# **Emnemodul: Advanced Process Control**

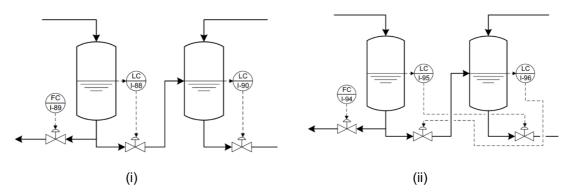
04. Dec. 2020. Time: 0915 - 1200.

Answer as carefully as possible, preferably using the available space. There are in total 6 questions. Answer <u>all</u> 6 questions. You may answer in Norwegian; however, English is preferred.

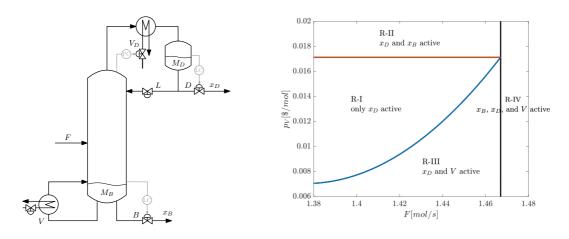
No printed or hand-written support material is allowed. A specific basic calculator is allowed.

# Problem 1 – General Questions (15%)

- a) State the first two steps of the top-down approach in the plantwide control procedure.
- b) What do you mean by consistent control structure? And when is a control structure called locally consistent?
- c) Are the following control structure locally and globally consistent and what is (are) the TPM(s)?



- d) Consider a distillation column with two degrees of freedom, namely *L* and *V*. The process has three constraints  $x_D \ge x_D^{min}$ ,  $x_B \ge x_B^{min}$ ,  $V \le V^{max}$ . Depending on the given feed rate *F*, and the energy prices  $p_V$ , the set of active constraints changes, as shown in the figure below.
  - 1. Where is the TPM for this process?
  - 2. Define what is meant by "bottleneck of a process". Where is the bottleneck in this process, and when is it reached?
  - 3. Would you change the TPM? Reason your answer.



## Problem 2 – Self-optimizing control (15%)

- a) Consider a process with  $n_d = 1$  disturbance,  $n_u = 1$  unconstrained degree of freedom, and  $n_y = 3$  measurements. We want to find a self-optimizing CV for the unconstrained DOF. The nullspace method is one method, which can be used to select a measurement combination  $\mathbf{c} = \mathbf{H}\mathbf{y}$  as the self-optimizing variable. For this method, answer the following questions:
  - 1. What is the minimum number of measurements that we need to apply the nullspace method? Can the null space method be used for the system above?
  - 2. The following table shows the optimal measurement values for the nominal case where the disturbance d = 1, and for a perturbed case where the disturbance d = 1.3. Using this information, calculate the optimal sensitivity matrix **F**, and calculate the selection matrix **H**. Does **H** have a unique solution? Reason your answer.

	d = 1	d = 1.3
<i>y</i> <sub>1</sub>	0.498	0.644
<i>y</i> <sub>2</sub>	0.502	0.656
<b>y</b> <sub>3</sub>	426.761	429.17

b) Consider the simple optimization problem with 2 degrees of freedom, namely  $u_1$  and  $u_2$ 

$$\min_{u_1, u_2} (u_1 - 2)^2 + (u_2 - 3)^2$$
  
s.t.  $u_1 + u_2 \le 4$ 

- 1. Write the KKT conditions for this problem. Is the constraint  $u_1 + u_2 \le 4$  active?
- 2. To translate this optimization problem into a feedback control problem, what should you control using  $u_1$  and  $u_2$ ?
- 3. For the unconstrained degrees of freedom, we want to control a linear gradient combination  $\mathbf{c} = \mathbf{N}^T \nabla_{\mathbf{u}} \mathbf{J}$ . How do you choose **N**? Compute **N** for the problem above?

#### Problem 3 – PID tuning (20%)

a) For a stable first order plus time delay process

$$G(s) = k \frac{1}{(\tau_1 s + 1)} e^{-\theta s}$$

show how the IMC design method results in a PI controller with a desired closed-loop time constant  $\tau_c$ .

In other words, derive the expression for the PI tuning parameters for a desired closed-loop time constant  $\tau_c$ .

b) Consider the following model for predicting the influence of two inputs on two outputs

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{-6e^{-0.75s}}{3.6s+1} & \frac{3.2e^{-s}}{2s+1} \\ \frac{-1.27e^{-0.75s}}{0.67s+1} & \frac{5e^{-s}}{5s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

- 1. Calculate the steady-state RGA for this problem and propose appropriate pairing
- 2. For your proposed pairing, design a pair of PI controllers using the SIMC tuning rules, with  $\tau_c$  equal to the corresponding time delay  $\theta$ .

