, T_{max}]$.

- the rate of event samplings in this interval is sufficiently large.

Proof: By the definition of the EBC system, the finite set of events $E = \{E_1, E_2, \ldots, E_n\}$ and the related set of event rates $R = \{r_1, .., , r_n\}$ are well defined for a large period of time. Hence, there exist the scalars $\alpha_1, \alpha_2, ..., \alpha_n$ related with each n possible matrix $\bar{A}_i$ $i = 0, \ldots, n$ in equation (8). Under the conditions defined in this corollary, the system is exponentially stable for any arbitrary switch in the interval $T_k \in [T_1, T_2]$, [18]. Consider the set of m events $E' \subseteq E$ related with $T_k \in [T_1, T_2]$. By Theorem 3, the set of m systems $\bar{A}_j$ derived from E' fulfills

$$\bar{A}_j^T P \bar{A}_j - P \leq (\alpha^{-2} - 1)P$$

for some $\alpha > 1$. In addition, if the rate of events in interval $T_k \in [T_1, T_2]$ is sufficiently large, the condition (11) in Theorem 3 will be fulfilled and the EBC system is stable. ■

A particular case derived of this Corollary is when the interval considered is $[T_{min}, T_{max}]$. In this case, the second condition is always fulfilled, but the obtaining of a controller fulfilling the first condition can be hard.

Remark 5. The event rate in a particular interval is directly related with the selection of δ_{limit} in each case.

Remark 6. The sampling mechanism can be enforced to assure the necessary rate in any interval.

The next section presents a design process valid for the proposed EBC system.

IV. Controller design and LMI based stability analysis

Consider a continuous controller and its discretization for a sampling rate interval $T_k \in [T_1, T_2]$, under event-triggered sampling (14). This scheme leads to a discrete representation of the controller changing at every event t_k. If all the interval $[T_{min}, T_{max}]$ is considered, the n possible system matrices in equation 8for each $T_k \in \mathcal{T}_s$ are:

$$\bar{A}_i = \begin{bmatrix} \phi_i - \Gamma_i D_i^c C & \Gamma_i C_i^c \\ -B_i^c C & A_i^c \end{bmatrix} \quad i = 0, \ldots, n$$

where matrices $(A_i^c, B_i^c, C_i^c, D_i^c)$ are obtained by discretization, following equivalent equations to (6). Let us consider $V_k = x_k^T P x_k$, for some $P > 0$. The system will be stable if the equations

$$\bar{A}_i^T P \bar{A}_i - P \leq -Q_i$$

are fulfilled for $i = 0, \ldots, n$ and $Q_i > 0$. However, by continuity in the discretization process, if the equations corresponding to T_{min} and T_{max} are fulfilled, the equivalent equation of every sampling time in interval $[T_{min}, T_{max}] \in \mathcal{T}_s$ will be fulfilled.

A more general scheme can be considered. The interval can be subdivided into N_I intervals $[T_l, T_{l+1}] \subset \mathcal{T}_s$ with $l = 0, \ldots, N_I - 1$, covering the full interval $[T_{min}, T_{max}]$, being $T_0 = T_{min}$ and $T_N = T_{max}$. Then, controllers which stabilize the closed-loop for any $T_k \in [T_l, T_{l+1}]$ for each such interval $[T_l, T_{l+1}]$ are enough, if the conditions of Corollary 4 are fulfilled. Therefore, N design problems must be solved, one for each time interval $[T_l, T_{l+1}]$, which can be expressed as the next LMIs

$$\begin{aligned} \bar{A}_l^T P_l \bar{A}_l - P_l &\leq -Q_l \\ \bar{A}_{l+1}^T P_l \bar{A}_{l+1} - P_l &\leq -Q_l' \quad l = 0, \ldots, N_I - 1 \end{aligned} \tag{15}$$

with $P_l > 0$, $Q_l > 0$ and $Q_l' > 0$. The simplest way to fulfill Corollary 4 is to force a unique P matrix, i.e. $P_l = P > 0$ $l = 0, \ldots, N_I - 1$,. This design problem can be expressed by mean of a set of LMIs to fulfill some specification.

However, the feasibility of the LMI problem considering an unique P matrix can be difficult to satisfy in general. A more open problem is to relax this condition allowing multiple P_l matrices. In such case, in order to achieve the second condition in Corollary 4, a supervisor assuring a minimum rate in each interval $[T_l, T_{l+1}]$, a dwell time, [18], is required. Note that several politics can be followed by this supervisor.

