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We would like you to consider this paper for submission in Computers and Chemical Engineering.

Best regards Sigurd Skogestad and Jrgen B. Jensen

Optimal operation of simple cooling cycles

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Abstract

In this paper we consider the selection of controlled variables for two vapour compression cycles. One is a conventional sub-critical ammonia cooling cycle and the other is a trans-critical CO_2 cooling cycle. The two cycles have quite different operational properties. For the ammonia cycle we find that several simple control structures gives acceptable performance in terms of achieving in practice the optimal thermodynamic efficiency. For the CO_2 cycle on the other hand, a combination of measurements is necessary to achieve self-optimizing control.

Key words: Operation, self-optimizing control, vapour compression cycle

1 Introduction

Vapour compression cycles are used both in homes, cars and in industry. The load and complexity varies, from small simple cycles, like a refrigerator or air-conditioner, to large complex industrial cycles, like the ones used in liquefaction of natural gas.

The simple cooling process illustrated in Figure 1 is studied in this paper. We will consider a) a conventional sub-critical ammonia cycle for cold storage ($T_C = -10\,°C$) and b) a trans-critical CO_2 cycle for cooling a home ($T_C = 20\,°C$). In the CO_2 cycle there is no saturation condition on the high pressure side and this is usually said to introduce one extra degree of freedom to the cycle (Kim et al., 2004). However, as shown by Jensen and Skogestad (2006) this "extra" degree of freedom is also available in a conventional sub-critical cycle if we allow for sub-cooling in the condenser. The sub-cooling will to some

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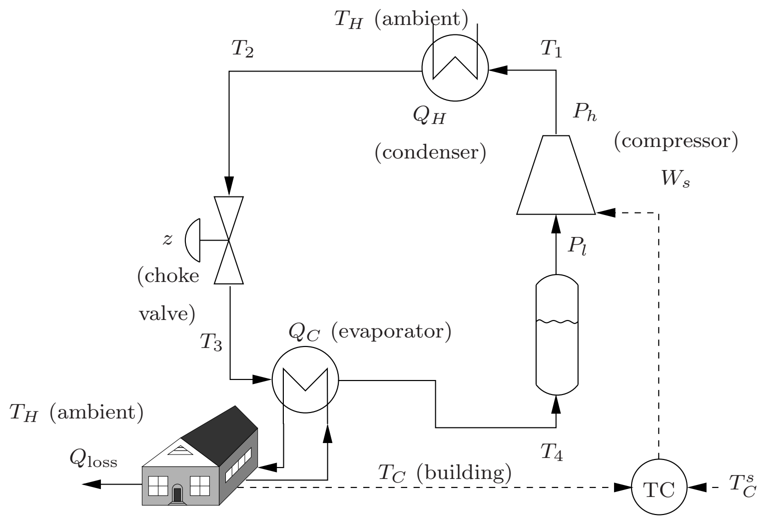
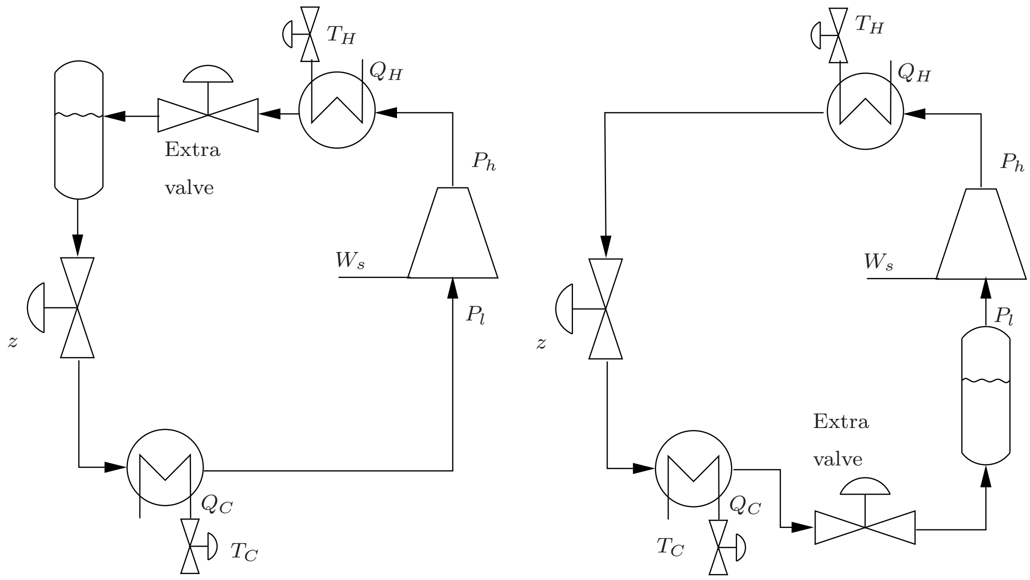


Fig. 1. Simple vapour compression cycle studied in this paper (shown for the ammonia case)



(a) Liquid receiver on high pressure side

(b) Liquid receiver on low pressure side (note that the extra valve is not optimal for this design)

Fig. 2. General vapour compression cycle with 5 manipulated variables (Jensen and Skogestad, 2006)

extent decouple the outlet temperature and the saturation pressure. More importantly, some sub-cooling is actually positive in terms of thermodynamic efficiency (Jensen and Skogestad, 2006).

Although there is a vast literature on the thermodynamic analysis of closed heating/cooling cycles, there are few authors who discuss the operation and control of such cycles. Some discussions are found in text books such as

Stoecker (1998), Langley (2002) and Dossat (2002), but these mainly deal with more practical aspects. Svensson (1994) and Larsen et al. (2003) discuss operational aspects. A more comprehensive recent study is that of Kim et al. (2004) who consider the operation of trans-critical CO_2 cycles.

This paper considers steady state operation, as this is most important for the operational costs, and the objective is to find which controlled variables to fix. The compressor power is used as the objective function (cost $J = W_s$) for evaluating optimal operation.

2 Degree of freedom analysis

In Figure 2 we show two general vapour compression cycles with a liquid receiver placed either at the a) high pressure side or b) low pressure side. From a control point of view, the two alternative configurations have five degrees of freedom (manipulated inputs):

(1) Compressor power W_s
(2) Choke valve opening z
(3,4) Effective heat transfer (UA) in each of the two heat exchangers (in Figure 2 shown to be adjusted by flow of hot and cold utility respectively, but could also use bypass, adjustable fans etc.)
(5) The "extra" valve placed either after the condenser (Figure 2(a); this valve is necessary to have sub-cooling with a high pressure liquid receiver) or after the evaporator (Figure 2(b); this valve is generally not optimal for simple cycles (Jensen and Skogestad, 2006))

Our cycle (Figure 1) has a liquid receiver on the low pressure side, and compared to Figure 2(b), four of the five inputs are at constraints or are already used for control purposes:

- W_s is indirectly given by the action of temperature controller which keeps a constant room temperature
- Maximum heat transfer (UA) is assumed in both heat exchangers
- There is no "extra" valve after the evaporator

We are then left with one degree of freedom (choke valve opening z) that should be used to optimize the operation. Note also that the liquid receiver after the evaporator gives no super-heating ($\Delta T_{\mathrm{sup}} = 0\,°\mathrm{C}$), which is optimal from a thermodynamic point of view.

3 Selection of controlled variable

We have one unconstrained degree of freedom (z) that should be used to optimize operation for all disturbances and operating points. We could envisage an on-line optimization scheme where one continuously optimizes the operation (minimize compressor power) by adjusting the valve (z). However, such schemes are quite complex and sensitive to uncertainty, so in practice one uses simpler schemes, where the valve is used to control some other variable. What should be controlled? Some candidate controlled variables to keep fixed are:

- Valve position z (that is, an open-loop policy where the valve is left in a constant position)
- High pressure (P_h)
- Low pressure (P_l)
- Temperature out of compressor (T_1)
- Temperature before valve (T_2)
- Degree of sub-cooling in the condenser [1] ($\Delta T_{\text{sub}} = T_2 - T_{sat}(P_h)$)
- Temperature approach in hot source heat exchanger ($T - T_H$)
- Temperature out of evaporator (T_4)
- Degree of super-heating in the evaporator ($\Delta T_{\text{sup}} = T_4 - T_{sat}(P_l)$)
- Liquid level in the receiver (V_l) to adjust the active charge in the rest of the system
- Liquid level in the condenser ($V_{l,con}$) or in the evaporator ($V_{l,vap}$)
- Pressure drop across the "extra" valve in Figure 2 [2]

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation (Skogestad, 2000). The selection of controlled variables is a challenging task if one should consider all possible measurements, so we will first use a simple screening process based on a linear model.

3.1 Linear analysis

To find promising controlled variables, the "maximum gain" rule (Halvorsen et al., 2003) will be used:
Prefer controlled variables with a large scaled gain $|G'|$ from the input (degree of freedom) to the output (controlled variable)
Procedure:

(1) Make a small perturbation in all disturbances (same fraction of expected

[1] Not relevant in the CO_2 cycle because of super-critical high pressure
[2] Not relevant for our design (Figure 1)

disturbance) and re-optimize the operation for each disturbance to find the optimal change in all variables ($\Delta y_{\text{opt}}(d_i)$). Compute from this the overall optimal variation:
$\Delta y_{\text{opt}} = \sqrt{\sum_{d_i} (\Delta y_{\text{opt}}(d_i))^2}$

(2) Scale with respect to the inputs such that all the inputs have equal effect on the objective function (not necessary in this case since there is only one manipulated input)
(3) Make a perturbation in the independent variables (u) to find the unscaled gain ($G = \frac{\Delta y}{\Delta u}$).
(4) Scale the gain with the optimal span $y = \Delta y_{\text{opt}} + n$, where n is implementation error, to obtain the scaled gain:
$G' = \frac{G}{\text{span } y}$

The worst-case loss $L = J(u,d) - J_{\text{opt}}(u,d)$ (the difference between the cost with a constant setpoint and re-optimized operation) is then (Skogestad and Postlethwaite, 2005, page 394):

$$L = \frac{\alpha}{2} \frac{1}{|G'|^2} \tag{1}$$

where $\alpha = |J_{uu}|$ is the Hessian of the cost function J. In our case $J = W_s$ (compressor work).

The most promising controlled variables must then be tested on the non-linear model using realistic disturbances to check for non-linear effects, including feasibility problems.

3.2 Combination of measurements

If the losses with a fixed single measurement is large, as for the CO_2 case study, then one may consider combinations of measurements as controlled variables. The simple null space method (Alstad and Skogestad, 2006) gives a linear combination with zero local loss for the considered disturbances,

$$c_{\text{combine}} = h_1 \cdot y_1 + h_2 \cdot y_2 + \dots \tag{2}$$

The minimum number of measurements y to be included in the combination is $n_y = n_u + n_d$. In our case $n_u = 1$ and if we want to consider combinations of $n_y = 2$ measurements then only $n_d = 1$ disturbance can be accounted for. With the extended null space method (Alstad and Skogestad, 2006) it is possible to consider additional disturbances. The loss is then not zero, and we will minimize the 2-norm of the effect of disturbances on the loss.

