

A Robust LPV Fault Detection Approach Using Parametric Eigenstructure Assignment

Fengming Shi and Ron J. Patton

Department of Engineering
University of Hull
Hull, HU6 7RX, UK

shi_fengming@163.com and R.J.Patton@hull.ac.uk

Abstract --In this paper, an eigenstructure assignment fault detection approach to linear time invariant (LTI) systems is extended to Linear Parameter Varying (LPV) systems. Fault detection filter design algorithms using eigenstructure assignment have been widely studied for LTI systems. However, LPV strategies are very useful for systems which have no unique equilibrium and are difficult to linearize. The parametric eigenstructure assignment approach is used to design an observer as a residual generator by viewing the varying parameters as fixed parameters in the design procedure. The residual observer feedback structure is implemented using a measured scheduling parameter. An example is given of actuator fault detection of a two-link manipulator system.

Key words: *Fault detection; LPV systems; Eigenstructure assignment, Fault residual generation*

I. INTRODUCTION

Safety and reliability are very important in control systems and these demanding requirements must be ensured at a reasonable level. Fault detection (FD) methodologies and techniques are important topics in systems engineering from the viewpoint of improving plant safety and reliability. The FD literature is vast and the topics addressed are essentially related to the different design methodologies proposed to tackle the FD problem [1-4]. Model-based FD techniques are the most popular and are receiving considerable attention. The ideas are to derive a mathematical model of the plant and to compute additional artificial signals that are checked, during the on-line operations, with the corresponding measured quantities. State observers are often considered as the role of on-line residual generation because of the fast detection rate [5].

The eigenstructure assignment approach to robust FD was first demonstrated in [6]. It has been shown that a well-defined residual signal can be completely de-coupled from the disturbance by assigning a suitable eigenstructure to an observer. In this way, robust fault detection is achieved. Parametric eigenstructure assignment approaches [7-14] opened a wide field to use the design freedom of eigenvalue placement to achieve other desired performance, such as structured disturbance decouple. Some optimization approaches were also considered in the FD methods with pole placement [15-17].

Many real systems cannot be modeled by linear models, for example when no unique equilibria exist. A feasible approach to handle the nonlinearity of such systems is to use linear parameter-varying (LPV) models to approximate the dynamic nonlinearity. The LPV strategy was first introduced in [18, 19] and the big advantage of LPV modeling is that powerful linear design tools for stability and performance can be extended and applied [20, 21], LMI methods for multiple-model FD have been studied in [22, 23]. Traditional multi-model eigenstructure assignment approaches use iterative methods based on optimizing the worst case performance and the initial condition is calculated e.g. by a linear quadratic regulator (LQR) or by an H infinity method [24, 25].

In this paper, a non-iterative robust fault detection approach is presented based on a state observer structure within the LPV framework. The LPV fault detection approach is an extension of the approach in LTI case. Using parametric eigenstructure assignment, the varying parameters are viewed as fixed parameters in the design procedure and the observer law is implemented with the varying parameters measured or estimated on line.

The remainder of the paper is organized as follows: Section II recalls the parametric eigenstructure assignment approach to LPV systems and FD approach for LTI system. A design procedure for robust FD is also proposed in this Section; Section III demonstrates the usefulness of the proposed approach by a means of a two-link manipulator example. Conclusions are given in Section IV.

II. PARAMETRIC EIGENSTRUCTURE ASSIGNMENT TO LPV SYSTEMS

Consider a stable LPV system in the following form:

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) + B_f(\theta(t))f(t) + Ed(t) \quad (1)$$

$$y(t) = C(\theta(t))x(t) + D(\theta(t))u(t), \quad (2)$$

where $x(t) \in R^n$, $u(t) \in R^r$ and $y(t) \in R^m$ are the state vector, the input vector and measured output vector, respectively. And $f(t)$ and $d(t)$ are the fault vector and disturbance signal, respectively. E is a known constant matrix and $A(\cdot), B(\cdot), C(\cdot), D(\cdot), B_f(\cdot)$ are known

continuous functions of a time-varying parameter vector $\theta(t)$ which satisfies:

$$\theta(t) = [\theta_1(t), \dots, \theta_{n_\theta}(t)]^T \in \Theta, \forall t \geq 0$$

where Θ is a compact set. Hereafter, the subscript t is omitted without causing confusion.

