

**Control structure design for optimal operation of  
heat exchanger networks**

|                          |  |
|--------------------------|--|
| Journal:                 | <i>AIChE Journal</i>   |
| Manuscript ID:           | AIChE-07-10714   |
| Wiley - Manuscript type: | Research Article   |
| Keywords:                | Control, Mathematical modeling, Process control, Simulation, process |
|                          |  |



Review only

# Control structure design for optimal operation of heat exchanger networks

Veerayut Lersbamrungsuk<sup>I</sup>

Sridharakumar Narasimhan<sup>II</sup>

Sigurd Skogestad<sup>II</sup>

Thongchai Srinophakun<sup>I\*</sup>

<sup>I</sup>Department of Chemical Engineering, Kasetsart University, Bangkok, 10160, Thailand

<sup>II</sup>Department of Chemical Engineering, Norwegian University of Science and Technology, 7491, Trondheim, Norway

Email: lekveerayut@yahoo.com  
narasimh@nt.ntnu.no  
skoge@chemeng.ntnu.no  
fengtcs@ku.ac.th

\*Correspondence concerning this article should be addressed to Thongchai Srinophakun at fengtcs@ku.ac.th

## Abstract

When only single bypasses and utility duties are used as manipulations, optimal operation of heat exchanger networks (HENs) can be categorized as an active constraint control problem. This work suggests a simple split-range control scheme to implement the optimal operation. The control structure can be found by solving an integer-linear programming (ILP) problem with two objective functions providing optimal split-range pairs (for tracking active constraints during the operation) and appropriate control pairings (for fast control action). A HEN case study is used to demonstrate the application of the proposed design technique. Dynamic simulation shows the ability to provide the optimal operation of the obtained control structure.

Keywords: Active constraint control, Control structure design, Heat exchanger networks, Optimal operation, Split-range control

## 1. Introduction

We are looking for simple ways of implementing (economic) optimal operation. In general, we first control the active constraints, and for the remaining unconstrained we look for good “self-optimizing” variables<sup>1</sup>. For some problems, including the heat exchanger network problem considered in this paper, there are no optimally unconstrained degrees of freedom, that is, all degrees of freedom should be used to satisfy active constraints. For heat exchanger networks, the active constraints are typically given target temperatures and zero or maximum heat exchanger duties. The issue in terms of implementing optimal operation is then to identify the active constraints and change the control policy accordingly. A naive (or at least rather complex) approach is to use on-line optimization. In this paper, *the approach is to use off-line optimization to identify all possible regions with different set of active constraints and then attempt to find a simple operation policy for switching between regions*. The approach taken here is to use *split-range control*, which probably is the simplest way of dealing with changes in active input constraints. In a previous paper<sup>2</sup>, we used a physically-based approach using structural information and an arithmetic sign for how the heat is transferred but this works only in simple cases. In this paper, we solve the problem by integer linear programming (ILP), which gives a solution in terms of split-range control if a feasible solution exists.

Heat exchanger networks (HENs) are widely used in chemical industries to reduce the utility consumption by energy interchange of hot and cold streams. However, without a good control strategy, the reduction may not be achieved in practice. Marselle et al.<sup>3</sup> proposed a method for control structure design based on graph theory and developed a policy to adjust flow distributions in the HEN to meet target temperatures with minimum utility consumption. Calandranis and Stephanopoulos<sup>4</sup> used the structural characteristics of HENs to develop an expert controller for allocating loads to available sinks. A method based on structural information using an arithmetic sign (directional effect between a manipulation and a controlled variable) to generate an optimal control policy was studied by Mathisen et al.<sup>5</sup>, Glemmestad et al.<sup>6</sup> and Lersbamrungsuk et al.<sup>2</sup>. Online and periodic optimizations for the operation of HENs were studied by Aguilera and Marchetti<sup>7</sup>, Glemmestad et al.<sup>8</sup> and González et al.<sup>9</sup>.

If only single bypasses and utility duties are used as manipulations, then the steady-state optimal operation of HENs can be formulated as a linear programming (LP) problem<sup>2,7</sup>. This assumes constant heat capacity for the streams and constant heat transfer coefficients, which is a reasonable assumption for many problems. The important implication in terms of operation is that the optimal solution is always at constraints. In most cases, the resulting active constraint solution can be implemented using a split-range control scheme<sup>2</sup>. In Lersbamrungsuk et al.<sup>2</sup>, the split-range control structure was found using the directional effect between a manipulation and a controlled variable<sup>5</sup> which may be unclear in some cases (e.g. the sign is  $[\pm]$  for a HEN with loops). Instead in this work we use an integer-linear programming (ILP) formulation to suggest an optimal control structure based on a split-range control scheme.

The paper is divided into seven sections. In the following section, a LP problem for the optimal operation of HENs is formulated. This results in an active constraints control problem. Next, an idea for switching between active constraint regions and an ILP for finding an optimal split-range control structure are described. The fourth section illustrates the application of the proposed ILP to a HEN case study. Dynamic simulations to demonstrate the ability for tracking active constraints of the obtained control structure are presented in the fifth section. Further discussion is shown in the sixth section. The last section is the conclusions.

## 2. Optimal operation of HENs

Consider heat exchanger networks where the objective is to maintain optimal operation in spite of the variations in the inlet temperature. Assume

- Constant heat capacity flowrate ( $mC_p$ ) for all streams
- Constant heat transfer coefficients (UA) for all heat exchangers

Further assume that the available degrees of freedom for control (operation) are

- Single bypasses (duties of individual exchangers,  $Q$ )
- Utility duties ( $Q_h, Q_c$ )

Under these assumptions, Aguilera and Marchetti<sup>7</sup> and Lersbamrungsuk et al.<sup>2</sup> show that the corresponding steady-state optimal operation of simple HENs can be formulated as a linear programming (LP) problem:

$$\min c^T x \quad (1a)$$

Subject to:

$$Ax \leq b \quad (1b)$$

$$A_{eq}x = b_{eq} \quad (1c)$$

The vector  $x$  consist of the inlet and outlet temperatures on the hot side ( $T_i^{hot,in}$  and  $T_i^{hot,out}$ ) and cold side ( $T_i^{cold,in}$  and  $T_i^{cold,out}$ ) of all the exchangers, as well as the duty of all exchangers ( $Q_i$ -process exchanger,  $Q_{ci}$ -cold utility exchanger and  $Q_{hi}$ -hot utility exchangers). The equality constraints include the process models, the internal connection, and given supply temperatures  $T_i^s$  and target temperatures  $T_i^t$ . The inequality constraints include the lower and upper bounds on the duty of the heat exchangers. The objective function (1a) allows for many problem formulations including maximum temperature problem. In this paper, the objective is to minimize the utility cost. In this case, all elements of the cost vector  $c$  are zero except the elements related to the duty of utility exchangers. The LP problem formulation for optimal operation of HENs is given in equation 2a-2m:

$$\text{Objective function: } \sum_i c_i Q_{ci} + \sum_j c_j Q_{hj} \quad i \in CU, j \in HU \quad (2a)$$

Subject to

Equality constraints:

a) Process models (energy balances): assuming constant heat capacity flowrates ( $mC_p$ )

for process exchanger  $i$ :

$$Q_i - (mC_p)_i^{cold} (T_i^{cold,out} - T_i^{cold,in}) = 0 \quad i \in PHX \quad (2b)$$

$$Q_i - (mC_p)_i^{hot} (T_i^{hot,in} - T_i^{hot,out}) = 0 \quad i \in PHX \quad (2c)$$

for cooler  $i$ :

$$Q_{ci} - (mC_p)_i^{hot} (T_i^{hot,in} - T_i^{hot,out}) = 0 \quad i \in CU \quad (2d)$$

for heater  $i$ :

$$Q_{hi} - (mC_p)_i^{cold} (T_i^{cold,out} - T_i^{cold,in}) = 0 \quad i \in HU \quad (2e)$$

b) Connecting equations

supply connection:

$$T_i^{hot,in} = T_i^s \quad i \in HXHS \quad (2f)$$

$$T_i^{cold,in} = T_i^s \quad i \in HXCS \quad (2g)$$

internal connection:

$$T_i^{hot,out} - T_j^{hot,in} = 0 \quad i \in HXHO, j \in HXHI \quad (2h)$$

$$T_i^{cold,out} - T_j^{cold,in} = 0 \quad i \in HXCO, j \in HXCI \quad (2i)$$

target connection:

$$T_i^{hot,out} = T_i^t \quad i \in HXHT \cup CUT \quad (2j)$$

$$T_i^{cold,out} = T_i^t \quad i \in HXCT \cup HUT \quad (2k)$$

Inequality constraints:

lower bound:

$$-Q_i \leq 0 \quad i \in PHX \cup CU \cup HU \quad (2l)$$

upper bound: assuming constant thermal efficiency ( $P_{h,i}$ ) and heat capacity flowrate ( $mC_p$ )

$$Q_i \leq P_{h,i} (mC_p)_i^{hot} (T_i^{hot,in} - T_i^{cold,in}) \quad i \in PHX \cup CU \cup HU \quad (2m)$$

where

PHX: set of all process-process heat exchangers

CU: set of cold utility exchangers

HU: set of hot utility exchangers

HXHT: subset of PHX with hot side outlet is a controlled target

HXCT: subset of PHX with cold side outlet is a controlled target

CUT: subset of CU with outlet is a controlled target

HUT: subset of HU with outlet is a controlled target

HXHO: subset of PHX with hot side outlet entering a hot side inlet of the adjacent exchanger