Table 1
Model equations

Heat exchangers
$Q = U \cdot \int \Delta T \, \mathrm{d}A = \dot{n} \cdot (h_{out} - h_{in})$
Valve
$\dot{n} = z \cdot C_V \sqrt{\Delta P \cdot \rho} \qquad h_{out} = h_{in}$
Compressor
$W_s = \dot{n}\,(h_{out} - h_{in}) = \dot{n}/\eta \cdot (h_s - h_{in})$

Table 2
Data for the ammonia case study

$T_H = 20\,°\mathrm{C}$
$T_C = T_C^s = -10\,°\mathrm{C}$
Condenser UA: $2500\,\mathrm{W\,K^{-1}}$
Evaporator UA: $3000\,\mathrm{W\,K^{-1}}$
Compressor: isentropic efficiency $\eta = 0.95$
Choke valve: $C_V = 0.0017\,\mathrm{m}^2$
Building: $UA_{\mathrm{loss}} = 500\,\mathrm{W\,K^{-1}}$

4 Ammonia case study

The cycle operates between air inside a building ($T_C = T_{\mathrm{room}}$) and ambient air ($T_H = T_{\mathrm{amb}}$). This could be used in a cold storage building as illustrated in Figure 1. The heat loss from the building is

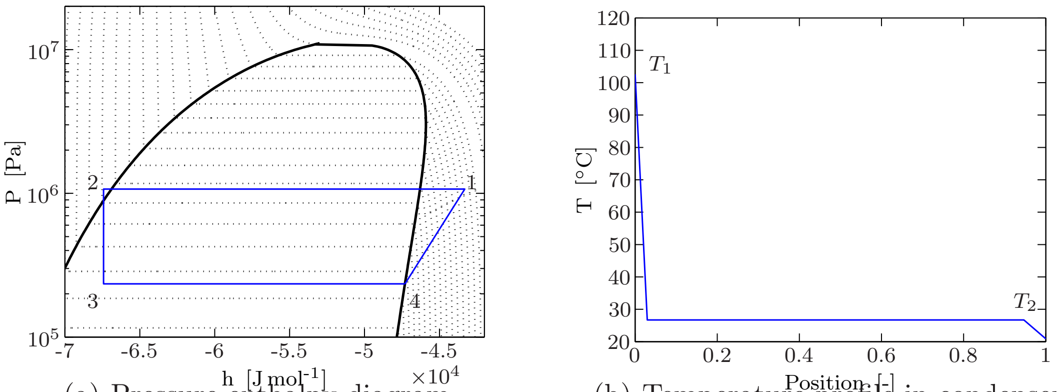
$$Q_{\mathrm{loss}} = UA_{\mathrm{loss}} \cdot (T_H - T_C) \tag{3}$$

The temperature controller shown in Figure 1 will indirectly give $Q_C = Q_{\mathrm{loss}}$. The nominal heat loss is 15 kW.

4.1 Modelling

The most important model equations are given in Table 1 and the data are given in Table 2. The heat exchangers are modelled assuming constant air temperature. The SRK equation of state is used for the thermodynamic calculations.

At nominal conditions the compressor power was minimized with respect to the degree of freedom (z). The optimal results are given in Table 3, and the corresponding pressure enthalpy diagram and temperature profile in the condenser are shown in Figure 3. Note that the sub-cooling out of the condenser is 5.8 °C. This saves about 2.0 % in compressor power (W_s) compared to the conventional design with saturation.



(a) Pressure enthalpy diagram (b) Temperature profile in condenser

Fig. 3. Optimal operation for the ammonia case study

Table 3
Optimal steady state for ammonia case study

W_s [kW]	2.975
z [-]	0.372
P_h [bar]	10.70
P_l [bar]	2.35
Q_H [kW]	17.96
$\dot{n}$ [mol s^{-1}]	0.745
ΔT_{sub} [°C]	5.80
T_1 [°C]	102.6
T_2 [°C]	20.9
T_3 [°C]	-15.0
T_4 [°C]	-15.0

4.3 Selection of controlled variables

We want to find a good controlled variable (see Section 3 for candidates) to keep fixed by adjusting the choke valve opening z.

4.3.1 Linear analysis of alternative controlled variables

The following disturbance perturbations[3] are used to calculate the optimal variation in the measurements y.

d_1: $\Delta T_H = \pm 10\,°\mathrm{C}$
d_2: $\Delta T_C^s = \pm 5\,°\mathrm{C}$
d_3: $\Delta UA_{\mathrm{loss}} = \pm 100\,\mathrm{W\,K^{-1}}$

The assumed implementation error (n) for each variable is given in Table 4 which summarizes the linear analysis and gives the resulting scaled gains in order from low gain (poor) to high gain (promising).

Table 4
Linear "maximum gain" analysis of controlled variables for ammonia case

			$\|\Delta y_{\mathrm{opt}}(d_i)\|$						
Variable (y)	Nom.	G	d_1 (T_H)	d_2 (T_C)	d_3 (UA_{loss})	$\|\Delta y_{\mathrm{opt}}\|$	n	span y	$\|G'\|$
P_l [bar]	2.35	0.00	0.169	0.591	0.101	0.623	0.300	0.923	0.00
T_4 [°C]	-15.0	0.00	0.017	0.058	0.010	0.061	1.00	1.06	0.00
ΔT_{sup} [°C]	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
T_1 [°C]	102.6	-143.74	38	17.3	6.2	42.2	1.00	43.2	3.33
P_h [bar]	10.71	-17.39	4.12	0.41	0.460	4.17	1.00	5.17	3.37
z [-]	0.372	1	0.0517	0.0429	0.0632	0.092	0.05	0.142	7.03
T_2 [°C]	20.9	287.95	10.4	0.20	0.300	10.4	1.00	11.4	25.3
V_l [m^3]	1.00	5.1455	9e-03	0.011	1.2e-03	0.0143	0.05	0.064	80.1
ΔT_{sub} [°C]	5.80	-340.78	2.13	1.08	1.08	2.62	1.50	4.12	82.8
$V_{l,con}$ [m^3]	0.67	-5.7	5.8e-03	2.4e-03	1.4e-03	0.0064	0.05	0.056	101.0
$T_2 - T_H$ [°C]	0.89	-287.95	0.375	0.174	0.333	0.531	1.50	2.03	141.8

Some notes about Table 4:

- P_l and T_4 can not be controlled because they both are indirectly fixed by the design:

$$Q_{\mathrm{loss}} = Q_C = (UA)_C \cdot (T_4 - T_C) \quad \text{and} \quad P_l = P_{sat}(T_4) \tag{4}$$

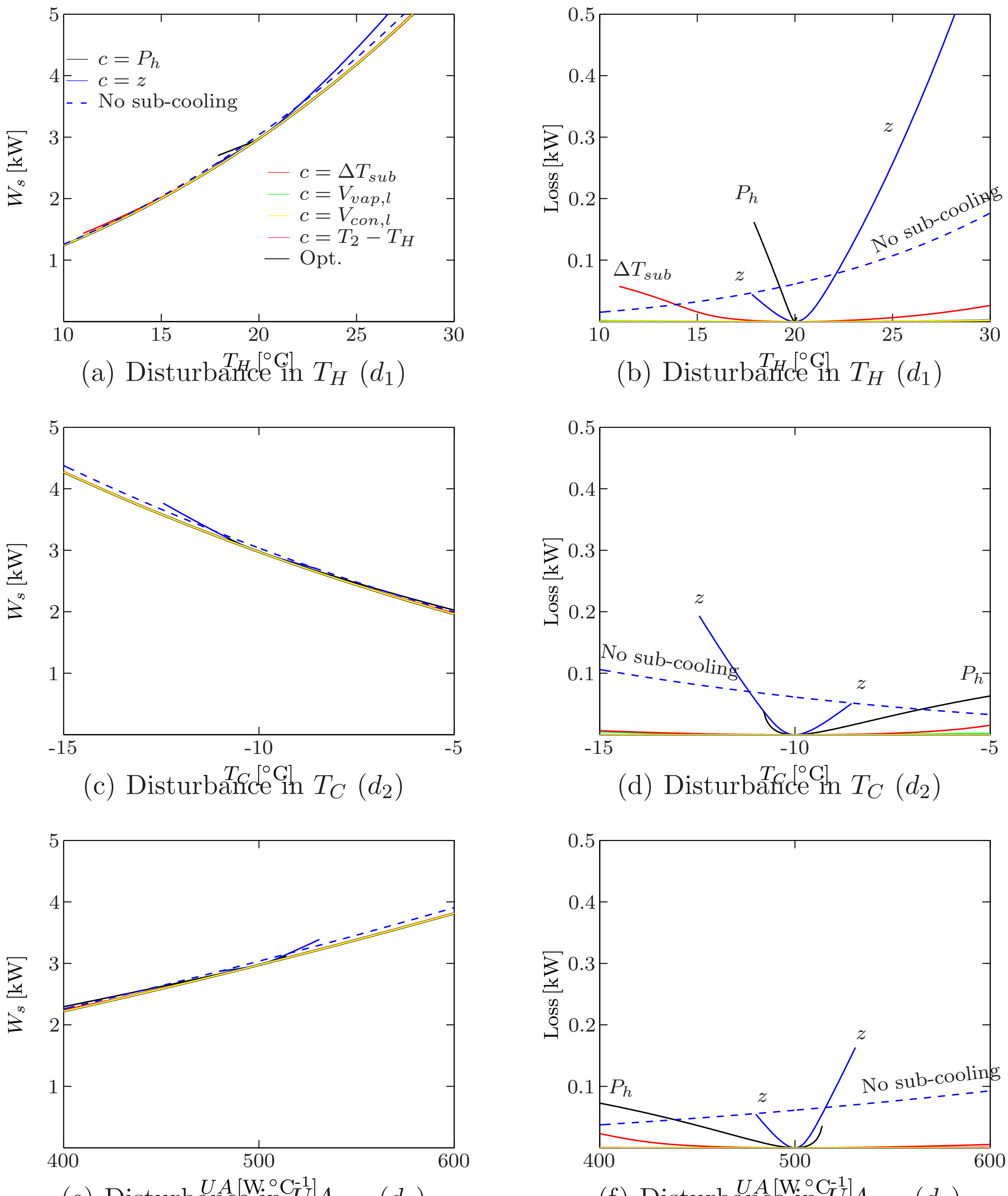
[3] In order to remain in the linear region, the optimal variations were computed for a disturbance of magnitude 1/100 of this, and the resulting optimal variations were then multiplied by 100 to get $\Delta y_{\mathrm{opt}}(d_i)$

- ΔT_{sup} can obviously not be controlled in our case because it is fixed at $0\,^\circ$C (by design of the cycle).
- The loss is proportional to the inverse of squared scaled gain (see Equation 1). This implies, for example, that a constant condenser pressure (P_h) would result in a loss in W_s that is $\left(\frac{82.8}{3.37}\right)^2 = 603$ times larger than a constant sub-cooling (ΔT_{sub}).
- The simple policies with a constant pressure (P_h) or constant valve position (z) are not promising with scaled gains of 3.37 and 7.03, respectively.
- The liquid level is sometimes controlled in a flooded evaporator (Langley, 2002). In our case (Figure 1) this corresponds to a constant level in the liquid receiver (V_l). This is indeed a good choice with a scaled gain of 80.1, but according to the linear analysis, the liquid level in the condenser ($V_{l,con}$) is even better with a scaled gain of 101.0 (also a scheme shown in Langley (2002)).
- Controlling the degree of sub-cooling in the condenser (ΔT_{sub}) is also promising with a scaled gain of 82.8, but the most promising is the temperature approach at the condenser outlet ($T_2 - T_H$) with a scaled gain of 141.8.
- The ratio between n and Δy_{opt} tells whether the implementation error or the effect of the disturbance is most important for a given control policy. For the most promising policies, the implementation error is most important.

4.3.2 Nonlinear analysis

The nonlinear model was subjected to the "full" disturbances to test more rigorously the effect of fixing alternative controlled variables. Figure 4 shows the compressor power W_s (left) and loss $L = W_s - W_s^{\text{opt}}$ (right) for disturbances in T_H (d_1), T_C (d_2) and UA_{loss} (d_3). W_s^{opt} is obtained by re-optimizing the operation for the given disturbances. As predicted from the linear analysis, control of P_h or z should be avoided as it results in a large loss and even infeasibility (a line that ends corresponds to infeasible operation). Controlling the degree of sub-cooling ΔT_{sub} gives small losses for most disturbances, but gives infeasible operation when T_H is low. Controlling the liquid level, either in the receiver or in the condenser, gives small losses in all cases. Another good policy is to maintain constant temperature approach out of the condenser ($T_2 - T_H$). This control policy was also the best in the linear analysis and has as far as we know not been suggested in the literature for ammonia cycles.