The observer dynamics used by the residual generator are described by:

$$\left. \begin{aligned} \dot{\hat{x}} &= (A(\theta) - K(\theta)C(\theta))\hat{x} + (B(\theta) - K(\theta)D(\theta))u + K(\theta)y \\ \hat{y} &= C(\theta)\hat{x} + D(\theta)u \\ r &= Q(\theta)(y - \hat{y}) \end{aligned} \right\} \quad (3)$$

where $r \in R^p$ is the residual vector, \hat{x} and \hat{y} are state and output estimation vectors. The matrix $Q(\theta) \in R^{p \times m}$ is the residual weighting factor.

A. LPV Parametric Eigenstructure assignment

Observer design is dual of state feedback controller design. Left Eigenstructure assignment of observer is dual of the right Eigenstructure assignment of state feedback controller. That means if the desired left eigenvector matrix of $A(\theta) - K(\theta)C(\theta)$ is $R^T(\theta)$, then the desired right Eigenvector matrix of $A^T(\theta) - C^T(\theta)K^T(\theta)$ is $R(\theta)$. That is to say the desired left Eigenvector of $A(\theta) - K(\theta)C(\theta)$ can be assigned by assigning the right eigenvector of $A^T(\theta) - C^T(\theta)K^T(\theta)$. By the definition, if the right eigenvector matrix of $A^T(\theta) - C^T(\theta)K^T(\theta)$ is $R(\theta)$, the eigenvalue matrix is $F(\theta)$. It follows that:

$$A^T(\theta)R(\theta) - C^T(\theta)K^T(\theta)R(\theta) = R(\theta)F(\theta) \quad (4)$$

Let $K^T(\theta)R(\theta) = W(\theta)$, Equation (4) can be rewritten as

$$A^T(\theta)R(\theta) - C^T(\theta)W(\theta) = R(\theta)F(\theta) \quad (5)$$

Hence, the problem to assign desired closed-loop eigenstructure to a system using a residual generator is to find a solution of (4).

Now, without proof, a theorem of parametric solution of Sylvester equation is introduced, and the proof details can be found in [8, 9, 26].

Theorem 1

Let $[A(\theta) B(\theta)]$ be controllable, and the matrix $B(\theta)$ be of full-column rank. If the desired closed-loop self-conjugate eigenvalue set be described as $\Lambda = \{\lambda_i(\theta): \lambda_i(\theta) \in \mathbb{C}, i = 1, 2, \dots, \tilde{n}, 1 \leq \tilde{n} \leq n\}$. The algebraic and geometric multiplicities of the eigenvalue λ_i are denoted by q_i and r_i , respectively and p_{ij} , q_i and r_i satisfy the relations:

$$\sum_{j=1}^{r_i} p_{ij} = q_i, \sum_{i=1}^{\tilde{n}} q_i = n$$

Then all the solutions of the Sylvester matrix equation [10]:

$$A(\theta)R(\theta) + B(\theta)W(\theta) = R(\theta)F(\theta)$$

are given by:

$$\begin{bmatrix} R_{ij}^k \\ W_{ij}^k \end{bmatrix} = \begin{bmatrix} N(\theta, \lambda_i(\theta)) & \dots & \frac{1}{(k-1)!} \frac{d^{k-1}}{d\lambda^{k-1}} N(\theta, \lambda_i(\theta)) \\ M(\theta, \lambda_i(\theta)) & \dots & \frac{1}{(k-1)!} \frac{d^{k-1}}{d\lambda^{k-1}} M(\theta, \lambda_i(\theta)) \end{bmatrix} \begin{bmatrix} f_{ij}^k(\theta) \\ \vdots \\ f_{ij}^1(\theta) \end{bmatrix} \quad (6)$$

$$k = 1, 2, \dots, p_{ij}, j = 1, 2, \dots, q_i, i = 1, 2, \dots, \tilde{n}'$$

where the $f_{ij}^k \in \mathbb{C}^r$ are arbitrarily chosen from parameter vectors. $N(\theta, \lambda(\theta))$ and $M(\theta, \lambda(\theta))$ are right co-prime matrix polynomials satisfying:

$$[\lambda(\theta)I - A(\theta)]^{-1}B(\theta) = N(\theta, \lambda(\theta))M^{-1}(\theta, \lambda(\theta)) \quad (7)$$

Then the observer gain can be calculated by:

$$K(\theta) = R^{-1}(\theta)W(\theta)$$

From the above theorem, it can be known that the desired eigenvectors and generalized eigenvectors can be parameterized by (6). By specially choosing the free parameters given in (6), solutions with desired properties can be obtained.