V. Example of application

In this section, an example of application of the tools described in the previous section is presented. The procedure applied in this example is summarized as follows:

- Considering the plant to control and the basic specifications, the parameters for the event sampling approach (14) are selected: interval $[T_{min}, T_{max}]$, n, δ_{limit}, K_p^s and K_i^s.
- Selection of the N_I intervals $[T_l, T_{l+1}] \subset \mathcal{T}_s$ covering the full interval $[T_{min}, T_{max}]$.
- Perform a controller design in the continuous domain applicable for each interval. A reasonable design constrain is that the bandwidth of the closed-loop system in each interval is directly related to the sampling times involved, that is, for higher sampling rates, larger bandwidth.
- Solve the LMI feasibility problem derived from equations (15). Note that those problems are solved in the discrete domain and the controller must be conveniently discretized, using a kind of emulation technique.

Resolving satisfactorily the last point, the EBC system can be implemented and performance tests done. Now, details about the example are presented.

A. Controller Design

The plant used in current analysis consist of a DC motor connected to a rotational-to-translational motion converter. The continuous plant dynamics relating the input voltage and the position has been modeled approximately by the following transfer function:

$$G(s) = \frac{1}{s}\frac{K}{(\tau_m s + 1)} \quad (16)$$

where, $K = 6536\, mm\, vol^{-1}$ and $\tau_m = 1/4.3$. So, a state-space representation is given by the following equations:

$$A_p = \begin{pmatrix} 0 & 1 \\ 0 & -1/\tau_m \end{pmatrix} \quad B_p = \begin{pmatrix} 0 \\ K/\tau_m \end{pmatrix} \quad (17)$$

$$C_p = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (18)$$

A standard PID controller can be enough for controlling this second order system. After design, the standard version must be rewritten as the equivalent representation in the state-space.

For a first example of application, the sampling interval defined by the event-based sampling scheme (14) is split in three zones, $N_I = 3$:

$$\overbrace{[T_{min}, T_1]}^{controller\,1} \bigcup \overbrace{[T_1, T_2]}^{controller\,2} \bigcup \overbrace{[T_2, T_{max}]}^{controller\,3} \quad (19)$$

being $T_{min} = 0.5msec.$, $T_1 = 1msec.$, $T_2 = 3msec.$, $T_{max} = 50msec.$. In addition, the other parameters needed in (14) are $K_p^s = 0.1$, $\delta_{limit} = 0.01$ and $K_i^s = 100$. The last one has been selected as reference, since an analysis of the effect of this parameter in the behavior of the EBC system has been done. This is presented below.

The EBC sampling scheme leads to a changing variable time between consecutive control actions, depending on the speed of the changes on the error signal. Therefore, three different controllers are proposed for different sampling times, corresponding to the three intervals in (19). For high sampling rates, faster controller is preferred, with a higher bandwidth. On the contrary, for low sampling rates, a control system with a lower bandwidth is enough for leading the system to the desired state.

The overall methodology for the full controller design procedure is presented in Figure (2), starting from the model of the system to be controlled. After obtaining a valid controller for each different region, the LMI feasibility problem is analyzed. The implementation consist on the use of event sampling strategy (14), followed by a decision algorithm which chooses the correct controller for each interval. Depending on the different sampling times T_k, appropriate discrete controller is used when applying the event sampling policy.

For each region, the next controller parameters are obtained, requiring in the design process different closed-loop bandwidths:

$$\begin{array}{lll} K_p^{(1)} = 0.0016737 & K_i^{(1)} = 0.0002193 & K_d^{(1)} = 0.0010633 \\ K_p^{(2)} = 0.0072328 & K_i^{(2)} = 0.0011645 & K_d^{(2)} = 0.0095654 \\ K_p^{(3)} = 0.0163627 & K_i^{(3)} = 0.0020823 & K_d^{(3)} = 0.0325810 \end{array} \quad (20)$$

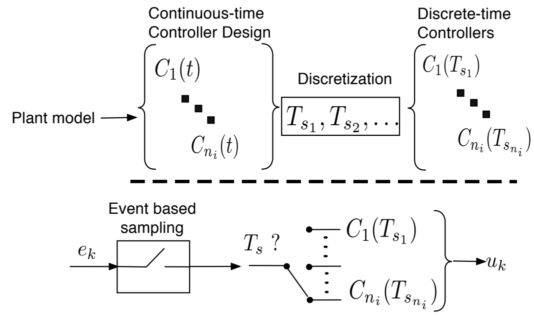

Figure 2. Proposed controller design methodology for the presented Event Based control.