A common design of vapour compression cycles, not discussed so far, is without sub-cooling in the condenser. In practice, this might be realized with the design in Figure 2(b) by adding a liquid receiver after the condenser, and using the choke valve to control this liquid level. The performance of this design ("No sub-cooling") is shown with the dashed line in Figure 4. The loss (right graphs) for this design is always nonzero, as it even at the nominal point has a loss of 0.06 kW loss, and the loss increases with the cooling duty of the cycle.



(a) Disturbance in T_H (d_1)
(b) Disturbance in T_H (d_1)
(c) Disturbance in T_C (d_2)
(d) Disturbance in T_C (d_2)
(e) Disturbance in UA_{loss} (d_3)
(f) Disturbance in UA_{loss} (d_3)

Fig. 4. Ammonia case: Compressor power (left) and loss (right) for different disturbances and controlled variables. A line that ends corresponds to infeasible operation.

Nevertheless, we note that the loss with this design is acceptable (less than about 0.2 kW) for all considered disturbances.

Figure 5 shows the sensitivity to implementation error for the four best controlled variables. Controlling a temperature difference at the condenser exit (either $T_2 - T_H$ or ΔT_{sub}) has a small sensitivity to implementation error. On the other hand, controlling either of the two liquid levels (V_l or $V_{l,con}$) might lead to infeasible operation for relatively small implementation errors. In both

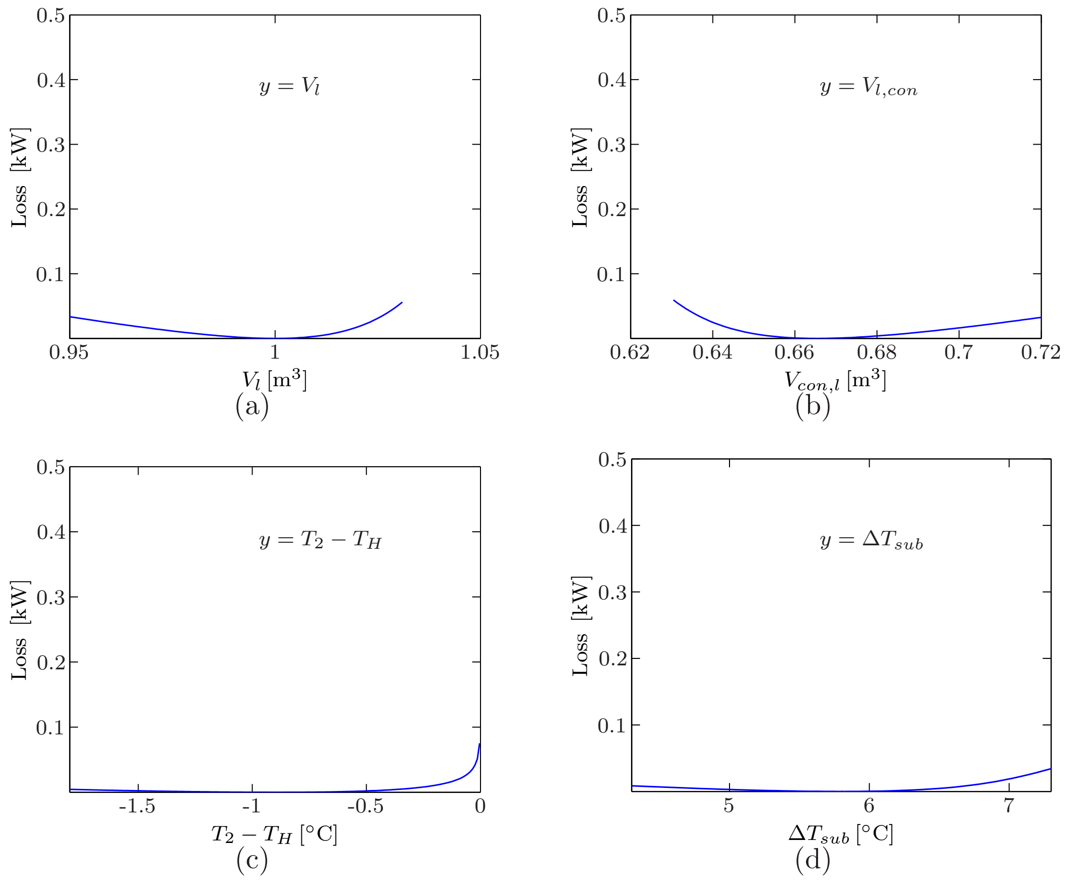


Fig. 5. Ammonia case: Loss as function of implementation error

cases the infeasibility is caused by vapour at the condenser exit. In practice, this vapour "blow out" may be "feasible", but certainly not desirable.

A third important issue is the sensitivity to the total charge of the system which is relevant for the case where we control the liquid level in the receiver ($c = V_l$). There is probably some uncertainty in the initial charge of the system, and maybe more importantly there might be a small leak that will reduce the total charge over time. Optimally the total charge has no steady state effect for the design we have chosen (it will only affect the liquid level in the receiver), but controlling the liquid level in the receiver will make the operation depend on the total charge. This is because we maintain a fixed level in the receiver and leaks will directly affect the charge to the rest of the system, so we have lost one of the positive effects of having the liquid receiver. The other control structures will not be affected by varying total charge.

4.3.3 Conclusion ammonia case study

For this case study, controlling the temperature approach at the condenser exit ($T_2 - T_H$) seems to be the best choice as the sensitivity to implementation error is very small (Figure 5), and the losses for all disturbances are small (Figure 4).

5 CO_2 case study

Nekså (2002) shows that CO_2 cycles are attractive for several applications, both from an efficiency point of view and also from an environmental perspective. Skaugen (2002) gives a detailed analysis of what parameters that affect the performance of a CO_2 cycles and discuss pressure control in these systems.

The simple cycle studied in this paper operates between air inside a room ($T_C = 20\,°\mathrm{C}$) and ambient air ($T_H = 30\,°\mathrm{C}$). This could be an air-conditioner for a home as illustrated in Figure 6(a). The heat loss out of the building is given by Equation 3, and the temperature controller shown in Figure 6(a) indirectly gives $Q_C = Q_{\text{loss}}$. The nominal heat loss is 4.0 kW.

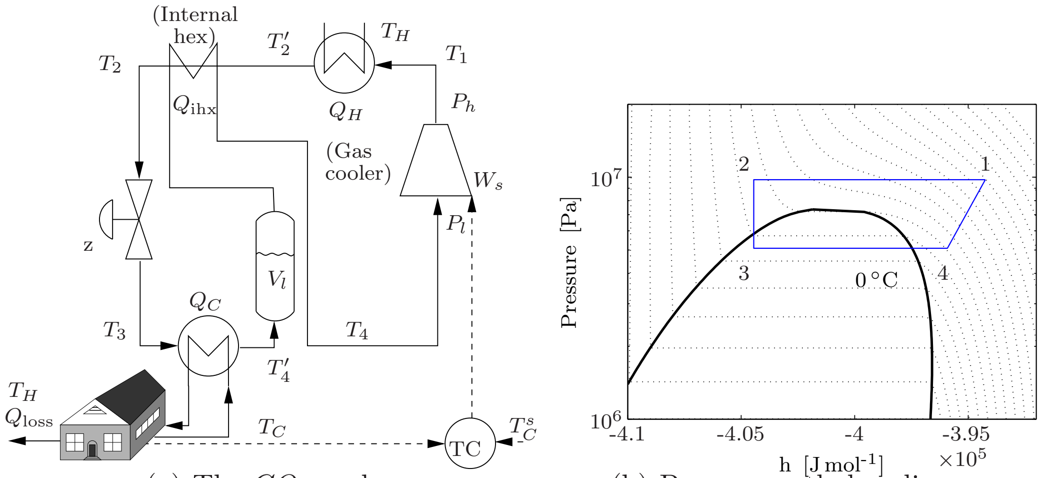
We consider a cycle with an internal heat exchanger, see Figure 6(a). This heat exchanger gives further cooling before the choke valve by super-heating the evaporator outlet. This has the advantage of reducing the expansion loss through the valve, but super-heat increases the compressor power. For the CO_2 cycle it has been found that the internal heat exchanger improves performance for some operating points (Domanski et al., 1994). For the nominal case, we find that the internal heat exchanger gives a reduction of 9.9 % in W_s. For the ammonia cycle, the effect of internal heat exchange is always negative in terms of efficiency.

5.1 Modelling

Table 1 shows the most important model equations and the data are given in Table 5. Constant air temperature (T_C) is assumed in the evaporator. The gas cooler and internal heat exchanger are modelled as counter-current heat exchangers with 6 control volumes. The Span-Wagner equation (1996) of state is used for the thermodynamic calculations.

5.2 Optimal operation

Some key values for optimal operation of the CO_2 cycle are summarized in Table 6 and the pressure enthalpy diagram is given in Figure 6(b). Figure 7 shows the optimal temperature profiles in the gas cooler and in the internal heat exchanger.



(a) The CO_2 cycle (b) Pressure enthalpy diagram

Fig. 6. The CO_2 cycle operates trans-critical and is designed with an internal heat exchanger

Table 5
Conditions for the CO_2 case study

Evaporator: $UA = 798\,\mathrm{W\,K^{-1}}$
Gas cooler: $UA = 795\,\mathrm{W\,K^{-1}}$
Internal heat exchanger: $UA = 153\,\mathrm{W\,K^{-1}}$
Compressor: isentropic efficiency $\eta = 0.75$
Ambient: $T_H = 30\,°\mathrm{C}$
Room: $T_C = T_C^s = 20\,°\mathrm{C}$
Room: $UA_{\text{loss}} = 400\,\mathrm{W\,°C^{-1}}$
Choke valve: $C_V = 1.21 \cdot 10^{-6}\,\mathrm{m}^2$

Note that when the ambient air goes below approximately $T_H = 25\,°\mathrm{C}$ the optimal pressure in the gas cooler is sub-critical. We will only consider trans-critical operation, so we assume that the air-conditioner is not used below 25 °C.

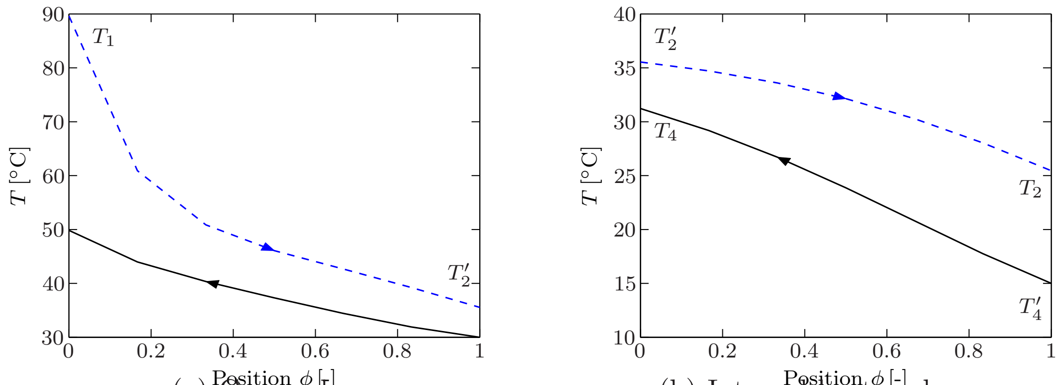
5.3 Selection of controlled variable

We want to find what the valve should control. In addition to the variables listed in Section 3, we also consider internal temperature measurements in the gas cooler and internal heat exchanger.