HXCO: subset of PHX with cold side outlet entering a cold side inlet of the adjacent exchanger

HXHI: subset of PHX with hot side inlet coming from a hot side outlet of the adjacent exchanger

HXCI: subset of PHX with cold side inlet coming from a cold side outlet of the adjacent exchanger

HXHS: subset of PHX with hot side inlet directly coming from a hot supply

HXCS: subset of PHX with cold side inlet directly coming from a cold supply

$$P_{h,i}: \text{thermal efficiency of exchanger } i, \quad P_{h,i} = \frac{NTU_{h,i} (1 - e^{(NTU_{c,i} - NTU_{h,i})})}{NTU_{h,i} - NTU_{c,i} e^{(NTU_{c,i} - NTU_{h,i})}}$$

$$NTU_{h,i} = \frac{(UA)_i}{(mC_p)_i^{hot}}, \quad NTU_{c,i} = \frac{(UA)_i}{(mC_p)_i^{cold}}$$

$(mC_p)_i^{cold}$  and  $(mC_p)_i^{hot}$ : heat capacity flowrate on cold and hot side (kW/°C) of exchanger  $i$

$(UA)_i$ : product of heat transfer coefficient and heat transfer area of exchanger  $i$  (kW/°C)

As shown in the formulation, one process exchanger generates five variables in the vector  $x$  (inlet and outlet temperatures of hot and cold side, and heat duty, see equation 2b-2c) while one utility exchanger generates three variables (inlet and outlet temperatures and heat duty, see equation 2d-2e). Therefore, for a HEN containing  $N_{hx}$  process exchangers,  $N_{cu}$  coolers and  $N_{hu}$  heaters, the number of variables ( $N_{var}$ ) in the vector  $x$  becomes:

$$N_{var} = 5N_{hx} + 3N_{cu} + 3N_{hu} \quad (3)$$

In terms of equality constraints, one process exchanger generates two equality constraints (removed heat on hot side and received heat on cold side, see equation 2b-2c), while one utility exchanger generates one equality constraint (removed heat to a cooler or received heat from a heater, see

equation 2d-2e). The number of connecting equations is the sum of the number of supply specification ( $N_s$ , see equation 2f-2g), number of internal variable connection between the adjacent heat exchangers ( $N_{int,connect}$ , equation 2h-2i), and number of target specification ( $N_t$ , see equation 2j-2k). Therefore, the number of equality constraints ( $N_{eq}$ ) is:

$$N_{eq} = 2N_{hx} + N_{cu} + N_{hu} + N_s + N_t + N_{int,connect} \quad (4)$$

Each process exchanger and utility exchanger generates two inequality constraints (see equation 2l-2m) and hence the number of inequality constraints ( $N_{ineq}$ ) is:

$$N_{ineq} = 2(N_{hx} + N_{cu} + N_{hu}) \quad (5)$$

**Theorem 1** The optimal operation problem of a simple HEN\* is a LP problem

*Proof:* Equation 2a-2m

\*A simple HEN in this context refers to a HEN with 1) constant heat capacity flowrates, 2) only single bypasses (duties on individual process heat exchangers) and utility duties as degrees of freedom (manipulations), and 3) constant UA values for the heat exchangers. Note that the process stream flowrates and stream-splits are not considered as degrees of freedom.

□.

The main “trick” used above to show that the optimal operation of a HEN is a LP problem is to introduce the thermal efficiency  $P_{h,i}$  which avoids introducing the logarithmic mean temperature difference (LMTD) in the model for the heat transfer. The efficiency factors are constant under the assumption of constant heat capacity flowrates ( $mC_p$ ) and constant heat transfer coefficients (UA).

**Corollary 1.1** The optimal operation of a simple HEN lies always at constraints

*Proof:* Property of a LP problem.

□.

An important property of a LP problem is that one optimal solution is always in a “corner”. This implies that after satisfying in equality constraints (i.e. target temperatures), it is optimal to use all remaining degrees of freedom to satisfy active constraints (i.e. fully closing or opening of some bypasses or utility duties). From this follows Theorem 1 and Corollary 1.1.

The above LP problem may have multiple solutions (but not always) if there are some free degrees of freedom that may not affect the utility cost. This occurs when the HEN contains some loops. An idea to handle multiple solutions of the LP is discussed in the discussion section.

Note that it is possible to extend the LP formulation to include, for example, inequality constraints on temperatures (rather than targets) and other objective functions, for example, maximum temperature. However, the results in this paper are based on the above formulation by use of split-range control.

The LP formulation implies that the optimal solutions are always at a constraint (vertex). The inequality constraints in the above formulation (see equations 2l and 2m) imply active constraints on manipulations (i.e. duties of individual process and utility exchangers). This means that after the necessary degrees of freedom (manipulations) are used for control of the target temperatures (equality constraints), it is optimal to keep all remaining manipulations at constraints. However, under the variation of operating conditions, the optimal vertex (set of active constraints) may change. For a given operating window, we may have several optimal vertices for active constraint regions. Hence, if one can track the right active constraints during the operation, optimality can be obtained. One solution is to use an online optimization technique (e.g. Arkun and Stephanopoulos<sup>10</sup>). Alternatively,

one may try to avoid an online optimization task by using some logic to determine switching between active constraint regions and combine this with decentralized control. A particular implementation of the latter using common split-range control is the focus of this paper.

### 3. Switching between active constraints

#### 3.1 Preliminaries

In this section we describe methods for possible implementation of the optimal policy by tracking the changing set of active constraints. We make the following assumptions:

- A1. Target temperatures are feasible for the given disturbance window (output constraints do not change).
- A2. The output constraints do not change and are always active. The optimal point is a vertex, i.e., at the intersection of constraints and hence, a certain number of inputs are at the constraints.

Under these assumptions, the optimal solution has the following properties:

- 1) The set of active constraints remains constant in a certain region of the disturbance space. The largest region in the disturbance space where the set of active constraints remains the same is known as critical region. Critical regions are polyhedral in shape for a LP and can be determined using off-line optimization or parametric programming tools<sup>11</sup>.
- 2) If there are two or more critical regions in the given disturbance window, from the definition of critical region, it follows that the set of constraints are different. Since the output constraints do not change, it follows that the set of input constraints are different in each critical region. At the interface between two neighboring critical regions, constraints corresponding to both critical regions are active (which is a degenerate LP solution). However, since this constitutes a set of measure zero (i.e., the probability of being exactly on the boundary is zero), it does not affect the controllability properties of the network on the whole.

Using these properties of the optimal solution, it is possible to operate the HEN optimally using the following procedure:

- 1) In a given critical region  $R_o$ , it is possible to operate the HEN optimally using a decentralized control structure where some manipulations are used to control the output constraints using SISO control loops with zero steady state error, for example, PI controllers. The remaining manipulations are maintained at the constraints.
- 2) If the disturbances are such that we have moved from  $R_o$  to a different region  $R_l$ , it is possible to implement the optimal policy in  $R_l$  by tracking the transition or change in active constraints.

For example, suppose we have a system with 3 manipulations and 2 controlled variables (target temperatures, T1 and T2). Clearly, we need 2 manipulations for control. Furthermore, since one optimal solution is always at input constraints, the remaining manipulation may be at constraints (saturated). For a given operating window, active constraint regions can be found using parametric programming and the results can be summarized as shown in Table 1.



Table 1 Set of active constraints for example process

| Region | MV <sub>1</sub> | MV <sub>2</sub> | MV <sub>3</sub> |
|--------|-----------------|-----------------|-----------------|
| 1      | S               | U               | U               |
| 2      | U               | U               | S               |
| 3      | U               | S               | U               |

U-Unsaturated manipulation (inactive constraint)

to be used for control of target temperatures

S-Saturated manipulation (active constraint)

Thus, in region 1, it is optimal to use MV<sub>2</sub> and MV<sub>3</sub> to control the outputs T1 and T2 respectively using SISO PI control loops and keep MV<sub>1</sub> at constraint. When moving into region 2, MV<sub>3</sub> saturates and so, the optimal policy is to keep MV<sub>3</sub> at the constraint and instead use MV<sub>1</sub> as a manipulation for control. Thus MV<sub>1</sub> and MV<sub>2</sub> are used for control in region 2. Likewise, in region 3, the optimal policy is to control T1 and T2 using MV<sub>1</sub> and MV<sub>3</sub> and keep MV<sub>2</sub> at constraint. It is possible to keep track of the regions by tracking the changes in active constraints. When the new region is determined, the optimal policy corresponding to the new region is implemented. We discuss two ways of implementing this policy:

#### Implementation 1: using switching logic

In this method, a switching logic based on the current state and change in some set of active constraints is used to determine the corresponding control law. The switching logic can be represented as:

##### Switching between regions 1 and 2

- MV<sub>2</sub> is inactive constraint, MV<sub>3</sub> becomes active constraint in region 2
- MV<sub>2</sub> is inactive constraint, MV<sub>1</sub> becomes active constraint in region 1

##### Switching between regions 1 and 3

- MV<sub>3</sub> is inactive constraint, MV<sub>2</sub> becomes active constraint in region 3
- MV<sub>3</sub> is inactive constraint, MV<sub>1</sub> becomes active constraint in region 1

##### Switching between regions 2 and 3

- MV<sub>1</sub> is inactive constraint, MV<sub>2</sub> becomes active constraint in region 3
- MV<sub>1</sub> is inactive constraint, MV<sub>3</sub> becomes active constraint in region 2

This switching logic with the 3 set of decentralized controllers (corresponding to regions 1, 2, and 3) can be used to implement the operating policy and is optimal in the presence of disturbances without the need to directly measure the disturbances and re-optimize the plant. The logic can be extended to more general situations using finite state machines.