B. LPV fault detection

The FD design must ensure that the residuals are close to zero in the fault-free situation whilst suitably deviating from zero in the presence of faults. A necessary condition for achieving disturbance de-coupling design is [4, 14].

$$Q(\theta)C(\theta)E = H(\theta)E = 0$$

If $C(\theta)E = 0$, any residual weighting matrix can satisfy this necessary condition.

The basic principle to assign the left eigenstructure for LTI case is given in [4, 12]. The theorem is introduced here.

Theorem 2

The sufficient conditions for satisfying the disturbance de-coupling requirement $G_{rd} = QC(sI - A + KC)^{-1}Ed = 0$ are:

- (1) $QCE = 0$
- (2) All rows of the matrix $H = QC$ are left eigenvectors of $(A - KC)$ corresponding to any eigenvalues.

A similar result for the LPV case is now given.

Theorem 3

The sufficient conditions for satisfying the disturbance de-coupling requirements for the system.

$$G_{rd} = Q(\theta)C(\theta)(sI - A(\theta) + K(\theta)C(\theta))^{-1}Ed(s) = 0$$

are:

- (1) $Q(\theta)C(\theta)E = 0$
- (2) All rows of the matrix $H(\theta) = Q(\theta)C(\theta)$ are left eigenvectors of $(A(\theta) - K(\theta)C(\theta))$ corresponding to any eigenvalues.

Noting that the above result is intuitively an extension of the LTI case, the proof is omitted here.

C. Design procedure

Following the previous arguments, a design procedure is proposed to design a robust residual generator to LPV system.

Step 1: Select the desired eigenvalues for the observer which can be parametric to obtain more design freedoms.

Step 2: Calculate the $N(\theta, \lambda(\theta))$ and $M(\theta, \lambda(\theta))$ using elementary transformation and the rational matrix factorization method.

Step 3: Check $\text{rank}(C(\theta)E)$, choose a basis for $\text{LKer}(C(\theta)E)$.

Step 4: Check $Q(\theta)C(\theta)$, set the desired *left* eigenvectors with some of the eigenvectors in a parametric form to keep the design freedom.

Step 5: Project the desired eigenvectors into the achievable subspace to get the achieved eigenvector matrix.

Step 6: Calculate the observer gain by $K(\theta) = W(\theta)R(\theta)^{-1}$. To simplify the structure, some parameters are chosen at this step.

Step 7: Verify the achieved eigenvalues and eigenvectors, and chose the remaining parameters based on the performance specifications.

III. AN EXAMPLE

A two-link robotic manipulator is considered to rotate in the vertical plane, whose position can be described by a 2-vector $\varphi = (\varphi_1, \varphi_2)^T$ of joint angles, and whose actuator inputs consist of a vector $u = (u_1, u_2)^T$ of torques applied at the manipulator joints as shown in Fig. 1. Using the vectors $\dot{\varphi}$ and $\ddot{\varphi}$ to denote the joint velocities and accelerations, respectively. The dynamics of this simple manipulator can be written in the more general form [27] as:

$$\Xi(\varphi)\ddot{\varphi} + O(\varphi, \dot{\varphi})\dot{\varphi} + g(\varphi) = u$$

where: $\Xi(\varphi) \in R^{2 \times 2}$ is the manipulator inertia tensor matrix, $O(\varphi, \dot{\varphi})\dot{\varphi} \in R^2$ is the vector function containing the Centripetal and Corioles torques, i.e. $O(\varphi, \dot{\varphi}) \in R^{2 \times 2}$ and $g(\varphi) \in R^2$ are the gravitational torques. The details of equations of motion and physical parameters as outlined in Table I are described in [23].