As discussed in more detail below, there are no obvious single measurements to control for this application. One exception is the holdup N on the high pressure side of the cycle. However, measuring the holdup of a super-critical

Table 6
Optimal operation for CO_2 case

W_s [W]	958
z [-]	0.34
P_h [bar]	97.61
P_l [bar]	50.83
Q_H [W]	-4958
Q_{ihx} [W]	889
$\dot{n}$ [mol s^{-1}]	0.57
T_1 [°C]	89.6
T_2 [°C]	25.5
T_3 [°C]	15.0
T_4 [°C]	31.2



(a) Gas cooler (b) Internal heat exchanger

Fig. 7. CO_2 case: Temperature profile in gas cooler and internal heat exchanger

fluid is not easy (one might use some kind of scale, but this will be to expensive in most applications). Thus, we will consider measurement combinations. First we will try to combine two measurements, and if this is not acceptable for all disturbances, we may try more measurements. Any two measurements can be combined, but we choose here to combine P_h and T_2. The reason is that P_h is normally controlled anyway for dynamic reasons, and T_2 is simple to measure and is promising from the linear analysis. Also, temperature corrected setpoint for pressure has been proposed before (Kim et al., 2004). Using the extended null space method, we find that the linear combination $c_{\text{combine}} = h_1 \cdot P_h + h_2 \cdot T_2$ with $h_2/h_1 = k = -8.532\,\text{bar}\,°\text{C}^{-1}$ minimizes the 2-norm of the three disturbances on the loss. This can be implemented in practice by controlling the corrected pressure

$$P_{h,combine} = P_h + k \cdot (T_2 - T_{2,\text{opt}}) \tag{5}$$

where $T_{2,\text{opt}} = 25.5\,°\text{C}$ and $k = -8.53\,\text{bar}\,°\text{C}^{-1}$. An alternative is to use a more physically based combination. For an ideal gas we have $N = \frac{PV}{RT}$, and since N_{gco} seems to be a good variable to control, we will include P/T in the gas cooler as a candidate controlled variable.

5.3.1 Linear method

We will here use the linear method to find promising controlled variables to check on the non-linear model. The following disturbances[4] are considered d_1: $\Delta T_H = \pm 10\,°\text{C}$, d_2: $\Delta T_C = \pm 5\,°\text{C}$ and d_3: ΔUA_{loss} from -100 to $+40\,\text{W}\,°\text{C}^{-1}$.

The linear results are summarized in Table 7. Some controlled variables (P_l, T_4', ΔT_{sub} and ΔT_{sup}) are not considered because they as discussed earlier can not be fixed or are not relevant for this cycle. From Table 7 the most promising controlled variables are the holdup in the gas cooler (N_{gco}) and the linear combination ($P_{h,\text{combine}}$). Fixing the valve opening z_s (no control) or the liquid level in the receiver (V_l) are also quite good. The ratio P/T in the gas cooler is not favourable with a small scaled gain. This is probably not surprising, because the fluid in the gas cooler is far from ideal gas so P/T is not a good estimate of the holdup N_{gco}.

Table 7
Linear "maximum gain" analysis of controlled variables for CO_2 case

			$\lvert\Delta y_{\text{opt}}(d_i)\rvert$						
Variable (y)	Nom.	G	d_1 (T_H)	d_2 (T_C)	d_3 (UA_{loss})	$\lvert\Delta y_{\text{opt}}\rvert$	n	span y	$\lvert G'\rvert$
P_h/T_2' [bar °C^{-1}]	0.32	-0.291	0.140	-0.047	0.093	0.174	0.0033	0.177	0.25
P_h [bar]	97.61	-78.85	48.3	-15.5	31.0	59.4	1.0	60.4	1.31
T_2' [°C]	35.5	36.7	16.27	-2.93	7.64	18.21	1	19.2	1.91
$T_2' - T_H$ [°C]	3.62	24	4.10	-1.92	5.00	6.75	1.5	8.25	2.91
z [-]	0.34	1	0.15	-0.04	0.18	0.24	0.05	0.29	3.45
V_l [m^3]	0.07	0.03	-0.02	0.005	-0.03	0.006	0.001	0.007	4.77
T_2 [°C]	25.5	60.14	8.37	0.90	3.18	9.00	1	10.0	6.02
$P_{h,\text{combine}}$ [bar]	97.61	-592.0	-23.1	-23.1	3.91	33.0	9.53	42.5	13.9
N_{gco} [kg]	4.83	-11.18	0.151	-0.136	0.119	0.235	0.44	0.675	16.55

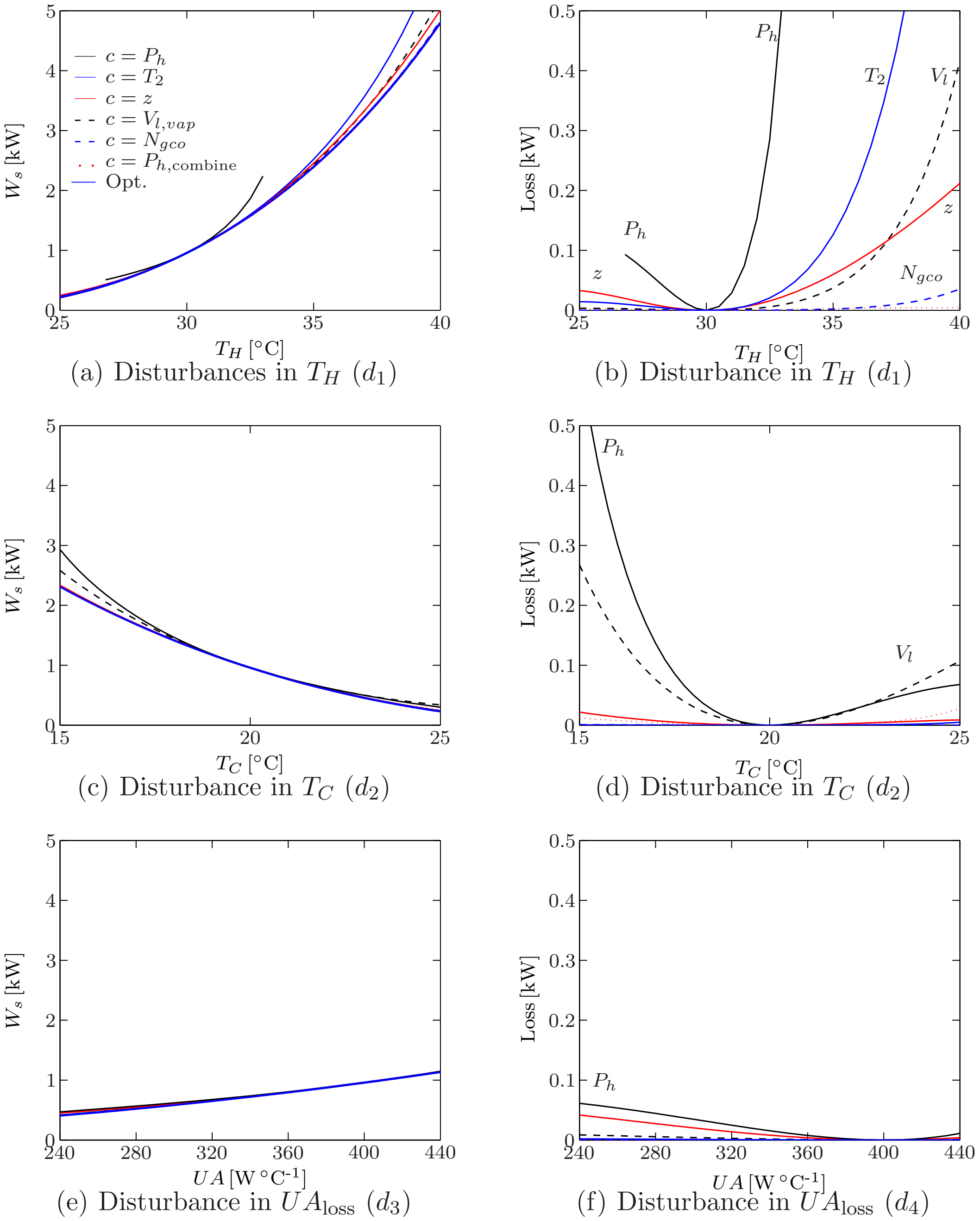
5.3.2 Non-linear analysis

Figure 8 shows the compressor power (left) and loss (right) for some selected controlled variables. We see that the two most important disturbances are the temperature T_H and T_C which gives larger losses than disturbance in the heat

[4] In order to remain in the linear region, the optimal variations were computed for a disturbance of magnitude 1/100 of this, and the resulting optimal variations were then multiplied by 100 to get $\Delta y_{\text{opt}}(d_i)$

loss out of the building. The nonlinear results confirms the linear gain analysis with $P_{h,\mathrm{combine}}$ and holdup in the gas cooler (N_{gco}) giving small losses.

Controlling the pressure P_h gives infeasible operation for small disturbances in the ambient air temperature (T_H).



(a) Disturbances in T_H (d_1)

(b) Disturbance in T_H (d_1)

(c) Disturbance in T_C (d_2)

(d) Disturbance in T_C (d_2)

(e) Disturbance in UA_{loss} (d_3)

(f) Disturbance in UA_{loss} (d_4)

Fig. 8. CO_2 case: Compressor power (left) and loss (right) for different disturbances and controlled variables. A line that ends corresponds to infeasible operation.

Another important issue is the sensitivity to implementation error. From Fig-

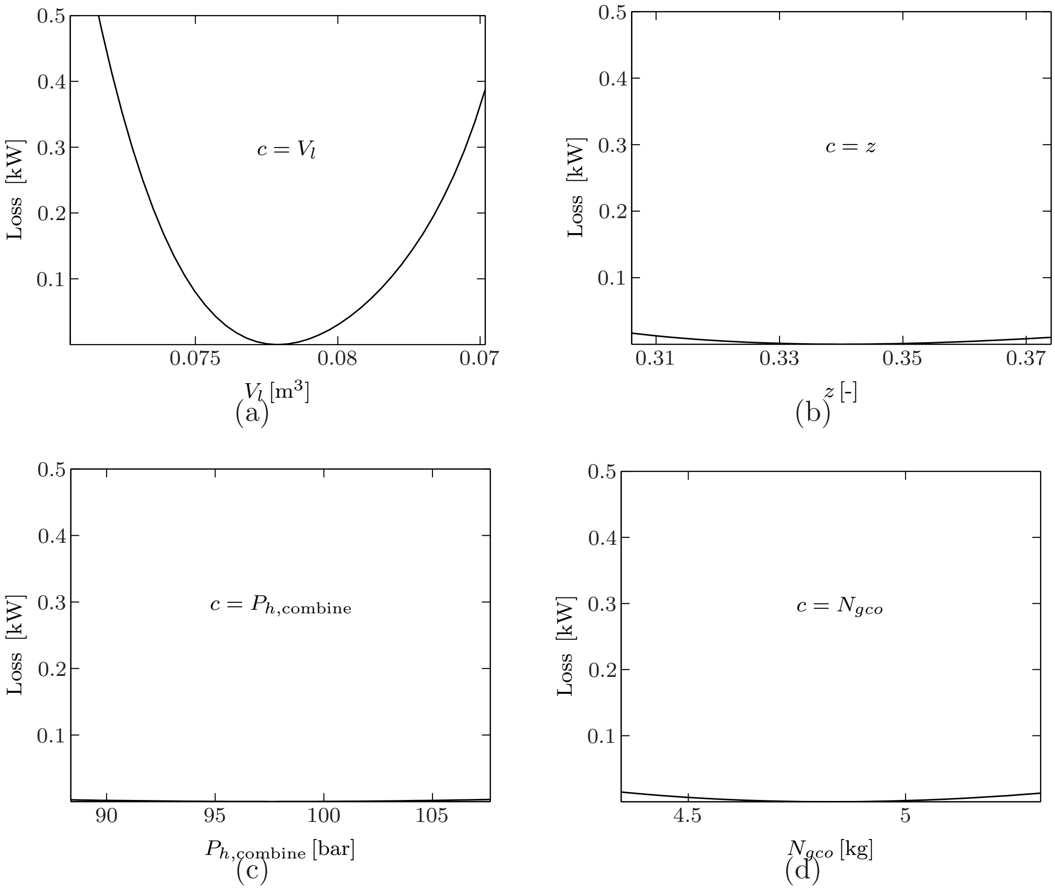


Fig. 9. CO_2 case: Loss as function of implementation error

ure 9 we see that the sensitivity to implementation error is very large for $c = V_l$. The three best are constant valve opening (z), constant holdup in the gas cooler (N_{gco}) and the linear combination ($P_{h,\text{combine}}$).