However, in general, the switching logic can become very complicated. In some circumstances, a simpler implementation is possible using a split range controller. In the remainder of this work, we focus on implementation of the optimal solution using a split range controller.

#### Implementation 2: using split range control

Split range controllers are commonly used to control two or more manipulations using a single controller. A technique using structural information (sign matrix) to find a control structure for optimal operation of HENs was proposed by Mathisen et al.<sup>5</sup> and Glemmestad et al.<sup>6</sup>. They commented that in most case the obtained control structure can be implemented in a split-range control manner. Depending on the directional effect of the manipulations, different control configurations are possible<sup>2</sup>. When 2 manipulations are used, we refer to one of them as primary and the other as secondary. The primary manipulation can be thought of as the manipulation that is used

to control a target under nominal conditions. However, the final choice of primary and secondary can be based on other considerations also. This flexibility will be exploited in the final control structure design.

In addition to the assumptions A1-A2, in order to obtain a decentralized control structure using split range control, further assumptions made are:

- A3. One split-range combination contains only two manipulations. Hence, each primary manipulation can have only one secondary manipulation. Note that this does not rule out the possibility that a variable that is treated as a secondary manipulation can be used in two or more split range controllers.
- A4. Only one saturation (upper or lower bounds) is allowed for each manipulation

A simple illustration will be provided for the above example. Assume that region 1 is the “primary” region. Then  $MV_2$  and  $MV_3$  are the “primary” manipulations used for control of the target temperatures. For optimality, the active constraint should be switched to  $MV_3$  when operation moves into region 2, and to  $MV_2$  in region 3. In terms of control, when moving to region 2,  $MV_1$  needs to take over the task of saturated  $MV_3$  (“ $MV_1$  is used as a secondary manipulation for  $MV_3$ ”), and when moving to region 3  $MV_1$  needs to take over the task of saturated  $MV_2$  (“ $MV_1$  is used as a secondary manipulation for  $MV_2$ ”). Hence, we should combine  $MV_2$  &  $MV_1$  and  $MV_3$  &  $MV_1$  as split-range pairs using  $MV_1$  as the secondary manipulation. This control system can be shown in Fig 1.

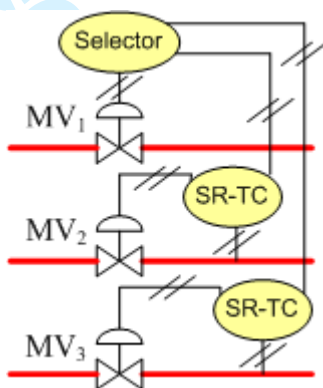


Fig 1 Control system of the example process  
(SR-TC = split-range temperature controller)

In the above example, the choices of the secondary manipulations for the primary manipulations could be determined by inspection. In general problems, with a large number of manipulations and regions of active constraints, this is not a trivial task. Hence, a systematic method of determining this pairing is necessary. Lersbamrungsuk et al.<sup>2</sup> showed how the optimal split-range control structure can be found by using the information of directional effect (sign element). However, when the sign is unclear, the control structure cannot guarantee optimality. We here present an optimization formulation that determines an optimal split range control structure.

### 3.2 ILP formulation to determine split-range control structure

Assuming that A1-A4 hold and the set of active constraints in the critical regions is known, an integer linear programming (ILP) formulation for the design of an optimal split-range control structure is shown in the problem P1 (see some definition and more details in Appendix).

Let  $x_{i,j}$  be a binary variable, where  $x_{i,i} = 1$  denotes that input  $i$  is a primary manipulation and  $x_{i,j} = 1$  denotes that input  $j$  is a secondary manipulation for input  $i$ . Determining the optimal control structure can be posed as the following optimization problem (P1):

**Problem P1**

Objective function I:  $\min J_I = \sum_i \sum_{j \neq i} x_{i,j} \quad i, j \in \mathbf{MV}$

(This objective is to minimize the number of “inter-connections” or “complexity” in the control structure.)

Subject to:

Constraint 1: Assign one primary manipulation to each control objective

$$\sum_i x_{i,i} = N_{CV} \quad i \in \mathbf{MV}$$

Constraint 2: A manipulation  $MV_i$  that always is an active constraint should not be used for other purposes

$$\begin{aligned} x_{i,i} &= 0 & i \in \mathbf{MVAAT} \\ \sum_{j \neq i} x_{i,j} &= 0 & i \in \mathbf{MVAAT}, j \in \mathbf{MV} \\ \sum_{j \neq i} x_{j,i} &= 0 & i \in \mathbf{MVAAT}, j \in \mathbf{MV} \end{aligned}$$

Constraint 3: A manipulation  $MV_i$  that is never an active constraint is used as a primary manipulation with no need of a secondary manipulation

$$\begin{aligned} x_{i,i} &= 1 & i \in \mathbf{MVINAT} \\ \sum_{j \neq i} x_{i,j} &= 0 & i \in \mathbf{MVINAT}, j \in \mathbf{MV} \\ \sum_{j \neq i} x_{j,i} &= 0 & i \in \mathbf{MVINAT}, j \in \mathbf{MV} \end{aligned}$$

Constraint 4: A manipulation  $MV_i$  that changes between being an active and inactive constraints may be a primary or secondary manipulations.

$$\begin{aligned} -x_{i,i} + \sum_{j \neq i} x_{i,j} &= 0 & i, j \in \mathbf{MVAT} \\ x_{j,j} + \sum_{i \neq j} x_{i,j} &\geq 1 & i, j \in \mathbf{MVAT} \\ M(x_{j,j} - 1) + \sum_{i \neq j} x_{i,j} &\leq 0 & i, j \in \mathbf{MVAT} \end{aligned}$$

Constraint 5: Possible and impossible split-range combination of manipulations

Constraints obtained from the information of active constraint regions  
(see example A1 in Appendix)

By solving the problem P1, one obtains split-range pairs that can provide optimal switching between active constraint regions. However, the solution of problem P1 may be non-unique. Hence, relative orders are introduced as an additional criterion for screening the set of poorly controllable structure solutions using lexicographic optimization. In lexicographic optimization, the objectives are arranged

in a decreasing order of preference and objectives with a higher preference are considered to be infinitely more important than those with lower orders. Among solutions that are optimal with respect to the first objective, solutions that are optimal with respect to the second objective are chosen. In this example, we assume that minimizing the sum of relative orders of control pairing (for fast control action) is the second objective. Let  $z_{k,j}$  be a binary variable that represents the relationship between controlled variable  $CV_k$  and manipulation  $MV_j$ ,

$z_{k,j} = 1$  denotes controlled variable  $CV_k$  is paired with manipulation  $MV_j$

$z_{k,j} = 0$  denotes controlled variable  $CV_k$  is not paired with manipulation  $MV_j$

Let  $r_{k,j}$  be a relative order between controlled variable  $CV_k$  and manipulation  $MV_j$ . However, for simplicity we here assume  $r_{k,j}$  as a number of exchanger units between controlled variable  $CV_k$  and manipulation  $MV_j$ . Thus, the secondary objective is to minimize  $J_{II}$  where

$$J_{II} = \sum_k \sum_j r_{k,j} z_{k,j}$$

Further details are presented in the Appendix.

Using the idea of lexicographic optimization, we first solve P1:

$$J_I^* = \min_x J_I(x), \quad x \in S$$

where  $S$  is the feasible set and then solve an associated problem P1':

$$\min_x J_{II}(x), \quad x \in S, \quad J_I = J_I^*(x)$$

which ensures that among minimized  $J_I$  solutions, the minimized  $J_{II}$  solutions are chosen. In principle, we need to solve 2 optimization problems in sequence. However, it is possible to solve P1 and P1' as a single optimization problem by minimizing a weighted objective function  $wJ_I + J_{II}$ , where  $w$  is a sufficiently large positive number chosen appropriately. Suggestions for choice of  $w$  are given in Sherali<sup>12</sup>, and Sherali and Soyster<sup>13</sup>. Hence, we solve the following problem P2:

### **Problem P2**

Objective function:  $J = \min(wJ_I + J_{II}) \quad i, j \in \mathbf{MV}, k \in \mathbf{CV}$

$$J_I = \sum_i \sum_{j \neq i} x_{i,j}$$

$$J_{II} = \sum_k \sum_j r_{k,j} z_{k,j}$$

Subject to:

Constraint 1-5

Constraint 6: Assign one manipulation ("pairing") to each control objective

$$\sum_j z_{k,j} = 1 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

Constraint 7: Only primary manipulations are paired with controlled variables.

$$-x_{j,j} + \sum_k z_{k,j} = 0 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

It can be seen that Constraints 6 and 7 do not alter the feasible set for P1. The ILP problem P2 consists of two objective functions with a weighting factor ( $w$ ) between the two. The first objective is used to minimize complexity when changing between active constraints whereas the second objective (controllability) is used to select the most controllable control structure. A large value of  $w$  will imply that the second objective (controllability) will only be considered when there are multiple solutions.