Since the presented event based methodology implies the use of discrete domain controllers, the PID is approximated using the backward approximation of the derivate as

$$G_{pid}(z) = K_p + K_d \frac{(1 - z^{-1})}{T_k(1 - \alpha z^{-1})} + K_i \frac{T_k}{(1 - z^{-1})}$$

which can be represented in the state-space

$$\begin{aligned} A_c &= \begin{pmatrix} 1+\alpha & -\alpha \\ 1 & 0 \end{pmatrix} \quad B_c = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ C_c &= \begin{pmatrix} K_1^c & K_2^c \end{pmatrix} \quad D_c = K_p + \frac{Kd}{T_k}(1-\alpha) + K_i T_k \\ K_1^c &= -\left((1+\alpha)K_p + 2(1-\alpha)\frac{Kd}{T_k} + \alpha K_i T_k\right) \\ K_2^c &= \alpha K_p + (1-\alpha) K_i T_k \end{aligned}$$

The parameter α is introduced to solve possible numerical issues. Now, three LMI feasibility problems following (15) can be proposed. Those equations admits some variations.

Indeed, the overall control design can be proposed as a LMI, [19]. In fact, the control representation in the state-space has been selected to facilitate this possibility. In any case, the controllers 20 fulfill the first condition of Corollary 4.

The switching between the different controllers is facilitated for the use of the same structure in all the cases. When an event is received, T_k is known and the appropriated controller gains can be applied. A supervisor acts in parallel in order to prevent continuous switching between different control intervals (second condition in Corollary 4). A minimum dwell time can be established if necessary.

For a first implementation of the proposed control approach, the choice of designing just three different regions is considered enough for comparison purposes. In addition, obtained results show better performance comparing to a single controller solution. The values of parameters T_{min}, T_{max} and T_i $(i = 1, 2)$ comes from the fact that the obtained discrete controllers must be stable, but also the performance of the system must be maintained inside reasonable limits.

B. Simulations results

In order to perform a correct study under a simulation test, the hybrid nature of the overall system must be adequately described. In this case, the tool used is Ptolemy II, a Java based software system valid for the use of heterogeneous mixtures of models of computation, [20]. This software allows the definition of components for the event based scheme proposed in this paper, including switching between different controllers, in addition to the continuous plant.

Using this simulation framework, the performance of the system when using a multiple EBC controller scheme has been compared to a single EBC controller solution. Both solutions use the same parameter values when applying (14).

The results of the simulations show the evolution of the *Mean Sampling Time*, the *Mean Error*, defined as $\langle |r - y| \rangle$, and the *Event Ratio* (Event ratio is defined as the mean number of produced control events for a given time interval). As long as event occurrences increases, this value will also increase.

While most parameters are maintained constant, the integral part of the event triggering level has been varied in the range $K_i^s \in [50, 200]$ (Figures 3, 4 and 5).

As can be expected, increasing K_i^s, which rises the effect of the integral part of the event sampling, the number of event occurrences increases. Thus, more control actions are generated, reducing the system error.

The multi-controller case presents higher mean sampling time and lower error and event ratio. In addition, as Figures 6 and 7 show, the multi-controller scheme presents an improved system behavior, for same event sampling parameters ($K_p^s = 0.1$, $\delta_{limit} = 0.01$ and $K_i^s = 100$). The oscillations are reduced by switching to a more suitable controller for higher sampling times. Therefore, the use of multiple controllers allows a better definition of the closed-loop dynamics, adding more degree of freedom.