5.3.3 Conclusion CO_2 case study

For this CO_2 cooling cycle we find that fixing the holdup in the gas cooler gives close to optimal operation. However, since the fluid is super-critical, holdup is not easily measured. In practice the best single measurement is a constant valve opening ("no control"). Another possibility is to use combinations of measurements. We obtained the combination $P_{h,\text{combine}} = P_h + k \cdot (T_2 - T_{2,\text{opt}})$ using the extended null space method. The loss compared with single measurements is significantly reduced and the sensitivity to implementation error is very small.

6 Discussion

We have in this paper assumed that there is no super-heating before the compressor. This can be achieved by having a liquid receiver after the evaporator as shown in Figure 1. However, in most conventional designs one requires a minimum degree of super-heating. Since super-heating is not thermodynamically efficient, the minimal degree of super-heating becomes an active constraint and is normally controlled by the thermostatic expansion valve (TEV). With this configuration, the "extra" valve in Figure 2(a) is the unconstrained degree of freedom that should optimize the operation. Otherwise the results from the present study hold.

We have assumed constant heat transfer coefficients in the heat exchangers. Normally, the heat transfer coefficient will depend on several variables such as phase fraction, velocity of the fluid and heat transfer. However, a sensitivity analysis (not included) indicates that the effect of varying heat transfer coefficients is small for the conclusions in this paper. For the CO_2 cycle we did some simulations using a constant air temperature in the gas cooler which might be used to represent a cross flow heat exchanger, and is an indirect way of changing the effective UA value. We found that the losses for a constant liquid level control policy ($c = V_l$) was slightly smaller compared with our case with a counter-current heat exchanger, but the analysis presented here is still valid and the conclusion that a combination of measurements is necessary to give acceptable performance, remains the same.

This paper has only considered steady-state operation. For dynamic reasons, in order to "stabilize" the operation, a degree of freedom is often used to control the high side pressure (P_h). However, the setpoint for the pressure may be used as a degree of freedom at steady-state, so this will not change the results of this study.

7 Conclusion

If we allow for sub-cooling in the condenser of a sub-critical vapour compression cycle, we find that there is one unconstrained degree of freedom that can be used to optimize the operation. For an ammonia cooling cycle, a good controlled variable is the temperature approach at condenser exit ($T_2 - T_H$), which is insensitive to implementation error and gives small loss for disturbances.

For the CO_2 cycle we find that the only single measurement that gives self-optimizing control is the holdup in the super-critical gas cooler (N_{gco}). Since this holdup is difficult to measure, and simple correlations are not sufficient, a combination of two measurements is necessary. We found that controlling the corrected pressure $P_{h,combine} = P_h + k \cdot (T_2 - T_{2\text{opt}})$ gives small losses. The linear combination is also very insensitive to implementation error.

Acknowledgment

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Chapter 1

Analysis of simple vapour compression cycles

June 7, 2006

Cycles for heating and cooling have traditionally been studied in detail when it comes to thermodynamics and design. However, there are few publications on their optimal operation which is the theme of this paper. One important issue is which variable to control, for example, degree of super-heating, pressure, liquid level or valve opening. Also, unlike open systems, the initial charge to the cycle may have a steady state effect, and it is discussed how different designs are affected by this factor. Numerical results are provided for an ammonia cycle.

1.1 Introduction

Cyclic processes for heating and cooling are widely used in many applications and their power ranges from less than 1 kW to above 100 MW. Most of these applications use the vapour compression cycle to "pump" energy from a low to a high temperature level.

The first application, in 1834, was cooling to produce ice for storage of food, which led to the refrigerator found in most homes Nagengast (1976). Another well-known system is the air-conditioner (A/C). In colder regions a cycle operating in the opposite direction, the "heat pump", has recently become popular. These two applications have also merged together to give a system able to operate in both heating and cooling mode.

A schematic drawing of a simple cycle is shown in Figure 1.1 together with a typical pressure-enthalpy diagram for a sub-critical cycle. The cycle works as follows:

The low pressure vapour (4) is compressed by supplying work W_s to give a high pressure vapour with high temperature (1). This stream is cooled to the saturation temperature in the first part of the condenser, condensed in the middle part and possibly sub-cooled in the last part to give the liquid (2). In the expansion choke, the pressure is lowered to its original value, resulting in a two-phase mixture (3). This mixture is vaporized and possibly super-heated in the evaporator (4) closing the cycle.

The coefficients of performance for a heating cycle (heat pump) and a cooling cycle (refrigerator, A/C) are defined as

$$COP_h = \frac{Q_h}{W_s} = \frac{\dot{n}(h_1 - h_2)}{\dot{n}(h_1 - h_4)} \qquad \text{and} \qquad COP_c = \frac{Q_c}{W_s} = \frac{\dot{n}(h_4 - h_3)}{\dot{n}(h_1 - h_4)} \tag{1.1}$$

respectively. Heat pumps typically have a COP of around 3 which indicates that 33% of the gained heat is addet as work (eg. electric power).

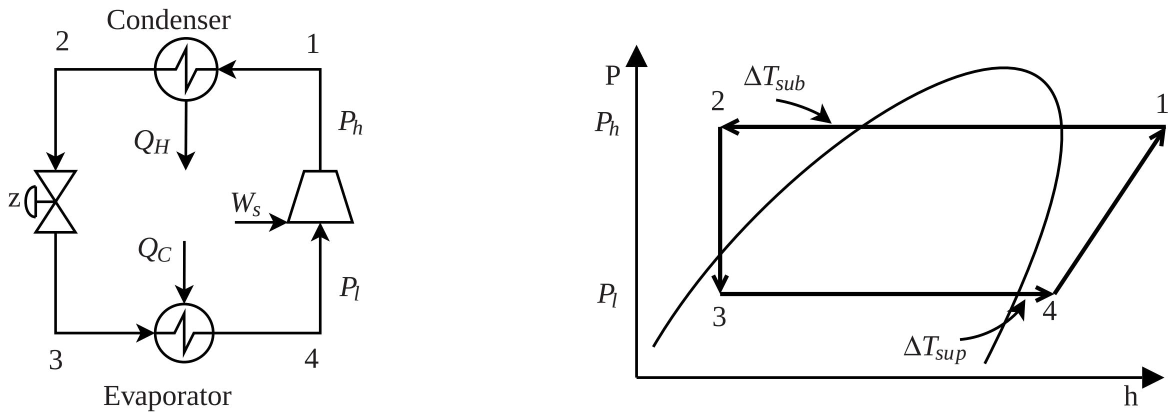


Figure 1.1: Schematics of a simple vapour compression cycle with typical pressure-enthalpy diagram indicating both sub-cooling and super-heating

In industrial processes, especially in cryogenic processes such as air separation and liquefaction of natural gas (LNG process), more complex cycles are used in order to improve the thermodynamic efficiencies. These modifications lower the temperature differences in the heat exchangers and include cycles with mixed refrigerants, several pressure levels and cascaded cycles. Our long term objective is to study the operation of such processes. However, as a start we need to understand the simple cycle in Figure 1.1.

An important result from this study is the degree of freedom analysis given in Section 1.2. This is more or less directly applicable to more complex designs. We find that the charge plays an important role in operation of cyclic processes. A cyclic process will not have boundary conditions on pressures, which for an open process will determine the pressure and holdup internally, so controlling the charge in the system is important as it indirectly sets the pressure level in the system.

Although there is a vast literature on the thermodynamic analysis of such cycles, there are few authors who discuss the operation and control of closed cycles. Some discussions are found in text books such as Stoecker (1998), Langley (2002) and Dossat (2002), but these mainly deal with more practical aspects. Svensson (1994) and Larsen et al. (2003) discuss operational aspects. A more comprehensive recent study is that of Kim et al. (2004) who consider the operation of trans-critical CO_2 cycles. They discuss the effect of "active charge" and consider alternatives for placing the receiver.

In the literature, it is generally taken for granted that there should be no sub-cooling and super-heating ($\Delta T_{sub} = 0\,^\circ$C and $\Delta T_{sup} = 0\,^\circ$C) in an optimal cycle. For example, (Stoecker, 1998, page 57) states that

> The refrigerant leaving industrial refrigeration condensers may be slightly sub-cooled, but sub-cooling is not normally desired since it indicates that some of the heat transfer surface that should be be used for condensation is used for sub-cooling. At the outlet of the evaporator it is crucial for protection of the compressor that there be no liquid, so to be safe it is preferable for the vapor to be slightly super-heated.

In this study, we find that super-heating is not optimal. However, contrary to popular belief, we find that with given equipment, sub-cooling in the condenser may give savings in energy usage (compressor power) in the order of 2%. It is normally assumed that the high pressure P_h and the hot source temperature T_H are directly coupled, but sub-cooling gives some de-coupling. The optimality of sub-cooling is discussed in Section 1.4. An ammonia case study is presented to obtain numerical results.

We consider only steady state operation in this paper, as this has the most influence on the operating costs.

1.2 Degrees of freedom in simple vapour compression cycles

1.2.1 Design versus operation

Table 1.1 shows typical specifications for the simple cycle in Figure 1.1 in design (find equipment) and in operation (given equipment). The five design specifications include the load, the two pressures, and the degree of sub-cooling and super-heating. Based on these five design specifications we may obtain the following four equipment parameters which can be adjusted during operation: compression work (W_s) valve opening (z) and effective areas (UA-values) for the two heat exchangers. Initially, we were puzzled because we could not identify the missing fifth equipment parameter to be adjusted during operation. However, we finally realized that we can manipulate the ”active charge” in the cycle, which indirectly sets the ”pressure level” in the cycle. By “active charge” is meant the mass accumulated in the process equipment, that is, mainly in the two heat exchangers in Figure 1.1. The fact that the charge is an independent variable is unique for closed systems since there is no boundary condition for pressure.

Table 1.1: Typical specifications in design and operation

	Given	#
Design	Load (e.g. Q_h), P_l, P_h, ΔT_{sup} and ΔT_{sub}	5
Operation	W_s (load), choke valve opening (z), UA in two heat exchangers and active charge	5

1.2.2 Active charge and holdup tanks

For the simple cycle in Figure 1.1 we have the following total material balance:

$$m_{tot} = \underbrace{m_{evap} + m_{con}}_{\text{Active charge}} + m_{valve} + m_{comp} + m_{tank} \tag{1.2}$$

With no filling, emptying or leaks, the total mass m_{tot} is fixed. We have not included a holdup tank in Figure 1.1, but in practice it is common to include a variable liquid level (tank; receiver) in the cycle. m_{tank} is then the overall mass in this tank(s). Normally the holdups in the valve and compressor are neglected and we get:

$$m_{tot} = \underbrace{m_{evap} + m_{con}}_{\text{Active charge}} + m_{tank} \tag{1.3}$$

With a given volume of the equipment, the "pressure level" is indirectly given by the active charge. With constant active charge, we assume that a change in m_{tank} (e.g. by filling or leaking) does not affect the operation of the cycle. This implies that the the tank(s) must contains both liquid and gas in equilibrium. Then we can move mass to or from the tank without affecting the pressure, and thus without affecting the rest of the cycle.

In addition to making operation independent of total charge in the system, the extra tank introduces an additional degree of freedom. This can be seen from Equation 1.3: With m_{tot} constant, we can by altering the liquid level in the tank (m_{tank}), change the active charge in the rest of the system (condenser and evaporator). This shows that the liquid level in the tank has an indirect steady state effect, and can therefor be used for control purposes, of course provided that we have means of changing it.

Although it is possible to introduce several tanks in a cycle, we only have one material balance for each cycle, so this will not add any steady-state degrees of freedom with respect to the total holdup.