### 4. A HEN case study

The HEN in Fig 2 is from the work of Aguilera and Marchetti<sup>7</sup> but we have modified it to use only single bypasses. The HEN contains two hot and two cold streams with four target outlet temperatures of stream H1, H2, C1 and C2 ( $T_{H1}^{out}$ ,  $T_{H2}^{out}$ ,  $T_{C1}^{out}$ , and  $T_{C2}^{out}$ ). The utility prices are 0.05 \$/kWh for hot utility h, 0.02 \$/kWh for cold utility c1 and 0.01 \$/kWh for cold utility c2.

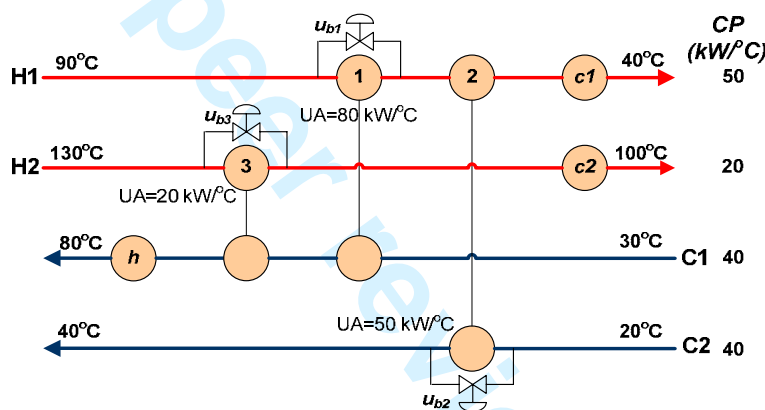


Fig 2 A HEN case study

There are six degrees of freedom (heat duties of all exchangers,  $Q$ ) and four equality constraints on the outlet temperatures. This leaves two degrees of freedom. Assuming the disturbance is the inlet temperature of each stream with the expected variation  $\pm 10$  °C for the inlet temperatures of stream H1, H2, and C1 and  $\pm 5$  °C for the inlet temperature of stream C2, this results in feasible optimal solutions. The resulting 5 active constraint regions are obtained by solving the LP optimization problem as shown in Table 2. As expected there are two active constraints (saturated manipulations) in each region.

Table 2 Set of active constraints in the case study

| Region | Manipulations  |                |                |                |          |                |
|--------|----------------|----------------|----------------|----------------|----------|----------------|
|        | $Q_{c1}$       | $Q_{c2}$       | $Q_h$          | $u_{b1}$       | $u_{b2}$ | $u_{b3}$       |
| 1      | S <sub>L</sub> | U              | S <sub>L</sub> | U              | U        | U              |
| 2      | S <sub>L</sub> | S <sub>L</sub> | U              | U              | U        | U              |
| 3      | U              | S <sub>L</sub> | U              | S <sub>L</sub> | U        | U              |
| 4      | U              | U              | S <sub>L</sub> | S <sub>L</sub> | U        | U              |
| 5      | U              | U              | S <sub>L</sub> | U              | U        | S <sub>U</sub> |

U-Unsaturated manipulation (inactive constraint),

S<sub>L</sub>-Saturated manipulation (active constraint) at the lower bound,

S<sub>U</sub>-Saturated manipulation (active constraint) at the upper bound

Table 2 demonstrates that manipulations  $Q_{c1}$ ,  $Q_{c2}$ ,  $Q_h$  and  $u_{b1}$  can become active constraints at the lower bounds (i.e. zero utility duties or fully close of bypasses) while manipulation  $u_{b3}$  can become an

active constraint at the upper bound (i.e. fully open of bypasses). The manipulation  $u_{b2}$  is never an active constraint (never saturated) and hence it should be used as a primary manipulation with no need of a secondary manipulation.

The software “GAMs” with the solver “CPLEX” was used to solve the ILP. The solution to problem P1 (minimizing complexity of optimal split-range pairs) in Table 3 shows that  $Q_{c1}$ ,  $Q_{c2}$ ,  $u_{b1}$  and  $u_{b2}$  are chosen to be primary manipulations (see diagonal elements with  $x_{i,i} = 1$ ) while  $Q_h$  and  $u_{b3}$  are chosen to be secondary manipulations (see diagonal elements with  $x_{i,i} = 0$ ).  $Q_h$  is the secondary manipulation for  $Q_{c2}$  ( $x_{2,3} = 1$ ) and  $u_{b3}$  is the secondary manipulation for  $Q_{c1}$  and  $u_{b1}$  ( $x_{1,6} = 1$  and  $x_{4,6} = 1$ ). However, the solution obtained from the problem P1 may not be unique. For example, by including a constraint  $x_{3,3} = 1$  (i.e. set  $Q_h$  as a primary manipulation) in problem P1, a different solution with the same value of objective function I ( $J_I=3$ ) is obtained as shown in Table 4.

Table 3 The values of  $x_{i,j}$  after solving the ILP problem P1 ( $J_I=3$ )

| Pri MV \ Sec MV | $Q_{c1}$ | $Q_{c2}$ | $Q_h$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|-----------------|----------|----------|-------|----------|----------|----------|
| $Q_{c1}$        | 1        |          |       |          |          | 1        |
| $Q_{c2}$        |          | 1        | 1     |          |          |          |
| $Q_h$           |          |          | 0     |          |          |          |
| $u_{b1}$        |          |          |       | 1        |          | 1        |
| $u_{b2}$        |          |          |       |          | 1        |          |
| $u_{b3}$        |          |          |       |          |          | 0        |

(the remaining entries are zero)

Table 4 The values of  $x_{i,j}$  after solving the ILP problem P1 with setting  $x_{3,3} = 1$  ( $J_I=3$ )

| Pri MV \ Sec MV | $Q_{c1}$ | $Q_{c2}$ | $Q_h$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|-----------------|----------|----------|-------|----------|----------|----------|
| $Q_{c1}$        | 1        |          |       |          |          | 1        |
| $Q_{c2}$        |          | 0        |       |          |          |          |
| $Q_h$           |          | 1        | 1     |          |          |          |
| $u_{b1}$        |          |          |       | 1        |          | 1        |
| $u_{b2}$        |          |          |       |          | 1        |          |
| $u_{b3}$        |          |          |       |          |          | 0        |

(the remaining entries are zero)

To handle the multiple solutions of problem P1, the second objective  $J_{II}$  (controllability purpose in terms of minimizing the sum of relative orders) is introduced and included in problem P2 for selecting the most controllable control structure. The additional information of relative orders is shown in Table 5. The values of binary variables  $x_{i,j}$  and  $z_{k,j}$  from solving the problem P2 are shown in Tables 6 and 7, respectively.

Table 5 Relative orders of the HEN in the case study

| CV \ MV        | $Q_{c1}$ | $Q_{c2}$ | $Q_h$    | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|----------------|----------|----------|----------|----------|----------|----------|
| $T_{H1}^{out}$ | 1        | $\infty$ | $\infty$ | 3        | 2        | $\infty$ |
| $T_{H2}^{out}$ | $\infty$ | 1        | $\infty$ | 3        | $\infty$ | 2        |
| $T_{C1}^{out}$ | $\infty$ | $\infty$ | 1        | 3        | $\infty$ | 2        |
| $T_{C1}^{out}$ | $\infty$ | $\infty$ | $\infty$ | 2        | 1        | $\infty$ |

Table 6 The values of  $x_{i,j}$  after solving the ILP problem P2 ( $J_I=3$ )

| Sec MV \ Pri MV | $Q_{c1}$ | $Q_{c2}$ | $Q_h$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|-----------------|----------|----------|-------|----------|----------|----------|
| $Q_{c1}$        | 1        |          |       | 1        |          |          |
| $Q_{c2}$        |          | 1        | 1     |          |          |          |
| $Q_h$           |          |          | 0     |          |          |          |
| $u_{b1}$        |          |          |       | 0        |          |          |
| $u_{b2}$        |          |          |       |          | 1        |          |
| $u_{b3}$        |          |          |       | 1        |          | 1        |

Table 7 The values of  $z_{k,j}$  after solving the ILP problem P2 ( $J_{II}=5$ )

| CV \ MV        | $Q_{c1}$ | $Q_{c2}$ | $Q_h$ | $u_{b1}$ | $u_{b2}$ | $u_{b3}$ |
|----------------|----------|----------|-------|----------|----------|----------|
| $T_{H1}^{out}$ | 1        |          |       |          |          |          |
| $T_{H2}^{out}$ |          | 1        |       |          |          |          |
| $T_{C1}^{out}$ |          |          |       |          |          | 1        |
| $T_{C2}^{out}$ |          |          |       |          | 1        |          |

Table 6 shows  $Q_{c1}$ ,  $Q_{c2}$ ,  $u_{b2}$  and  $u_{b3}$  are chosen to be primary manipulations while  $Q_h$  and  $u_{b1}$  are chosen to be secondary manipulations. Table 7 shows the appropriate control pairing,  $T_{H1}^{out} - Q_{c1}$ ,  $T_{H2}^{out} - Q_{c2}$ ,  $T_{C1}^{out} - u_{b3}$  and  $T_{C2}^{out} - u_{b2}$  (see  $z_{1,1} = z_{2,2} = z_{3,6} = z_{4,5} = 1$ ). The control structure for optimal operation of the HEN in this case study is shown in Fig 3.