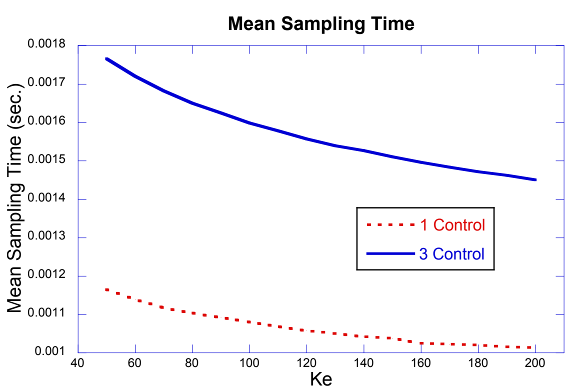

Figure 3. Mean Sampling Time vs parameter K_i^s, for both control schemes

A remarkable fact is that, in both cases, those results are qualitatively similar to the ones presented in previous works (e.g. [4]). They show a reduction in the needed resources, specially mean sampling time, without a notorious system performance reduction comparing to a pure periodic case.

Increasing the number of controllers, i. e., increasing the number of ranges in (19), the performance of the system is improved, but it also requires a more complex implementation. For the system under control, presented results are considered good enough and the addition of extra regions does not give significant advantages.

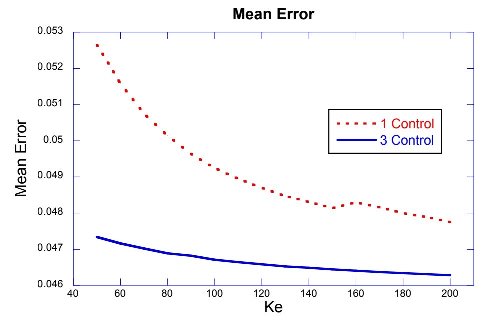

Figure 4. Mean Error vs parameter K_i^s, for both control schemes

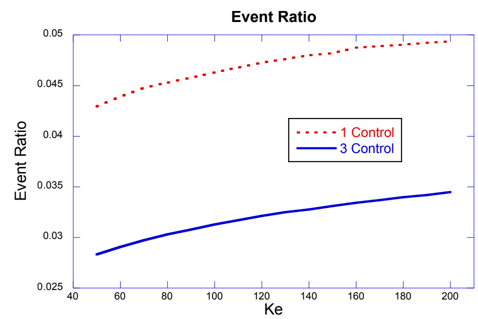

Figure 5. Event Ratio vs parameter K_i^s, for both control schemes

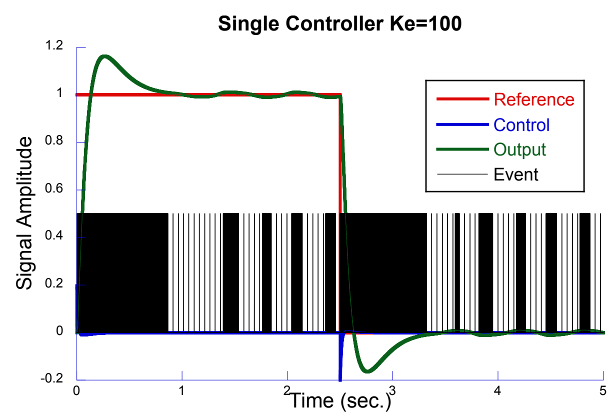

Figure 6. System output and event occurrences for single controller

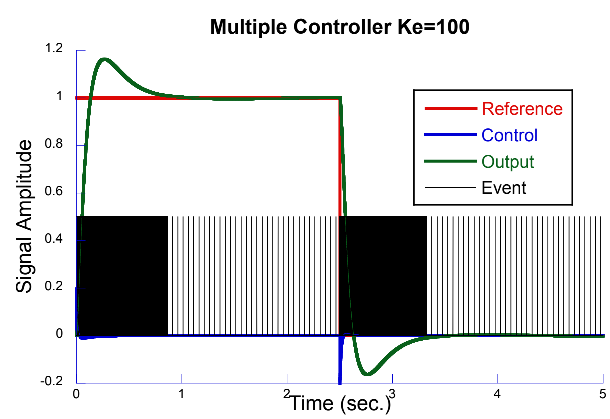

Figure 7. System output and event occurrences for multiple controller

VI. CONCLUSIONS

In this work, a design methodology for a class of event based controllers has been presented. The stability of the closed-loop system is guaranteed by applying ADS theory and solving several LMI problems Moreover, the proposed scheme allows the use of different controllers depending on the event rate, leading to a better closed-loop dynamics definition. So, desired dynamics specifications can be met more efficiently.