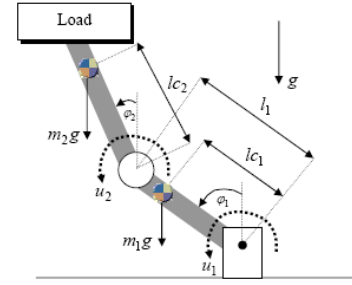


Figure 1. Two-link manipulator structure

TABLE I. PARAMETER VALUES FOR THE TWO LINK MANIPULATOR SYSTEM

Parameters	I_1	I_2	l_1	lc_1	lc_2	m_1	m_2	g
Values	0.833	0.417	1.0	0.5	0.5	10.0	5.0	9.80
Units	$\text{Kg} \cdot \text{m}^2$	$\text{Kg} \cdot \text{m}^2$	m	m	m	Kg	Kg	m/s^2

A. LPV model of two link manipulator

A polytopic LPV representation of this model system is used in this example. It is important to note that in this study the quadratic terms $O(\varphi, \dot{\varphi})$ are not considered because they are not bounded. As shown later by the Simulink result, it turns out that the two-link manipulator works well, even if these bounds are not known *a priori*. Considering this limitation the system dynamics can be described as:

$$\Xi(\varphi)\ddot{\varphi} + g(\varphi) = u$$

where

$$\Xi(\varphi) = \begin{bmatrix} m_{11} & m_{12} \cos(\varphi_1 - \varphi_2) \\ m_{21} \cos(\varphi_1 - \varphi_2) & m_{22} \end{bmatrix}$$

$$g(\varphi) = \begin{bmatrix} k_{11} \sin(\varphi_1) \\ k_{12} \sin(\varphi_2) \end{bmatrix}$$

$$m_{11} = m_1 lc_1^2 + m_2 l_1^2 + I_1, m_{12} = m_2 l_1 lc_2,$$

$$m_{21} = m_2 l_1 lc_2, m_{22} = m_2 lc_2^2 + I_2$$

$$k_{11} = -[m_1 lc_1 + m_2 l_1]g, k_{12} = -m_2 g lc_2$$

The nonlinear term in $\Xi(\varphi)$ is clearly a bounded function:

$$\rho_1 = \cos(\varphi_1 - \varphi_2), -1 \leq \rho_1 \leq 1$$

To facilitate a state-space formulation, the vector field $g(\varphi)$ with $\varphi \in R^2$ can be rearranged in the form of $G(\varphi)\varphi$ and function $\phi_2(\varphi)$ can now be defined which is bounded,

$$\sin(\varphi_1) = \left(\frac{\sin(\varphi_1)}{\varphi_1} \right) \varphi_1 = \rho_2(\varphi) \cdot \varphi_1$$

$$-0.2 \leq \rho_2 \leq 1.$$

Similarly,

$$\sin(\varphi_2) = \left(\frac{\sin(\varphi_2)}{\varphi_2} \right) \varphi_2 = \rho_3(\varphi) \cdot \varphi_2$$

$$-0.2 \leq \rho_3 \leq 1$$

To define the two-link system state space representation, let:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} \text{ and } W_b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The LTV state space equation is as follows:

$$\dot{x} = A(\varphi)x + B(\varphi)u + Ed + R_1f$$

where

$$A(\varphi) = \Pi^{-1} \begin{bmatrix} 0 & I \\ -G(\varphi) & 0 \end{bmatrix}, B(\varphi) = \Pi^{-1}W_b,$$

$$\Pi = \begin{bmatrix} I & 0 \\ 0 & \Xi(\varphi) \end{bmatrix}, \Xi(\varphi) = \begin{bmatrix} m_{11} & m_{12}\rho_1 \\ m_{21}\rho_1 & m_{22} \end{bmatrix}$$

$$G(\varphi) = \begin{bmatrix} k_{11}\rho_2 & 0 \\ 0 & k_{12}\rho_3 \end{bmatrix}$$

$$-1 \leq \rho_1 \leq 1, -0.2 \leq \rho_2 \leq 1, -0.2 \leq \rho_3 \leq 1$$

Assume that only $\varphi_1(t)$ and $\varphi_2(t)$ are measurable, so that:

$$C(\varphi) = C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Assume further that the system is disturbed by a zero-mean Gaussian random disturbance $d(t)$ with variance magnitude and with disturbance distribution vector:

$$E = [1 \ 0 \ 0 \ 0]^T$$

The considered fault is an actuator fault acting on the second actuator, so that the fault distribution vector is

$$B_f = [0 \ 0 \ 0 \ 1]^T$$

The proof that Π is non-singular follows from $\Xi(\varphi)$. As Π is block diagonal, its determinant is given by $\Xi(\varphi)$. It is thus only required to show that $m_{11}m_{22} \neq m_{12}m_{21}$. $m_{12} = m_{21}$ (by symmetry) and $m_{11} > m_{22}$ since $I_1 > I_2$ and $m_1 > m_2$, hence Π is non-singular.

B. Observer based residual Generator Design

Following the proposed procedure, the residual generator design is shown in this subsection.

Step 1: The desired observer eigenvalues are set to be parametric to obtain more design freedom as:

$$\Lambda = \{\lambda_1 \ \lambda_2 \ \lambda_3 \ \lambda_4\}.$$

Step 2: Using elementary transformation and the rational matrix factorization method, the following are obtained:

$$N(\theta, s) = \begin{bmatrix} s & 0 \\ 0 & s \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$M(\theta, s) = \begin{bmatrix} s^2 + \frac{6.5346\rho_2}{0.0833 + \rho_1^2 - \rho_1} & \frac{-9.8(2\rho_1 - 1)\rho_2}{0.0833 + \rho_1^2 - \rho_1} \\ \frac{-2.45 * (2\rho_1 - 1)\rho_3}{0.0833 + \rho_1^2 - \rho_1} & s^2 + \frac{2.45\rho_3}{0.0833 + \rho_1^2 - \rho_1} \end{bmatrix}.$$

Step 3: Note that

$$CE = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

So $p = 1$, and a basis for $LKer(CE)$ may be taken as

$$\xi^T = [0 \ 1]$$

Hence $Q = [0 \ \alpha]$

Step 4: Then noting that:

$$QC = [0 \ \alpha \ 0 \ 0],$$

one desired left eigenvector is $[0 \ \alpha \ 0 \ 0]^T$ and other left eigenvectors can be chosen arbitrarily only to satisfy $\det(L) \neq 0$. So, other parameters are given in a parametric way from:

$$f_i^T = [x_{i1} \ x_{i2}], i = 2,3,4$$

Using the parametric eigenstructure assignment approach, the first desired eigenvector is projected into the allowable subspace by setting:

$$f_1^T = [N^T(\lambda_1)N(\lambda_1)]^{-1}N^T(\lambda_1)C^T\alpha\xi = \begin{bmatrix} 0 \\ \lambda_1 \\ \alpha \frac{\lambda_1}{\lambda_1^2 + 1} \end{bmatrix}$$

Step 5: The achieved eigenvector matrix can be obtained by (6) as given in **Theorem 1**.

Step 6: To simplify the calculation, some parameters are chosen at this step. If the parameters are chosen as:

$$x_{21} = 1, x_{22} = 0, x_{31} = 0,$$

$$x_{32} = 1, x_{41} = 0, x_{42} = 1.$$

Using $K(\theta) = W(\theta)R(\theta)^{-1}$. The observer gain is obtained as in (8).

$$K = \begin{bmatrix} (\lambda_3 + \lambda_2) & \frac{(\lambda_1 - \lambda_3)(\lambda_3 - \lambda_4)}{\lambda_2 - \lambda_3} \\ 0 & (\lambda_1 + \lambda_4) \\ -\lambda_3\lambda_2 + \frac{6.53446\rho_2}{K_o} & \lambda_3\lambda_2 + \frac{-2.45(2\rho_1 - 1)\rho_3}{K_o} \\ \frac{-9.8(2\rho_1 - 1)\rho_2}{K_o} & \lambda_1\lambda_4 + \frac{8.06642\rho_3}{K_o} \end{bmatrix} \quad (8)$$

$$K_o = (-0.764 + 2.5\rho_1^2 - 2.5\rho_1)$$

Step 7: The achieved closed-loop eigenvalues are:

$$\Lambda = \{\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4\}.$$

To stabilize the observer system, let $real(\lambda_i) < 0, i = 1, 2, 3, 4$. The required transient response performance can be achieved by suitably choosing the parametric eigenvalues.