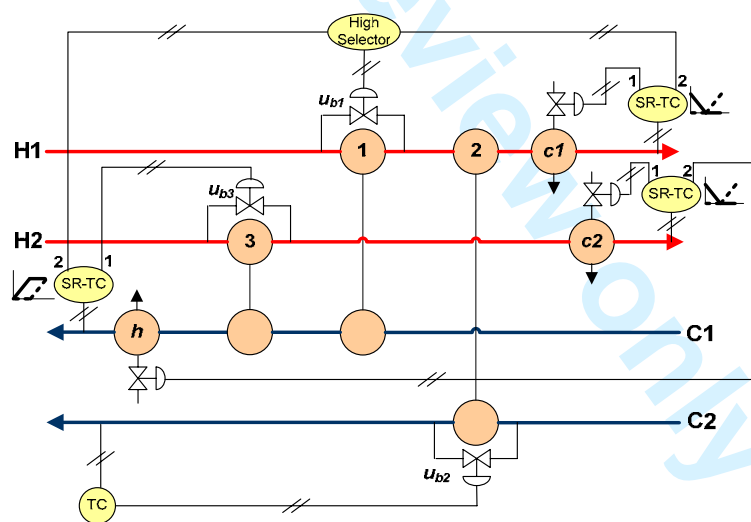


Fig 3 An optimal control structure for the HEN in the case study

From Fig 3, the SR-TC block represents a split-range controller with port 1 to send a signal to a primary manipulation and port 2 to send a signal to a secondary manipulation. The split-range signal of each split-range controller can be obtained by considering the information of active constraint regions (Table 2). For example, the split-range signal of the pair of  $Q_{c1}$  and  $u_{b1}$  is (thick line - primary manipulation, dot line - secondary manipulation) because  $Q_{c1}$  and  $u_{b1}$  switch alternately to their lower constraints. Furthermore, because  $u_{b1}$  is chosen as the secondary manipulation for more than one primary manipulation (i.e. either  $Q_{c1}$  or  $u_{b3}$ ), a selective controller is used to select the secondary signal from control loops.

## 5. Dynamic simulation

The HEN in the case study with the suggested control structure is tested by performing dynamic simulation on Aspen Dynamics v12.1. The information of disturbances and active constraints of the system at each period are shown in Table 8. Fig 4b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulations (see Fig 4d and 4e). The input saturation problem is solved by switching ability to use a secondary manipulation when a primary manipulation is saturated. Furthermore, the optimality (in term of utility cost) is also given as shown in Fig 4c that the graph of utility cost can track the optimal line. This consequence comes from the ability of the control structure to track the right active constraint during the operation (see Fig 4d and 4e, and set of active constraint in Table 8).

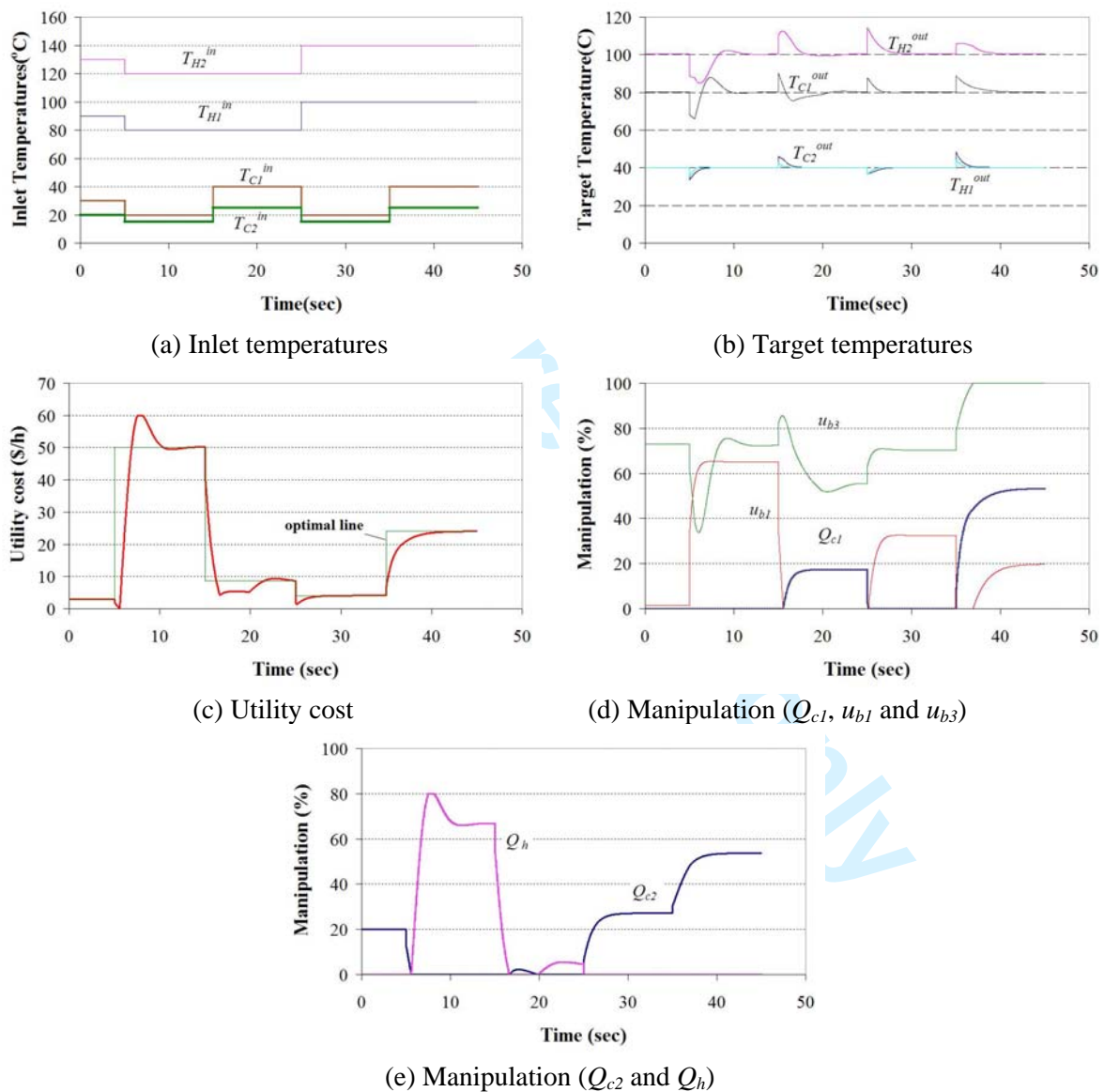


Fig 4 Dynamic simulation of the HEN in case study with the suggested control structure



Table 8 Disturbances and active constraints for each period

| Time (sec)   | Disturbance          |                      |                      |                      | Active constraint |                |                |                |          |                |
|--------------|----------------------|----------------------|----------------------|----------------------|-------------------|----------------|----------------|----------------|----------|----------------|
|              | $\Delta T_{H1}^{in}$ | $\Delta T_{H2}^{in}$ | $\Delta T_{C1}^{in}$ | $\Delta T_{C2}^{in}$ | $Q_{c1}$          | $Q_{c2}$       | $Q_h$          | $u_{b1}$       | $u_{b2}$ | $u_{b3}$       |
| less than 5  | 0                    | 0                    | 0                    | 0                    | S <sub>L</sub>    |                | S <sub>L</sub> |                |          |                |
| 5-15         | -10                  | -10                  | -10                  | -5                   | S <sub>L</sub>    | S <sub>L</sub> |                |                |          |                |
| 15-25        | -10                  | -10                  | 10                   | 5                    |                   | S <sub>L</sub> |                | S <sub>L</sub> |          |                |
| 25-35        | 10                   | 10                   | -10                  | -5                   | S <sub>L</sub>    |                | S <sub>L</sub> |                |          |                |
| more than 35 | 10                   | 10                   | 10                   | 5                    |                   |                | S <sub>L</sub> |                |          | S <sub>U</sub> |

S<sub>L</sub>-Saturated manipulation (active constraint) at the lower bound,

S<sub>U</sub>-Saturated manipulation (active constraint) at the upper bound

## 6. Discussion

We have already seen that the split-range implementation (solution to Problem P1) of the LP optimal utility cost solution may be non-unique. This was the main reason for including the controllability in term of the relative order as a secondary objective (Problem P2). However, it is also possible that the LP optimal solution itself is non-unique. This may happen if there are loops in the network<sup>14</sup> because of the possibility to shift duty around loops without affecting the utility cost. Also in this case, same secondary objective may be added into the LP optimization, but one should be careful to avoid changing the optimal solution including the range of feasible solutions. The number of loops ( $N_{loops}$ ) is given by<sup>14</sup>

$$N_{loops} = N_{units} - R - N_U$$

where  $N_{units}$  is the number of process exchangers and utility exchangers,  $R$  is the dimensional space spanned by the manipulations in the inner HEN to the outer HEN, and  $N_U$  is the number of utility types.