Rule 1.1 *In each closed cycle, we have one degree of freedom related to the active charge, which may be indirectly adjusted by introducing a variable liquid level (tank; receiver) in the cycle.*

Rule 1.2 *In each closed cycle, there will be one liquid holdup that does not need to be explicitly controlled, because the total mass is fixed. This is usually selected as the largest liquid volume in the closed system. The remaining liquid levels (holdups) must be controlled.*

Remark 1 Rule 1.2 does not mean that we cannot control all the liquid volumes in the system (including the largest one), but it just states that it is not strictly necessary. In fact, controlling all the liquid volumes, provides a way for explicitly controlling the active charge in the cycle, which may be a good option in some cases.

Remark 2 Introducing additional liquid tanks may be useful for operation, but at least for pure fluids, these will not introduce any additional steady-state degrees of freedom because we can move mass from one tank to another without affecting operation. Also, to avoid that tanks fill up or empty, these additional levels will need to be controlled (Rule 1.2), either by self-regulation or feedback control.

Remark 3 In *mixed refrigerant* cycles two tanks may be used to indirectly change the composition of the circulating refrigerant. In this case the two tanks have different composition so moving mass from one tank to another does affect operation. This is utilized in the auto-cascade process (Neeraas et al. (2001)). For more complex cycles the maximum number of degrees of freedom related to tank holdups is the number of components in the refrigerant.

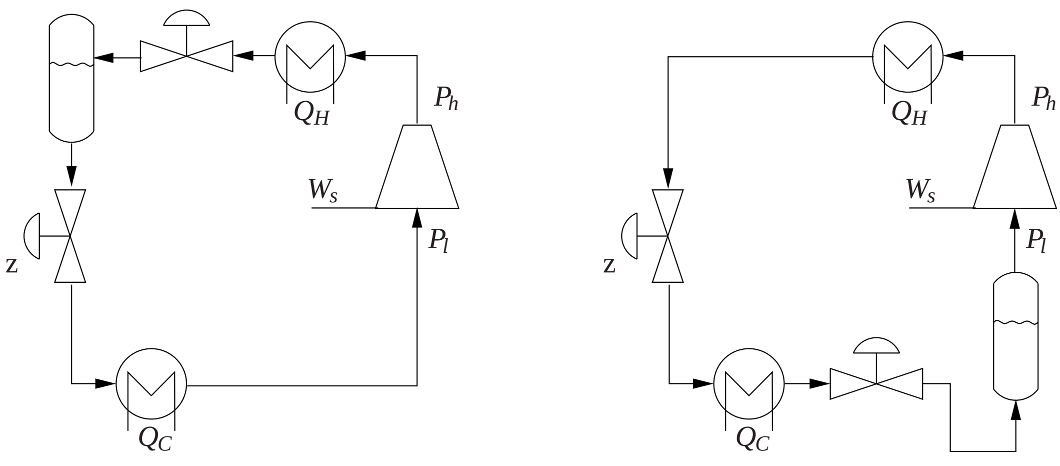
Adjusting the holdup with an extra valve

Kim et al. (2004) discuss alternative locations for the variable tank holdup (liquid receiver). In Figure 1.2, we show cycles for the cases where the tank is placed (a) on the high pressure side after the condenser and (b) on the low pressure side after the evaporator. Other placements and combinations are possible, but these are only variations of these two and will not add any steady-state degrees of freedom for pure refrigerants.

The most obvious way of introducing a means for adjusting the tank holdup, is to add an extra valve before the tank as shown in Figure 1.2.

In Figure 1.2(a), the tank is located at an intermediate pressure P_m after the condenser. In this case the extra valve is on the same side as the expansion valve (choke), so the pressure drop over the extra valve will not effect the efficiency of the cycle. The pressure P_m is assumed to be the saturation pressure at the tank temperature, an exit stream from the condenser will then have to be sub-cooled. Thus, with the tank after the condenser (Figure 1.2(a)), the pressure drop across the valve may be used to adjust the degree of sub-cooling in the condenser. As discussed below, it is possible to eliminate the valve, but if we then keep the tank we can not get sub-cooling. As found later in this paper, some sub-cooling appears to be optimal in most cases.

Another possibility is to place the tank after the evaporator, as shown in Figure 1.2(b). However, in this case the valve introduces a pressure drop which must be compensated by increasing the compression power, so a valve here is generally not optimal.



(a) Liquid tank and valve at high pressure side

(b) Liquid tank and valve at low pressure side. Note that the extra valve is **not** optimal

Figure 1.2: Simple cycle with variable active charge

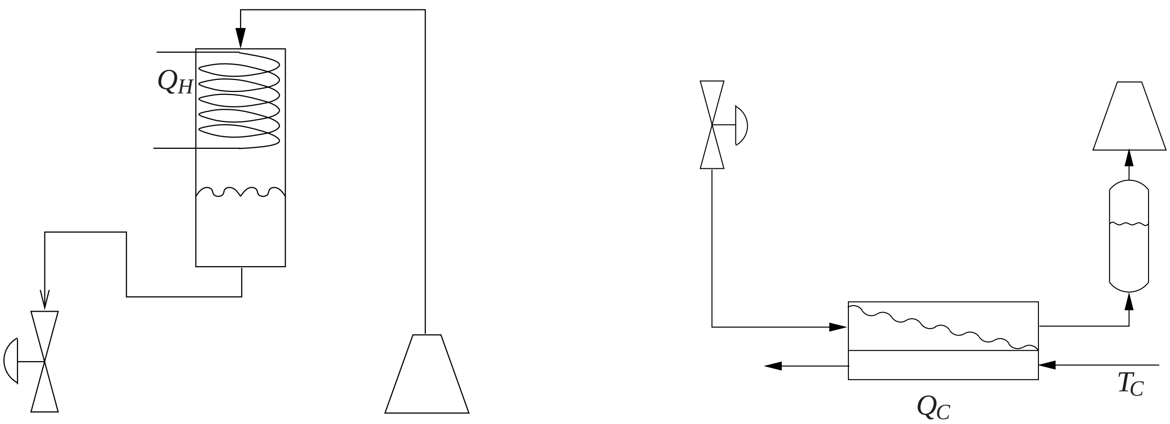
Extra valve removed

In most practical cases the extra valves in Figure 1.2(a) and 1.2(b) are removed. What effect does this have?

- High pressure tank without valve (see Figure 1.3(a) where the tank and condenser are merged together): Without the valve, we will have at steady state the same thermodynamic state at the exit of the condenser as at the exit from the tank. Thus, the exiting stream from the condenser will be saturated liquid. As we will show, this is not generally optimal. Thus, in this design we have used a degree of freedom ("no valve") to set the degree of sub-cooling to a non-optimal value. Nevertheless, this design is commonly used in most applications.

- Low pressure tank without valve (see Figure 1.3(b)): With a liquid tank after the evaporator we get saturated vapour to the compressor. Fortunately, this is generally optimal for the cycle as a whole, because the inlet temperature to the compressor should be as low as possible to minimize vapour volume and save compression power. Thus, in this design we have used a degree of freedom ("no valve") to set the degree of superheating to the optimal value.

So in the case of high pressure liquid tank we get a sub-optimal design if we remove the valve, whereas for the low pressure tank we get an optimal design if the extra valve is removed. It is also possible to remove the tanks as discussed later.



(a) Condenser without sub-cooling (b) Evaporator without super-heating

Figure 1.3: Condenser and evaporator with valve removed and saturation at outlet

1.2.3 Degrees of freedom for operation

During operation the equipment is given. Nevertheless, we have some operational or control degrees of freedom.

1 The compression power W_s. We assume here that it is used to set the "load" for the cycle.

2, 3 Effective heat transfer area (UA). There are two degrees of freedom related to adjusting the heat transfer, which may thought of as adjusting (reducing) the effective UA value in each heat exchanger. This may be done in many ways, for example, by introducing bypasses or using flooded condenser or evaporator. However, we generally find that it is optimal to maximize the effective UA. Thus, these degrees of freedom are not considered in the following.

4 Adjustable choke valve (z); see Figure 1.1

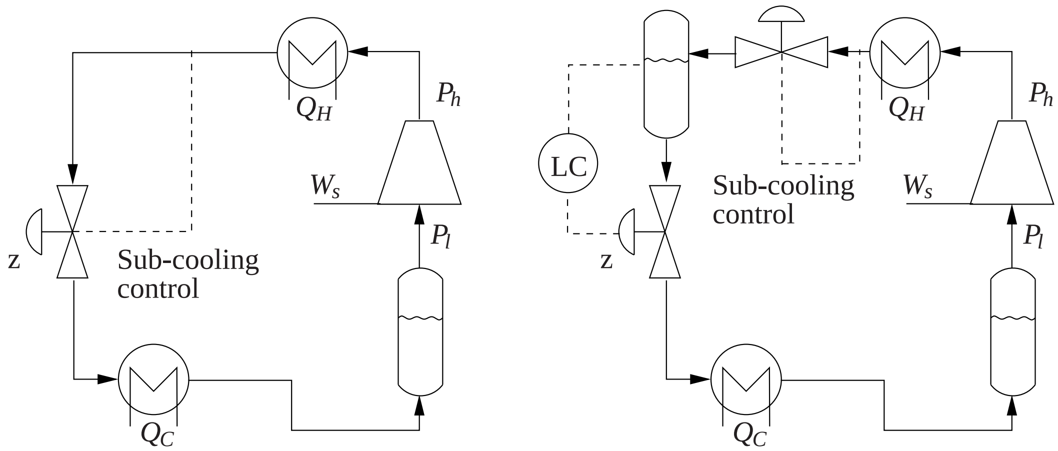
5 Adjustable active charge.

In summary, with a given load we are in practice left with two steady state degrees of freedom. These are the choke valve opening and the active charge. These may be used to set the degree of super-heating and degree of sub-cooling. The pressure levels (P_h and P_l) are indirectly determined by the given (maximum) value of the heat transfer $Q = UA\Delta T$ as determined by the two UA values.

1.3 Discussion of some designs

As discussed in more detail in Section 1.4, we find that the thermodynamic efficiency is optimized by having no super-heating and some sub-cooling.

1.3.1 Optimal designs



(a) Optimal 1 (b) Optimal 2 with extra tank after the condenser

Figure 1.4: Two potentially optimal designs

Two potentially optimal designs are shown in Figure 1.4. The reason we say “potentially optimal” is because they will only be optimal if we use the optimal value for the sub-cooling.

In Figure 1.4(a) we have a low pressure tank (receiver) between the evaporator and compressor which ensures that the vapour entering the compressor is saturated. A demister may be added to avoid that liquid droplets enter the compressor. The choke valve may be used to control the degree of sub-cooling (ΔT_{sub}) as shown in Figure 1.4(a). Also other control policies are possible, for example, keeping the choke valve position constant or controlling the pressure, but controlling ΔT_{sub} was found by Jensen and Skogestad (2005) to be a good self-optimizing controlled variable.

In Figure 1.4(b) we have added a high pressure tank and valve after the condenser. Thermodynamically this design is equivalent to Figure 1.4(a), but the addition of the tank may prevent that we get two-phase flow with vapour "blow out" through the choke. In this case, it seems reasonable to use the "new" valve to control the sub-cooling as shown in Figure 1.4(b). We now have two adjustable holdups, so from Rule 1.2 one of them must be controlled. In Figure 1.4(b), we show the case where the choke valve is used to control the level in the high pressure tank, but alternatively we could control the level in the low pressure tank.

A third potentially optimal design (not shown) would be to remove the valve in Figure 1.4(b), and instead add a sub-cooling heat exchanger before the choke. This may also be accomplished by having only one heat exchanger where the liquid level covers some of the heat transfer area and there is little mixing in the liquid phase.