The principal advantage of the resulting closed-loop system is that the EBC control structure lets a reduced control rate. This characteristic makes the proposed scheme very adequate for networked environments, for saving energy and for reducing actuators' stress. In fact, this methodology has been chosen since LMI and ADS theory have been successfully applied to NCS systems in the literature. Therefore, the extension of the approach for EBC systems in a networked environment will follow a natural way.

In addition, the simulations in the example show that an improved system performance can be obtained by designing different controllers for several sampling time intervals, as opposed to use a single controller independent of the sampling period of each control action. The reduction of the number of control actions, comparing with a periodic sampling scheme, is still guaranteed. However, the improvement of the dynamics obtained with multiple EBC controllers requires, in general, an increased event rate, comparing with the single EBC controller case.

Future work will consist in the extension of the presented control strategy to a networked environment.

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REFERENCES

[1] P. Otanez, J. Moyne, and D. Tilbury, "Using deadbands to reduce communication in networked control systems," in *American Control Conference, 2002. Proceedings of the 2002*, vol. 4, pp. 3015 – 3020 vol.4, 2002.

[2] K. J. Astrom, *Analysis and design of nonlinear control systems*, ch. Event based control. Springer Verlag, 2007.

[3] X. Wang and M. Lemmon, "Event design in event-triggered feedback control systems," in *Proceedings of IEEE Conference on Decision and Control 2008*, vol. 1, December 2008.

[4] T. Henningsson, E. Johannesson, and A. Cervin, "Sporadic event-based control of first-order linear stochastic systems," *Automatica*, vol. 44, no. 11, pp. 2890 – 2895, 2008.

[5] M. Rabi and J. S. Baras, "Level-triggered control of a scalar linear system," in *Proceedings of 15th IEEE Mediterranean Conference on Control and Automation*, vol. 1, (Athens, Greece), June 27-29 2007.

[6] J. Jugo and M. Eguiraun, "Experimental implementation of a networked input-output sporadic control system," in *Proceedings of the 2010 IEEE International Conference on Control Applications*, pp. 1779 – 1784, September 2010.

[7] S. Durand and N. marchand, "Further results on event-based pid controller," in *Proceedings of European Control Conference 2009*, vol. 1, (Budapest), 2009.

[8] V. Vasyutynskyy and K. Kabitzsch, "Deadband sampling in pid control," in *Industrial Informatics, 2007 5th IEEE International Conference on*, vol. 1, pp. 45 –50, 2007.

[9] J. H. S. W. P. M. H. Heemels and P. P. J. van den Bosch, "Analysis of event-driven controllers for linear systems," *Int. Journal of Control*, vol. 81, no. 4, pp. 571–590, 2008.

[10] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2008.

[11] D. Y. Songlin Hu, "Event-triggered control design of linear networked systems with quantizations," *ISA Transactions*, vol. 51, pp. 153–162, January 2012.

[12] D. Lehmann and J. Lunze, "Extension and experimental evaluation of an event-based state-feedback approach," *Control Engineering Practice*, vol. 19, no. 2, pp. 101 – 112, 2011.

[13] M. Donkers and W. Heemels, "Output-based event-triggered control with guaranteed l_∞-gain and improved event-triggering," in *Decision and Control, 2010., Proceedings of the 49th IEEE Conference on*, pp. 3246–51, abendua 2010.

[14] S. Boyd, L. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. SIAM Society for Industrial and Applied Mathematics, 1994.

[15] A. Hassibi, S. Boyd, and J. How, "Control of asynchronous dynamical systems with rate constraints on events," in *Decision and Control, 1999. Proceedings of the 38th IEEE Conference on*, vol. 2, pp. 1345 –1351 vol.2, 1999.

[16] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Control Systems Magazine*, vol. 21, pp. 84–99, 2001.

[17] M. Á. G. José Sánchez and S. Dormido, "On the application of different event-based sampling strategies to the control of a simple industrial process," *Sensors*, vol. 9, pp. 6795–6818, 2009.

[18] D. Liberzon, *Switching in Systems and Control*. Birkhauser, 2003.

[19] P. G. Carsten Scherer and M. Chilali, "Multiobjective output-feedback control via lmi optimization," *IEEE Transactions on Automatic Control*, vol. 42, pp. 896–911, July 1997.

[20] *Ptolemy II, http://ptolemy.eecs.berkeley.edu/ptolemyII/index.htm.*