In the network of the case study, we have  $N_{units}=6$ ,  $R=3$  and  $N_U=3$ , so  $N_{loops}=0$ . Thus there are no loops and the LP optimal solution is unique. Note that if the two cold utilities have the same cost, then  $N_U=2$  and we would have  $N_{loops}=1$ , and the optimal solution might in some cases be non-unique because of the possibility to have duty shift between the two coolers without affecting the utility cost. Next, we will consider a trivial example with one "internal" loop.

Example 1: A trivial HEN with a loop

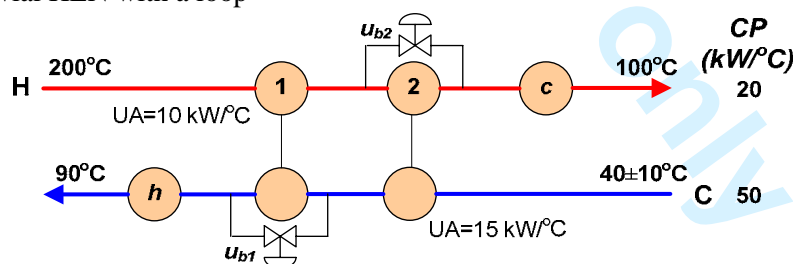


Fig 5 A trivial HEN with a loop

The HEN in Fig 5 contains one hot stream and one cold stream with two target temperatures (outlet temperatures of two streams). Each process exchanger has a single bypass. This network has  $N_{units}=4$ ,  $R=1$ ,  $N_U=2$ , and  $N_{loops}=4-1-2=1$ . This implies that there is one degree of freedom that may be used for some purposes without affecting the optimum of utility cost due to the duty shift between exchangers 1 and 2.

Consider a disturbance of  $\pm 10$  °C in the inlet temperature of the cold stream C ( $T_C^{in}$ ). The optimization result and active constraint regions (see the left side of Tables 9 and 10) from the LP utility cost

optimization problem shows that the manipulation  $Q_h$  (duty of the heater) is never saturated and hence  $Q_h$  has no need of a secondary manipulation. For the control pairing, to get a direct effect,  $Q_h$  is used to control  $T_C^{out}$  (the outlet temperature of the cold stream C) while  $Q_c$  (duty of cooler) is used to control  $T_H^{out}$  (the outlet temperature of hot stream H). However, because  $Q_c$  can be saturated in some operating conditions, it requires a secondary manipulation which may be  $u_{b1}$  or  $u_{b2}$  (bypasses of exchangers 1 or 2) or probably both. If the result on the left side of Table 9 is considered, the choice of secondary manipulation is not quite clear because both  $u_{b1}$  and  $u_{b2}$  are in use. However, for the reason of direct effect,  $u_{b2}$  seems to be a better choice. In general, this solution can be found by performing a two-step optimization (i.e. lexicographic optimization) with first solving for the utility cost, and then maximizing the duty of exchanger 1 ( $Q_1$ ) according to the optimal utility cost in the first step. This results in the LP solution with 2 active constraints as shown on the right side of Tables 9 and 10.

Table 9 Optimization result of the HEN in example 1

| Disturbance       | Minimize utility cost<br>(without handling multiple solutions) |          |                  |                  | Minimize utility cost and maximize $Q_1$<br>(with handling multiple solutions) |          |                  |                  |
|-------------------|--|----------|------------------|------------------|--|----------|------------------|------------------|
|                   | $u_{b1}$   | $u_{b2}$ | $Q_c(\text{kW})$ | $Q_h(\text{kW})$ | $u_{b1}$   | $u_{b2}$ | $Q_c(\text{kW})$ | $Q_h(\text{kW})$ |
| $\Delta T_C^{in}$ |  |          |                  |                  |  |          |                  |                  |
| 0                 | 0.3171*  | 0.1966*  | 0                | 500              | 0  | 0.2424   | 0                | 500              |
| -10               | 0.6002*  | 0.3612*  | 0                | 1000             | 0  | 0.4635   | 0                | 1000             |
| +10               | 0  | 0        | 48               | 48               | 0  | 0        | 48               | 48               |

\*multiple optimal solutions due to duty shift between exchangers in loops

Table 10 Active constraint regions of the HEN in example 1

| Region | Minimize utility cost<br>(without handling multiple solutions) |                |                  |                  | Minimize utility cost and maximize $Q_1$<br>(with handling multiple solutions) |                |                  |                  |
|--------|--|----------------|------------------|------------------|--|----------------|------------------|------------------|
|        | $u_{b1}$   | $u_{b2}$       | $Q_c(\text{kW})$ | $Q_h(\text{kW})$ | $u_{b1}$   | $u_{b2}$       | $Q_c(\text{kW})$ | $Q_h(\text{kW})$ |
| 1      | U*   | U*             | S <sub>L</sub>   | U                | S <sub>L</sub>   | U              | S <sub>L</sub>   | U                |
| 2      | S <sub>L</sub>   | S <sub>L</sub> | U                | U                | S <sub>L</sub>   | S <sub>L</sub> | U                | U                |

U – Unsaturated manipulation (inactive constraint),

S<sub>L</sub> – Saturated manipulation (active constraint) at the lower bound

\*multiple optimal solutions

Note that the information of active constraints needed in constraint 5 of the ILP should be obtained from the solution in the right side of Table 10. For example, if the information of active constraints on the left side of Table 10 is used in the ILP, there will be no feasible solution.

The result of active constraints regions from the solution in the right side of Table 10 shows that  $Q_c$  and  $u_{b2}$  switch alternately to be an active constraint and hence should be combined as a split-range pair. Moreover, because  $u_{b1}$  is always an active constraint, it should be assigned at the constraint for optimality. For this simple HEN, the optimal split-range control structure can be obviously found without the need of the ILP as shown in Fig 6.

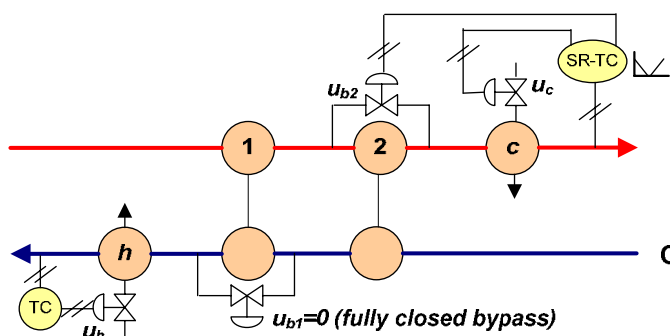


Fig 6 A trivial HEN with an optimal split-range control structure (SR-TC = split-range temperature controller)

## 7. Conclusions

When only single bypasses and utility duties are used as manipulations, optimal operation of HENs can be formulated as a linear programming implying the operation lies always at some input constraints. However, under the change of operating condition, the active constraints may change. This motivates the need of a control strategy with the ability to track active constraints under the change of operating condition. In this work, we focused on a decentralized control structure with the ability to provide appropriate switching between active constraints regions in a given operating window. This results in an optimal split-range control structure which can be found by solving an integer linear programming.

It is possible for some HENs to have no optimal split-range control structure (i.e. no feasible solution of the ILP). Hence, a study on a technique for switching between active constraint regions should be further investigated. We expect that this technique is not only able to be applied for constraint (vertex) optimal operation problem, but also for unconstrained (non-vertex) optimal operation problem (e.g. simplifying an online optimization task).

## 8. Acknowledgement

Financial support from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program (Grant No. PHD/0145/2547) and the support from the department of chemical engineering, Norwegian University of Science and Technology, Trondheim, Norway during the visit of Mr. Veerayut Lersbamrungsuk are gratefully acknowledged.

## 9. Literature Cited

1. Skogestad S. Plantwide control: the search for the self-optimizing control structure. *Journal of Process Control*. 2000;10:487-507.
2. Lersbamrungsuk V, Skogestad S, Srinophakun T. A simple strategy for optimal operation of heat exchanger networks. *International conference on modeling in chemical and biological engineering sciences*, Bangkok, Thailand, 2006.
3. Marselle DF, Morari M, Rudd DF. Design of resilient processing plants-II Design and control of energy management systems. *Chemical Engineering Science*. 1982;37:259-270.
4. Calandranis J, Stephanopoulos G. (1988). A structural approach to the design of control systems in heat exchanger networks. *Computer and Chemical Engineering*. 1988;12:651-669.
5. Mathisen KW, Morari M, and Skogestad S. Optimal operation of heat exchanger networks. *Process systems engineering (PSE'94)*, Kyongju, Korea, 1994.
6. Glemmestad B, Mathisen KW, Gundersen T. Optimal operation of heat exchanger networks based on structural information. *Computer and Chemical Engineering*. 1996;20(suppl):S823-S828.