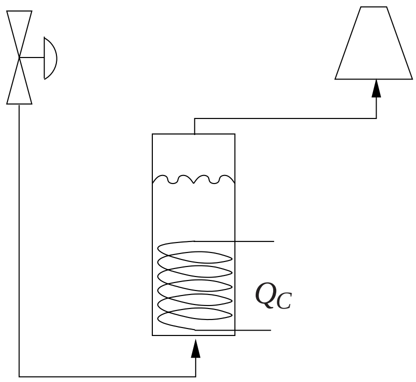


Figure 1.5: Flooded evaporator

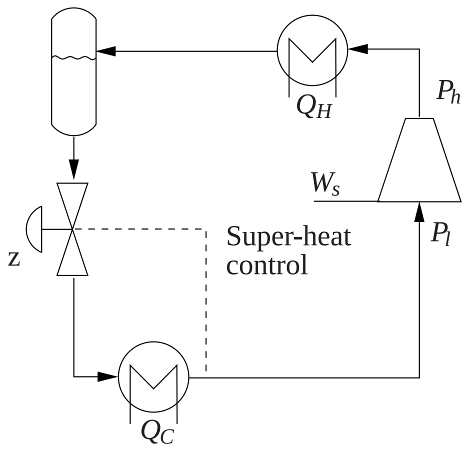
To avoid super-heating, we have in Figure 1.4(a) and 1.4(b) a tank after the evaporator. This tank will give saturated vapour out of the evaporator at steady state, and also by trapping the liquid it will avoid that we get liquid to the compressor during transient operation. To avoid super-heating we must have vapour-liquid equilibrium in the tank. This may be achieved by letting the vapour bubble through the tank. An alternative design is to integrate the heat exchanger and the tank as shown in Figure 1.5. This design is equivalent thermodynamically, but it may not be optimal because the effective heat transfer coefficient (U) may be lower.

1.3.2 Non-optimal designs

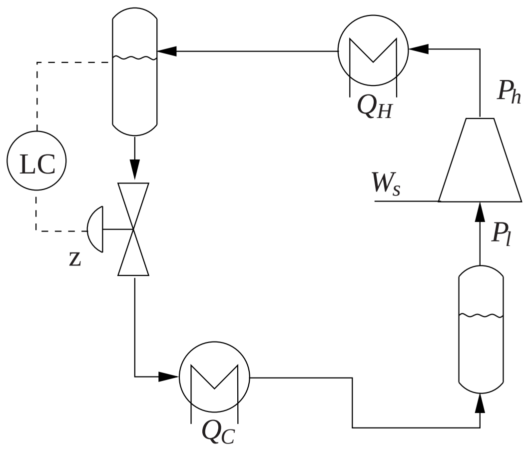
Figure 1.6(a) shows the design used in most applications. In practice the tank and condenser are often integrated as shown in Figure 1.3(a). This design has two errors compared to the optimal solution: 1) There is no sub-cooling in the condenser and 2) there is super-heating in the evaporator. The super-heat control is in practice accomplished with a thermostatic expansion valve (TEV).

In Figure 1.6(b) we have two liquid tanks, one after the evaporator and one after the condenser. This design is better since there is no super-heating in the evaporator, but one error remains: There is no sub-cooling in the condenser. Note that we need to control one of the liquid levels in accordance with Rule 1.2.

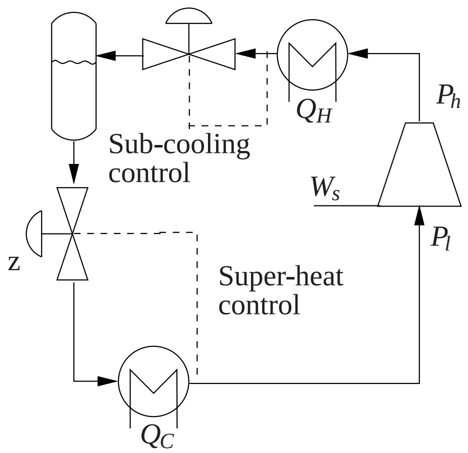
Another non-optimal design is shown in Figure 1.6(c). Here we have introduced the possibility for sub-cooling, but we have super-heating which is generally not optimal.



(a) Non-optimal 1. This design has two errors: 1) No sub-cooling and 2) Super-heating



(b) Non-optimal 2. This design has one error: No sub-cooling



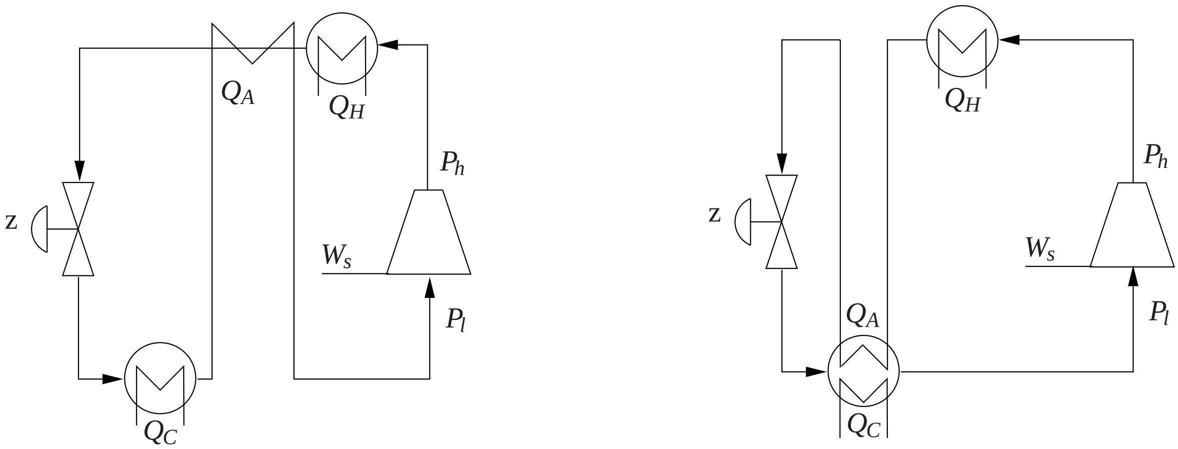
(c) Non-optimal 3, This design has one error: Super-heating

Figure 1.6: Three non-optimal designs

1.3.3 Internal heat exchange

Internal heat exchange has so far been excluded. There are two possibilities as shown in Figure 1.7. In Figure 1.7(a) we add a heat exchanger to super-heat the vapour entering the compressor and sub-cool the liquid before expansion. The sub-cooling is positive because of reduced expansion losses, whereas the super-heating is undesirable because compressor power increases. Depending on the properties of the fluid , this design may be desirable in some cases, even for pure refrigerants (Radermacher (1989)). In the ammonia case study presented below it is not optimal with internal heat exchange, but for a trans-critical CO_2 cycle it is optimal (Nekså et al. (1998)).

In Figure 1.7(b) the liquid out of the condenser is sub-cooled by heat exchange with the evaporator. For pure fluids this has no effect (apart from the fact that increased heat transfer area is needed). However, for mixed refrigerants it may be beneficial, and this configuration is frequently used in LNG processes utilizing mixed refrigerants.



(a) Internal heat exchange after evaporator (sometimes beneficial also for pure fluids)

(b) Internal heat exchange inside evaporator (no effect for pure fluids)

Figure 1.7: Two possible configurations of internal heat exchange

1.4 Optimality of sub-cooling

We have several times made the claim that sub-cooling may be optimal. To justify this controversial claim, we start by considering a specific example.

1.4.1 Ammonia case study

The objective is to cool a storage building by removing heat (Q_C) as illustrated in Figure 1.8. The cycle operates between a cold medium of air inside the building ($T_C = T_{room}$) and hot medium of ambient air ($T_H = T_{amb}$) removing 20 kW of heat (Q_C) from the building. Some data for the cycle:

- Ambient temperature $T_H = 25\,^\circ$C

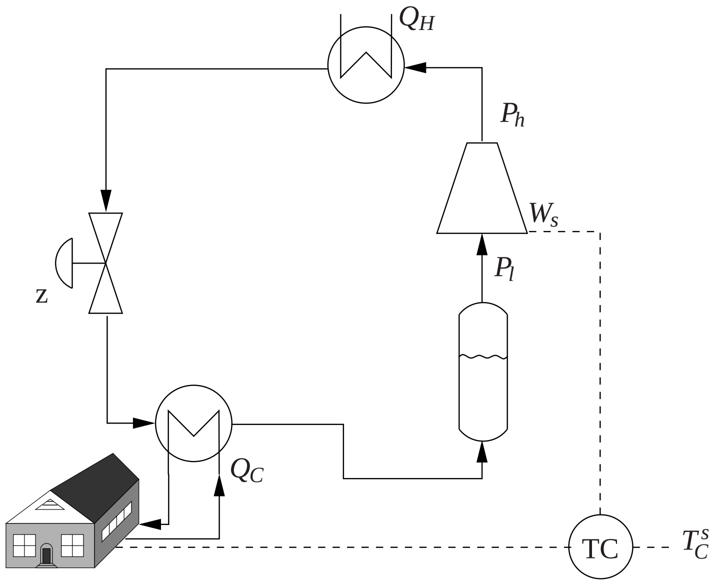
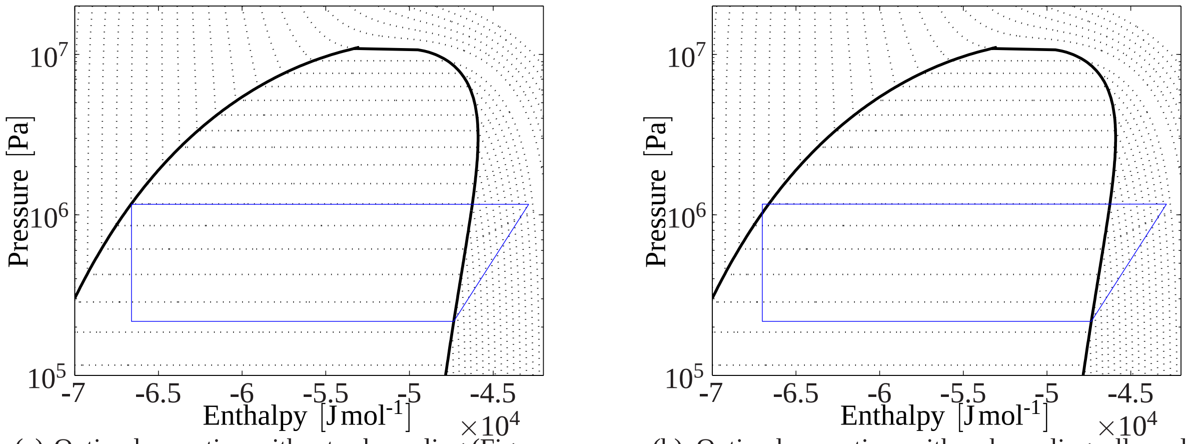


Figure 1.8: Cold warehouse with ammonia refrigeration unit

- Indoor temperature set point $T_C^s = -12\,^{\circ}\mathrm{C}$
- Isentropic efficiency for compressor is 95 %
- Heat transfer coefficients (U) are 1000 and 500 W m^{-2} K^{-1} for the evaporator and condenser, respectively
- Heat exchangers with areas given in Table 1.2
- Thermodynamic calculations are based on SRK equation of state

The steady state heat loss from the building is 20 kW and the load Q_C is indirectly adjusted by the temperature controller which adjusts the compressor work (W_s) to maintain $T_C = T_C^s$.