The achieved transfer function matrix between the residual and disturbance is:

$$G_{rd}(s) = [0]$$

It is apparent that the disturbance is completely decoupled because the transfer function matrix between the residual and disturbance is zero.

The transfer function between the residuals and faults is:

$$G_{rf}(s) = \frac{\alpha}{s^2 - (\lambda_1 + \lambda_4)s + \lambda_1\lambda_4}$$

So, the steady-state gain matrix $G_{rf}(0)$

$$G_{rf}(0) = \frac{\alpha}{\lambda_1\lambda_4}$$

From the above it is easy to choose suitable values of λ_1 and λ_4 to achieve a desired transient response and set $\alpha = \lambda_1\lambda_4$ to obtain good steady-state fault estimation.

The above analysis shows that the disturbances are decoupled completely and the residual is sensitive to the fault. This implies that the proposed design approach has achieved the desired goals. In the next subsection, some Simulink results are given.

C. Simulation result

The open-loop two-link manipulator is unstable. Therefore, a constant controller is designed first using an observer state feedback structure while the estimated state is provided by the designed residual generator. The LPV system is simulated with a step and sinusoidal signals as shown in Figs. 2 & 3. Noting that the initial estimation is not good because both the two-link manipulator and observer systems are not in steady state and the state estimation error is large during the transient phase. The fault estimation is close to the real fault signal after 2 seconds as shown both in Figs. 2 & 3. The LPV fault estimator can provide good estimation performance when the real system is in a steady state.

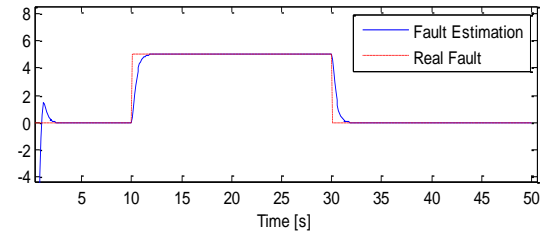


Figure 2. Fault estimation result for step fault signal

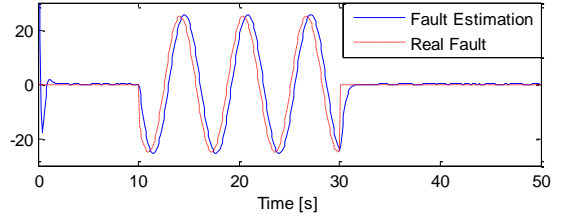


Figure 3. Fault estimation result for sinusoidal signal

IV. CONCLUSION

This paper proposes an LPV fault detection approach using eigenstructure assignment which is robust in the sense of disturbance decoupling. The disturbances which can be considered to represent modeling uncertainty can be completely decoupled if the disturbance distribution and output matrix satisfy a rank condition. If this is not the case the disturbance can be decoupled as much as possible by suitable choice of design freedom. A two-link manipulator case is studied to show the usefulness of the proposed design procedure. The Simulink results show that for one fault case, the designed residual generator works well. Future studies will be concerned with multi-fault cases and how to use the detected fault information to accommodate faults and improve the system reliability.

V. REFERENCES

- [1] R. J. Patton, "Robustness in model-based fault diagnosis: The 1995 situation," *Annual Reviews in Control*, vol. 21, pp. 103-123, 1997.
- [2] H. Inseok, K. Sungwan, K. Youdan, and C. E. Seah, "A Survey of Fault Detection, Isolation, and Reconfiguration Methods," *IEEE Trans. on Control Systems Technology*, vol. 18, pp. 636-653, 2010.
- [3] S. X. Ding, P. Zhang, T. Jeansch, E. L. Ding, P. Engel, and W. Gui, "A survey of the application of basic data-driven and model-based methods in process monitoring and fault diagnosis," in *IFAC World Congress*, Milano, Italy, 2011.
- [4] J. Chen and R. J. Patton, *Robust model-based fault diagnosis for dynamic systems*. London: Kluwer Academic Publishers, 1999.
- [5] Y. Zhang and J. Jiang, "Bibliographical review on reconfigurable fault-tolerant control systems," *Annual Reviews in Control*, vol. 32, pp. 229-252, 2008.
- [6] R. J. Patton and J. Chen, "Robust fault detection using eigenstructure assignment: a tutorial consideration and some new results," in the *30th IEEE Conference on Decision and Control*, 1991, pp. 2242-2247.
- [7] G. Duan, G. W. Irwin, and G. Liu, "Disturbance attenuation in linear systems via dynamical compensators: a parametric eigenstructure assignment approach," *IEE Proceedings on Control Theory and Applications*, vol. 147, pp. 129-136, 2000.
- [8] G. Cai, C. Hu, and G. Duan, "Eigenstructure assignment for linear parameter-varying systems with applications," *Mathematical and Computer Modelling*, vol. 53, pp. 861-870, 2011.