7. Aguilera N, Marchetti JL. Optimizing and controlling the operation of heat exchanger Networks. *AIChE Journal*. 1998;44:1090-1104.
8. Glemmestad B, Skogestad S, Gundersen T. Optimal operation of heat exchanger networks. *Computer and Chemical Engineering*. 1999;23:509-522.
9. González AH, Odloak D, Marchetti JL. Predictive control applied to heat-exchanger networks. *Chemical Engineering and Processing*. 2006;45:661-671.
10. Arkun Y, Stephanopoulos G. Studies in the synthesis of control structures for chemical processes: part IV. design of steady-state optimizing control structures for chemical process units. *AIChE Journal*. 1980;26:975-991.
11. Kvasnica M, Grieder P, Baotic M. Multi-parametric toolbox (MP). <http://control.ee.ethz.ch/~mpt>, 2004.
12. Sherali HD. Equivalent weights for lexicographic multi-objective programs: Characterizations and computations. *European Journal of Operational Research*, 1982;11:367-379.
13. Sherali HD, Soyster AL. Preemptive and non-preemptive multi-objective programming: Relationships and counter examples. *Journal of Optimization Theory and Application*. 1983;39:173-186.
14. Glemmestad B. Optimal operation of integrated processes: studies on heat recovery systems. PhD thesis. NTNU Trondheim, Norway, 1997.

## Appendix

### ILP formulation for the design of an optimal split-range control structure

**Definition A1** Set of controlled and manipulated variables

**CV**: set of controlled variables,  $\mathbf{CV} = \{ CV_1, CV_2, \dots, CV_{N_{CV}-1}, CV_{N_{CV}} \}$

**MV**: set of manipulations,  $\mathbf{MV} = \{ MV_1, MV_2, \dots, MV_{N_m-1}, MV_{N_m} \}$

**MVAAT**: subset of **MV** with manipulations which are always active constraints (saturated at upper or lower bounds)

**MVINAT**: subset of **MV** with manipulations which are always inactive constraints (never saturated)

**MVAT**: subset of **MV** with manipulations which change between being active and inactive constraints

**Definition A2** Primary and secondary manipulations

Primary manipulation: A manipulation that is used for controlling an output (target), except when it is saturated.

Secondary manipulation: A manipulation that is used to take over the task of a saturated primary manipulation.

**Definition A3** Relationship between primary and secondary manipulations

Let  $x_{i,j}$  (where  $i,j \in \mathbf{MV}$ ) be a binary variable which represents the relationship between manipulation  $MV_i$  and manipulation  $MV_j$  as shown in Table A1.

Table A1 Relationship between primary and secondary manipulations

| Sec MV \ Pri MV               | MV <sub>1</sub> | MV <sub>2</sub> | ... | ... | MV <sub>N<sub>m</sub>-1</sub> | MV <sub>N<sub>m</sub></sub> |
|-------------------------------|-----------------|-----------------|-----|-----|-------------------------------|-----------------------------|
| MV <sub>1</sub>               | $x_{1,1}$       | $x_{1,2}$       | ... | ... | $x_{1,N_m-1}$                 | $x_{1,N_m}$                 |
| MV <sub>2</sub>               | $x_{2,1}$       | $x_{2,2}$       | ... | ... | $x_{2,N_m-1}$                 | $x_{2,N_m}$                 |
| ⋮                             | ...             | ...             | ... | ... | ...                           | ⋮                           |
| ⋮                             | ...             | ...             | ... | ... | ...                           | ⋮                           |
| MV <sub>N<sub>m</sub>-1</sub> | $x_{N_m-1,1}$   | $x_{N_m-1,2}$   | ... | ... | ...                           | ⋮                           |
| MV <sub>N<sub>m</sub></sub>   | $x_{N_m,1}$     | $x_{N_m,2}$     | ... | ... | $x_{N_m,N_m-1}$               | $x_{N_m,N_m}$               |

for  $i=j$ ,

$x_{i,i} = 1$  implies manipulation MV<sub>i</sub> is a primary manipulation

$x_{i,i} = 0$  implies manipulation MV<sub>i</sub> is a secondary manipulation or unused

for  $i \neq j$ ,

$x_{i,j} = 1$  implies manipulation MV<sub>j</sub> is a secondary manipulation for MV<sub>i</sub>

$x_{i,j} = 0$  implies manipulation MV<sub>j</sub> is not a secondary manipulation for MV<sub>i</sub>

Number of variables  $x_{i,j} = N_m \times N_m$ , where  $N_m$  is number of manipulations

**Definition A4:** Relative order between manipulations and controlled variables

Let  $r_{k,j}$  be a relative order between controlled variable CV<sub>k</sub> and manipulation MV<sub>j</sub>. For a system with  $N_{CV}$  controlled variables and  $N_m$  manipulations, relative order matrix can be shown in Table A2.

Table A2 Relative order matrix

| CV \ MV                        | MV <sub>1</sub>  | MV <sub>2</sub>  | ... | ... | MV <sub>N<sub>m</sub>-1</sub> | MV <sub>N<sub>m</sub></sub> |
|--------------------------------|------------------|------------------|-----|-----|-------------------------------|-----------------------------|
| CV <sub>1</sub>                | $r_{1,1}$        | $r_{1,2}$        | ... | ... | $r_{1,N_m-1}$                 | $r_{1,N_m}$                 |
| CV <sub>2</sub>                | $r_{2,1}$        | $r_{2,2}$        | ... | ... | $r_{2,N_m-1}$                 | $r_{2,N_m}$                 |
| ⋮                              | ...              | ...              | ... | ... | ...                           | ⋮                           |
| ⋮                              | ...              | ...              | ... | ... | ...                           | ⋮                           |
| CV <sub>N<sub>CV</sub>-1</sub> | $r_{N_{CV}-1,1}$ | $r_{N_{CV}-1,2}$ | ... | ... | ...                           | ⋮                           |
| CV <sub>N<sub>CV</sub></sub>   | $r_{N_{CV},1}$   | $r_{N_{CV},2}$   | ... | ... | $r_{N_{CV},N_m-1}$            | $r_{N_{CV},N_m}$            |

However, for simplicity we here assume  $r_{k,j}$  as a number of exchanger units between controlled variable CV<sub>k</sub> and manipulation MV<sub>j</sub>

**Definition A5:** Relationship between controlled variables and manipulations

Let  $z_{k,j}$  be a binary variable that represents the relationship between controlled variable CV<sub>k</sub> and manipulation MV<sub>j</sub>

$z_{k,j} = 1$  denotes controlled variable CV<sub>k</sub> is paired with manipulation MV<sub>j</sub>

$z_{k,j} = 0$  denotes controlled variable CV<sub>k</sub> is not paired with manipulation MV<sub>j</sub>

where  $k \in \mathbf{CV}$ ,  $j \in \mathbf{MV}$

For a system with  $N_{CV}$  controlled variables and  $N_m$  manipulations, relationship between controlled variables and manipulations can be shown in Table A3.

Number of variables  $z_{k,j} = N_{CV} \times N_m$

Table A3 Relationship between controlled variables and manipulations

| CV \ MV                        | MV <sub>1</sub>  | MV <sub>2</sub>  | ... | ... | MV <sub>N<sub>m</sub>-1</sub> | MV <sub>N<sub>m</sub></sub> |
|--------------------------------|------------------|------------------|-----|-----|-------------------------------|-----------------------------|
| CV <sub>1</sub>                | $z_{1,1}$        | $z_{1,2}$        | ... | ... | $z_{1,N_m-1}$                 | $z_{1,N_m}$                 |
| CV <sub>2</sub>                | $z_{2,1}$        | $z_{2,2}$        | ... | ... | $z_{2,N_m-1}$                 | $z_{2,N_m}$                 |
| ⋮                              | ...              | ...              | ... | ... | ...                           | ⋮                           |
| ⋮                              | ...              | ...              | ... | ... | ...                           | ⋮                           |
| CV <sub>N<sub>CV</sub>-1</sub> | $z_{N_{CV}-1,1}$ | $z_{N_{CV}-1,2}$ | ... | ... | ...                           | ⋮                           |
| CV <sub>N<sub>CV</sub></sub>   | $z_{N_{CV},1}$   | $z_{N_{CV},2}$   | ... | ... | $z_{N_{CV},N_m-1}$            | $z_{N_{CV},N_m}$            |

The ILP formulation consists of two objective functions. The objective function I and constraints 1-5 are used to identify optimal split-range pairs for tracking active constraints (optimality purpose). The objective function II and constraints 6-7 are additionally used to select the best control structure with fast control action (controllability purpose).