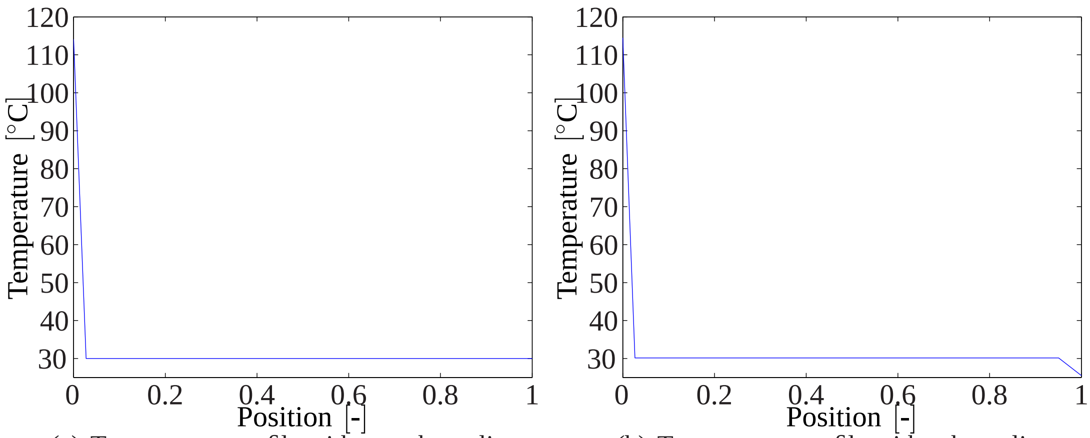
The equipment is given, so we have two remaining steady state degrees of freedom, which may be viewed as the degree of sub-cooling (ΔT_{sub}) and the degree of super-heating (ΔT_{sup}). The results from the optimization with and without sub-cooling are summarized in Table 1.2. We find that super-heating is not optimal, but contrary to popular belief, we find for this ammonia cycle, that sub-cooling by 4.66 °C reduces the compression work W_s by 1.74 %. The high pressure P_h is increases by 0.45 %, but this is more than compensated by a 2.12 % reduction in flowrate. The sub-cooling increases the condenser charge M_{con} by 5.01 % in optimal operation. Figure 1.9 shows the corresponding pressure enthalpy diagram for the two cases. Figure 1.10 shows the temperature profile in the condenser for the two cases. Similar results are obtained if we use other thermodynamic data, if we change the compressor efficiency or if we let UA be smaller in the sub-cooling zone.



(a) Optimal operation without sub-cooling (Figure 1.6(a))

(b) Optimal operation with sub-cooling allowed (Figure 1.4)

Figure 1.9: Ph-diagrams with and without sub-cooling



(a) Temperature profile without sub-cooling

(b) Temperature profile with sub-cooling

Figure 1.10: Temperature profile in condenser

1.4.2 Explanation

Physically, the reason for the improvement in efficiency by sub-cooling is that the irreversible loss through the choke is smaller because less vapour is formed. This more than compensates for the increased irreversible loss in the condenser. To understand this in more detail consider Figure 1.11 which shows a conceptual pressure enthalpy diagram of a typical vapour compression cycle. We have indicated a cycle without sub-cooling (solid line) and the same cycle with sub-cooling (dotted line). Note that since we in the latter case have a higher condenser pressure (and therefor also a higher temperature in the condensing section) we will with given equipment (UA-values) have a higher heat transfer, which gives a lower outlet temperature. The condenser outlet will follow the line "Con. out" with increasing pressure. The line will asymptotically approach the hot source temperature T_H and we want to find the optimal operating point on this line.

If we consider moving from one operating point to another we require an increase in the

Table 1.2: Optimal operation with and without sub-cooling

	No sub-cooling	Optimal
W_s [W]	4648	4567
Q_C [kW]	20	20
Flow [mol s^{-1}]	1.039	1.017
M_{con} * [kmol]	17.72	18.61
ΔT_{sub} [°C]	0.00	4.66
ΔT_{sup} [°C]	0.00	0.00
ΔT_{min} [°C]	5.00	0.491
P_h [bar]	11.63	11.68
P_l [bar]	2.17	2.17
A_{con} [m^2]	8.70	8.70
A_{vap} [m^2]	4.00	4.00

*Evaporator charge has no effect

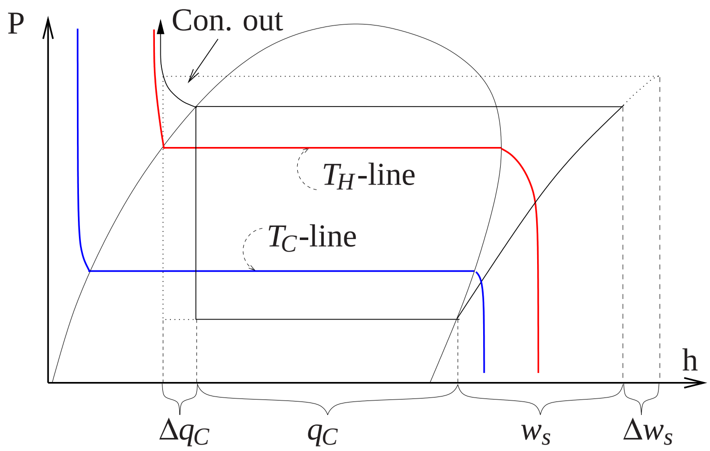


Figure 1.11: Pressure-enthalpy diagram for a cycle with and without sub-cooling

COP for the change to be optimal:

$$\Delta COP = \frac{q_C + \Delta q_C}{w_s + \Delta w_s} - \frac{q_C}{w_s} > 0 \tag{1.4}$$

$$COP \cdot \Delta w_s < \Delta q_C \tag{1.5}$$

where $q_C \cdot \dot{n} = Q_C$ and $w_s \cdot \dot{n} = W_s$. We assume that Q_C [J s^{-1}] is given, and that $\dot{n}$ and q_C may vary. We use ΔT_{sub} as the independent variable and introduce differentials. The requirement for improving efficiency is then from Equation 1.5:

$$\left(\frac{\partial q_C}{\partial \Delta T_{sub}}\right)_{UA} > COP \cdot \left(\frac{\partial w_s}{\partial \Delta T_{sub}}\right)_{UA} \tag{1.6}$$

According to Equation 1.6, for an initial COP of 3, the evaporator should have more than 3 times increase in specific duty compared with the compressor to give improved performance. In Figure 1.11 we have that $\Delta q_C \approx \Delta w_s$, so the optimal degree of sub-cooling is clearly less than that indicated by this Figure . Note however, that the "Con. out" line is much flatter for smaller Δq_C, so a small degree of sub-cooling may be optimal. The optimum is located at the degree of sub-cooling where the inequality in Equation 1.6 turns into an equality. In the case study we found that the optimum (25.49 °C) is closer to T_H (25 °C) than the saturation temperature (30.15 °C).

Similar considerations on optimizing the pressure P_h have been made earlier for trans-critical CO_2-cycles (Kim et al. (2004)). However, for sub-critical cycles it has been assumed that the pressure is fixed by a saturation condition.

1.4.3 Discussion of sub-cooling: Why not found before?

The above results on optimality of sub-cooling is contrary to previous claims and popular belief. Why has this result not been found before?

Reason 1: Not allowed by design

The design of the condenser is often as shown in Figure 1.3(a), where the liquid drips down into a liquid reservoir below the condenser as the droplets forms. In this design it is not possible to have sub-cooling.

Reason 2: Infinite area case

If one assumes an infinite heat transfer area, then it is not optimal with sub-cooling. In this case the temperature at the condenser outlet is equal to the hot source temperature T_H. Neglecting the effect of pressure on liquid enthalpy, the enthalpy is also given. We then find that $\Delta q_C = 0$ and sub-cooling is not optimal as illustrated in Figure 1.12.

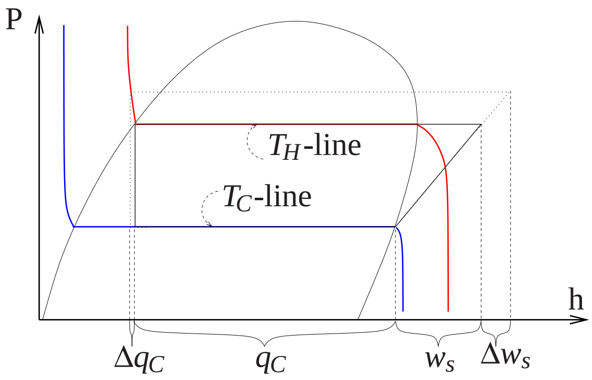


Figure 1.12: Pressure-enthalpy diagram for infinite area case where condenser outlet is at hot source temperature T_H

In practice, the enthalpy depends slightly on pressure (as indicated by the curved constant temperature lines in Figure 1.12) so Δq_C might be larger than zero, but this effect is too small to change the conclusion that sub-cooling is non-optimal with infinite area.

Reason 3: Specifying HRAT

The minimum approach temperature (ΔT_{min} or HRAT) is commonly used as a specification for design of processes with heat exchangers. The idea is to specify ΔT_{min} in order to get a reasonable balance between minimizing operating (energy) costs (favored by a small ΔT_{min}) and minimizing capital costs (favored by a large ΔT_{min}). Although specifying ΔT_{min} may be reasonable for obtaining initial estimates for stream data and areas, it should not be used for obtaining optimal design data - and especially not stream data (temperatures). This follows because specifying ΔT_{min} will, similarly as the infinite area case, result in an optimum with no sub-cooling. This can be seen by letting the T_C-line and T_H-line in Figure 1.12 represent lines for $T_C - \Delta T_{min}$ and $T_H + \Delta T_{min}$ respectively. The condenser outlet will then be given by $T_C + \Delta T_{min}$ and again we get that $\Delta q_C = 0$ neglecting the effect of pressure on liquid enthalpy.

Another way of understanding the difference is that we end up with two different optimization problems for design (Equation 1.7) and operation (Equation 1.8).

$$\begin{aligned} \min \quad & W_s \\ \text{such that} \quad & T_C - T_C^s = 0 \\ & \Delta T - \Delta T_{\min} \geq 0 \end{aligned} \tag{1.7}$$

$$\begin{aligned} \min \quad & W_s \\ \text{such that} \quad & T_C - T_C^s = 0 \\ & A_{\max} - A \geq 0 \end{aligned} \tag{1.8}$$

For the ammonia case study, solving 1.7 with $\Delta T_{min} = 5\,^\circ$C gives the data for “No sub-cooling” in Table 1.2. Setting the resulting areas as A_{max}, and solving the optimization problem 1.8 results in A=A_{max} and the data for “Optimal” in Table 1.2. We see that specifying ΔT_{min} gives no sub-cooling, whereas fixing the heat exchanger areas to the same value gives 4.66 $^\circ$C sub-cooling.

1.5 Selection of controlled variable

We have found that it is generally optimal to have no super-heat ($\Delta T_{sup} = 0\,^\circ$C) and some sub-cooling ($\Delta T_{sub} > 0\,^\circ$C). In practice, no super-heating is easily obtained by use of a design with a low pressure tank as shown in Figure 1.3(b) and Figure 1.4. It is less clear how to get the right sub-cooling. In Figure 1.4 we show a strategy where a valve is used to control the degree of sub-cooling ΔT_{sub}. However, the optimal value of ΔT_{sub} will vary during operation, and also ΔT_{sub} may be difficult to measure and control, so it is not clear that this strategy is good. More generally, we could envisage an on-line optimization scheme where one continuously optimizes the operation (maximizes COP) by adjusting the valves. However, such schemes are quite complex and sensitive to uncertainty, so in practice one uses simpler schemes, like the one in Figure 1.4, where the valves are used to control some other variable. Such variables could be:

- Valve position setpoint z_s (that is, the valve is left in a constant position)
- High pressure (P_h)
- Low pressure (P_l)
- Temperature out of condenser (T_2)
- Degree of sub-cooling ($\Delta T_{sub} = T_2 - T_{sat}(P_h)$)
- Temperature out of evaporator (T_4)
- Degree of super-heating ($\Delta T_{sup} = T_4 - T_{sat}(P_l)$)
- Liquid level in storage tank (to adjust charge to rest of system)
- Pressure drop across the extra valve if the design in Figure 1.4(b) is used

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation Skogestad (2000). The selection of controlled variables is outside the scope of this paper and is presented elsewhere (Jensen and Skogestad (2005)).

1.6 Conclusion

The "active charge" in a closed cycle has a steady state effect. This is unlike open systems, where we have boundary conditions on pressure. This steady state degree of freedom related to the "active charge" may be used to optimize operation of vapour compression cycles. The key to obtain the extra degree of freedom is to allow for sub-cooling in the condenser. So far it has been assumed that one should avoid sub-cooling in the condenser to maximize the efficiency. However, we find that some sub-cooling may be desirable. For the ammonia case study we get savings in the order of 2%, with the same heat transfer areas, by allowing sub-cooling in the condenser.

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