- [9] G. Duan, "Solution to matrix equation $AV + BW = EVF$ and eigenstructure assignment for descriptor systems," *Automatica*, vol. 28, pp. 639-642, 1992.
- [10] G. R. Duan, "Solutions of the equation $AV+BW=VF$ and their application to eigenstructure assignment in linear systems," *IEEE Trans. on Automatic Control*, vol. 38, pp. 276-280, 1993.
- [11] G.-R. Duan, "On the solution to the Sylvester matrix equation $AV+BW=EVF$," *Automatic Control, IEEE Transactions on*, vol. 41, pp. 612-614, 1996.
- [12] R. J. Patton and J. Chen, "On eigenstructure assignment for robust fault diagnosis," *International Journal of Robust and Nonlinear Control*, vol. 10, pp. 1193-1208, 2000.
- [13] G. P. Liu and R. J. Patton, "Parametric state feedback controller design of multivariable systems," *International Journal of Control*, vol. 61, pp. 1457-1464, 1995/06/01 1995.
- [14] G. P. Liu and R. Patton, *Eigenstructure Assignment for Control System Design*: John Wiley& Sons, Inc., 1998.
- [15] H. Wang, J. Wang, and J. Lam, "Worst-Case Fault Detection Observer Design: Optimization Approach," *Journal of Optimization Theory and Applications*, vol. 132, pp. 475-491, 2007.
- [16] H. Wang and J. Lam, "Robust fault detection for uncertain discrete-time systems " *Journal of Guidance, Control and Dynamics*, vol. 25, pp. 291-301, 2002.
- [17] H. Wang, J. Wang, and J. Lam, "An optimization approach for worst-case fault detection observer design," in *American Control Conference*, 2004.
- [18] J. S. Shamma and M. Athans, "Analysis of gain scheduled control for nonlinear plants," *IEEE Transactions on Automatic Control*, vol. 35, pp. 898-907, 1990.
- [19] W. J. Rugh, "Analytical framework for gain scheduling," *IEEE Trans. on Control Systems*, vol. 11, pp. 79-84, 1991.
- [20] W. J. Rugh and J. S. Shamma, "Research on gain scheduling," *Automatica*, vol. 36, pp. 1401-1425, 2000.
- [21] D.J.Leith and W. E.Leithead, "Survey of Gain-Scheduling Analysis and Design," *Journal of Control*, vol. 73, pp. 1001--1025, 1999.
- [22] L. Chen and R. J. Patton, "Polytope LPV estimation for non-linear flight control," presented at the World Congress, Milano, Italy, 2011.
- [23] R. Patton, L. Chen, and S. Klinkhieo, "An LPV pole-placement approach to friction compensation as an FTC problem," *J. of Applied Mathematics and Computer Science*, vol. 22, pp. 149-160, 2012.
- [24] J. Magni, "Multimodel eigenstructure assignment in flight-control design," *Aerospace Science and Technology*, vol. 3, pp. 141-151, 1999.
- [25] C. Döll, Y. Le Gorrec, G. Ferreres, and J. F. Magni, "A robust self-scheduled missile autopilot: design by multi-model eigenstructure assignment," *Control Engineering Practice*, vol. 9, pp. 1067-1078, 2001.
- [26] G. Duan, G. Wu, and W. Huang, "Eigenstructure Assignment for Linear Time-Varying Systems," *Science in China (Series A, English Edition)*, vol. 34, pp. 246-256, 1991.
- [27] S. B. Niku, *Introduction to Robotics: Prentice Hall Professional Technical Reference*, 2001.