**Objective function I:** Minimizing the number of “inter-connection” or “complexity” of control structure (unnecessary relationships between primary and secondary manipulations)

$$\text{Objective function I: } \min J_I = \sum_i \sum_{j \neq i} x_{i,j} \quad i, j \in \mathbf{MV}$$

**Objective function II:** Minimizing the sum of relative order of the control pairs

$$\text{Objective function II: } \min J_{II} = \sum_k \sum_j r_{k,j} z_{k,j} \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

**Constraint 1:** Assign one primary manipulation to each control objective

Number of primary manipulation is equal to number of controlled variables ( $N_{CV}$ )

$$\sum_i x_{i,i} = N_{CV} \quad i \in \mathbf{MV}$$

Number of equations = 1

**Constraint 2:** A manipulation MV<sub>i</sub> that always is an active constraint should not be used for other purposes

Manipulation MV<sub>i</sub> is not used for control

$$x_{i,i} = 0 \quad i \in \mathbf{MVAAT}$$

Manipulation  $MV_i$  has no need of a secondary manipulation

$$\sum_{j \neq i} x_{i,j} = 0 \quad i \in MVAAT, j \in MV$$

Manipulation  $MV_i$  is not used as a secondary manipulation

$$\sum_{j \neq i} x_{j,i} = 0 \quad i \in MVAAT, j \in MV$$

Number of equations = 3 x number of members in **MVAAT**

**Constraint 3:** A manipulation  $MV_i$  that is never an active constraint is used as a primary manipulation with no need of a secondary manipulation

Manipulation  $MV_i$  is a primary manipulation

$$x_{i,i} = 1 \quad i \in MVINAT$$

Manipulation  $MV_i$  has no need of a secondary manipulation

$$\sum_{j \neq i} x_{i,j} = 0 \quad i \in MVINAT, j \in MV$$

Manipulation  $MV_i$  is not used as a secondary manipulation

$$\sum_{j \neq i} x_{j,i} = 0 \quad i \in MVINAT, j \in MV$$

Number of equations = 3 x number of members in **MVINAT**

**Constraint 4:** A manipulation  $MV_i$  that changes between being an active and inactive constraint may be a primary or secondary manipulation.

- $MV_i$  may have a need or no need of a secondary manipulation

if  $MV_i$  is chosen to be a primary manipulation that can be saturated (active constraint), then a secondary manipulation is needed

$$\text{if } x_{i,i} = 1 \text{ then } \sum_{j \neq i} x_{i,j} = 1 \quad i, j \in MVAT$$

if  $MV_i$  is not chosen to be a primary manipulation, then it has no need of a secondary manipulation

$$\text{if } x_{i,i} = 0 \text{ then } \sum_{j \neq i} x_{i,j} = 0 \quad i, j \in MVAT$$

the above two statements can be written

$$-x_{i,i} + \sum_{j \neq i} x_{i,j} = 0 \quad i, j \in MVAT$$

- $MV_i$  can be or cannot be used as a secondary manipulation

if  $MV_j$  is chosen to be a primary manipulation, then it is not used as a secondary manipulation for the other manipulations

$$\text{if } x_{j,j} = 1 \text{ then } \sum_{i \neq j} x_{i,j} = 0 \quad i, j \in \mathbf{MVAT}$$

if  $MV_j$  is chosen to be a secondary manipulation, then it is used for at least one primary manipulation

$$\text{if } x_{j,j} = 0 \text{ then } \sum_{i \neq j} x_{i,j} \geq 1 \quad i, j \in \mathbf{MVAT}$$

the above two statements can be written

$$x_{j,j} + \sum_{i \neq j} x_{i,j} \geq 1 \quad i, j \in \mathbf{MVAT}$$

$$\text{and } M(x_{j,j} - 1) + \sum_{i \neq j} x_{i,j} \leq 0 \quad i, j \in \mathbf{MVAT}$$

where  $M$  = a positive integer which is greater than the number of members in  $\mathbf{MVAT}$

Number of equations = 3 x number of members in  $\mathbf{MVAT}$

**Constraint 5:** Possible and impossible split-range combination of manipulations (these constraints are obtained from the information of active constraint regions)

Constraint 5A: Impossible split-range combination of manipulations

*“Impossible pair: two manipulations which are active constraints (saturated) at the same time cannot be combined as a split-range pair”*

For an active constraint region  $R$ , we have

$$\sum_i \sum_{j \neq i} x_{i,j} = 0 \quad i, j \in \mathbf{MVAT}^{A,R}$$

where  $\mathbf{MVAT}^{A,R}$  is the subset of  $\mathbf{MVAT}$  with manipulations being active constraints in region  $R$ .

Constraint 5B: Possible split-range combination of manipulations

*“Possible pair: two manipulations which are not active (inactive) constraint at the same time may be combined as a split-range pair”*

For an active constraint region  $R$ , we have

$$x_{j,j} + \sum_{i \neq j} x_{i,j} \geq 1 \quad i \in \mathbf{MVAT}^{I,R}, j \in \mathbf{MVAT}^{A,R}$$

$$x_{i,i} + \sum_{j \neq i} x_{j,i} \geq 1 \quad i \in \mathbf{MVAT}^{I,R}, j \in \mathbf{MVAT}^{A,R}$$

where  $\mathbf{MVAT}^{I,R}$  is the subset of  $\mathbf{MVAT}$  with manipulations being inactive constraints in region  $R$



**Constraint 6:** Assign one manipulation to each control objective

$$\sum_j z_{k,j} = 1 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

Number of equations =  $N_{CV}$

**Constraint 7:** Only primary manipulations are paired with controlled variables.

If  $MV_j$  is a primary manipulation, it must be paired with a controlled variable

$$\text{If } x_{j,j} = 1 \text{ then } \sum_k z_{k,j} = 1 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

If  $MV_j$  is not a primary manipulation, it must not be paired

$$\text{If } x_{j,j} = 0 \text{ then } \sum_k z_{k,j} = 0 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

therefore,

$$-x_{j,j} + \sum_k z_{k,j} = 0 \quad k \in \mathbf{CV}, j \in \mathbf{MV}$$

Number of equations =  $N_m$

**Example A1:** A trivial example to illustrate constraint 5

Suppose we have a system with 6 manipulations and 3 controlled variables. For optimality, the number of active manipulations during the operation is  $6-3=3$ . The information of set of active constraints within an operating window is shown in Table A4.

Table A4 Set of active constraints of the system in example A1

| Region | MV <sub>1</sub> | MV <sub>2</sub> | MV <sub>3</sub> | MV <sub>4</sub> | MV <sub>5</sub> | MV <sub>6</sub> |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1      | S <sub>L</sub>  | U               | S <sub>L</sub>  | U               | U               | S <sub>U</sub>  |
| 2      | U               | S <sub>L</sub>  | S <sub>L</sub>  | U               | U               | S <sub>U</sub>  |
| 3      | U               | S <sub>L</sub>  | U               | S <sub>L</sub>  | U               | S <sub>U</sub>  |

\*U-Unsaturated manipulation (inactive constraint),

S<sub>L</sub>-Saturated manipulation (active constraint) at the lower bound,

S<sub>U</sub>-Saturated manipulation (active constraint) at the upper bound

The results in Table A4 shows  $MV_5$  is never an active constraint (satisfy constraint 3) while  $MV_6$  is always an active constraint (satisfy constraint 2). Therefore,  $\mathbf{MVAT} = \{MV_1, MV_2, MV_3, MV_4\}$ .

For region 1, we have  $\mathbf{MVAT}^{A,R=1} = \{MV_1, MV_3\}$  and  $\mathbf{MVAT}^{I,R=1} = \{MV_2, MV_4\}$  and hence the constraints extracted from this region are

*Impossible pair:*

$$x_{1,3} + x_{3,1} = 0$$

*Possible pair:*

$$x_{1,1} + x_{2,1} + x_{4,1} \geq 1$$

$$x_{2,2} + x_{1,2} + x_{3,2} \geq 1$$

$$x_{3,3} + x_{2,3} + x_{4,3} \geq 1$$

$$x_{4,4} + x_{1,4} + x_{3,4} \geq 1$$

For region 2 and 3, we have

$$x_{2,3} + x_{3,2} = 0$$

$$x_{1,1} + x_{2,1} + x_{3,1} \geq 1$$

$$x_{2,2} + x_{1,2} + x_{4,2} \geq 1$$

$$x_{3,3} + x_{1,3} + x_{4,3} \geq 1$$

$$x_{4,4} + x_{2,4} + x_{3,4} \geq 1$$

$$x_{2,4} + x_{4,2} = 0$$

$$x_{1,1} + x_{2,1} + x_{4,1} \geq 1$$

$$x_{2,2} + x_{1,2} + x_{3,2} \geq 1$$

$$x_{3,3} + x_{2,3} + x_{4,3} \geq 1$$

$$x_{4,4} + x_{1,4} + x_{3,4} \geq 1$$

□.

Combining objective function I and II and constraints 1-7, the ILP formulation to find an optimal split-range control structure is:

$$\text{Objective function} = \min \left( \underbrace{w \sum_i \sum_{j \neq i} x_{i,j}}_{J_I} + \underbrace{\sum_k \sum_j r_{k,j} z_{k,j}}_{J_{II}} \right) \quad i, j \in \mathbf{MV}, k \in \mathbf{CV}$$

Subject to: Constraints 1 to 7

From the formulation, the process information required for the ILP are 1) set of active constraints within the specified operating window, and 2) relative orders.