## Problem 4 – Advanced Control (15%)

a) Consider the plant

$$G(s) = \frac{-0.6}{s-3}$$

- 1. Is the plant open-loop stable? Reason your answer.
- 2. We now wish to control this plant using a proportional controller  $K(s) = K_c$ . Determine the maximum value of the proportional gain  $K_c$  that can be used such that the closed-loop system is stable.
- b) Consider the plant

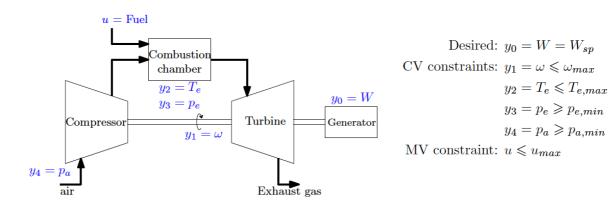
$$G(s) = \frac{3(-2s+1)}{(10s+1)(5s+1)}$$

which we wish to control using a proportional controller  $K(s) = K_c$ . Determine the minimum and maximum value of the proportional gain  $K_c$  such that the closed-loop system is stable. (Hint: It should be sufficient to check for the signs of the coefficients of the characteristic polynomial of the closed-loop system.

c) Given a state space system  $x_{k+1} = Ax_k + Bu_k$ , with an initial state  $x_0$  and an input sequence  $\bar{u} = [u_0, u_1, ..., u_{N-1}]^T$ , the model predictions can be given as  $\bar{x} = \Phi x_0 + \Gamma \bar{u}$ , where  $\bar{x} = [x_1, x_2, ..., x_N]^T$ . How does the expression of  $\Phi$  and  $\Gamma$  look like?

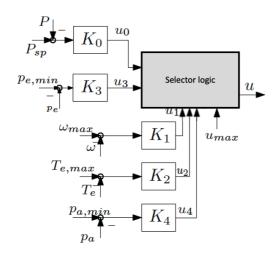
## Problem 5 – Classical Control elements (15%)

Consider a gas turbine engine in Fig. 2 with one MV (degree of freedom), namely the fuel injection rate u. The objective is to control the engine power  $y_0 = W$  to a desired setpoint, subject to a max-constraint on the rotational speed  $\omega$ , a max-constraint on the engine temperature  $T_e$ , a min-constraint on the engine pressure  $p_e$ , a min-constraint on the air inlet pressure  $p_a$ , and a max-constraint on the fuel injection rate u (MV),



The process models from the input u to the different outputs is given by:

	$y_0$	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>	<i>y</i> <sub>3</sub>	$y_4$
u	$\frac{3}{3s+1}e^{-0.75s}$	$\frac{1.2}{6s+1}e^{-0.2s}$	$\frac{15}{10s+1}e^{-0.3s}$	$\frac{3}{2s+1}$	$\frac{-2.3}{8s+1}e^{-0.7s}$

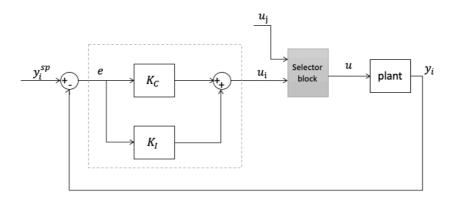


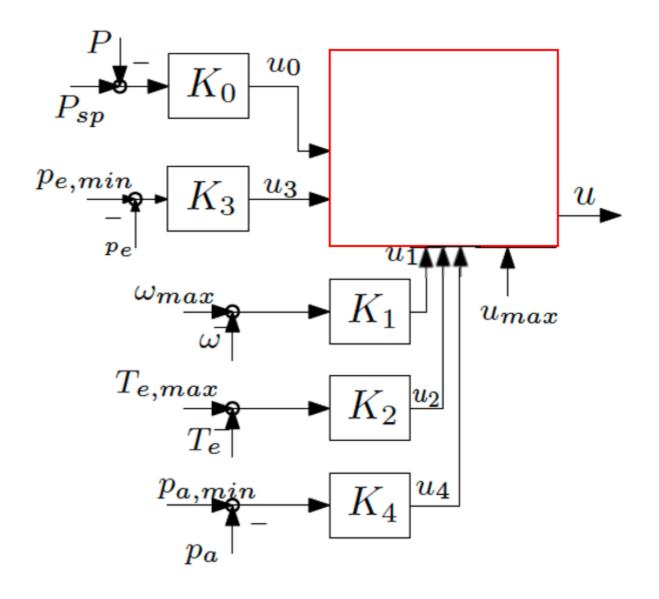
There is a PI controller to control each of the outputs using the single MV, as shown in the figure below, and one needs to select a value from the different controllers using maximum, minimum or mid selector blocks. However, the engineer is not sure how to design the selector block to automatically switch between the different controllers.

You are now called upon to design the selector logic blocks to automatically switch between the different controllers.

- a) Using the systematic design procedure for selector design, how would you group the different constraints? Justify your answer.
- b) Looking at the values computed by the different controllers  $u_1, ..., u_4$  can you explain under what condition the constraints are guaranteed to be feasible?
- c) You are now told that the constraint on the rotational speed is a hard constraint, and can never be given up. How would you design the selector logic block to guarantee that this constraint never becomes infeasible. Draw the proposed selector design inside the red block to connect the arrows in the figure below.

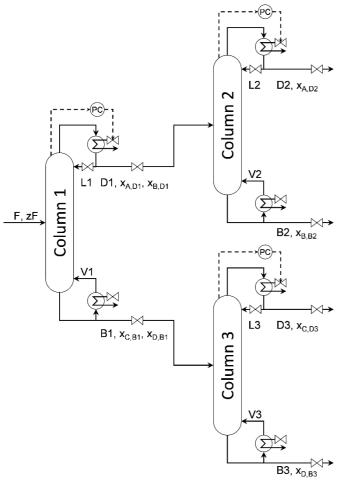
d) The controllers also need an anti-windup to avoid integral windup when not selected. Below is a block diagram of a PI controller without anti-windup. Complete the block-diagram to show how antiwindup is implemented.





# Problem 6 – Plantwide control (20%)

Consider a sequence of distillation columns for separating four components A, B, C, and D as shown in the figure below.



The feed and its composition are given by the upstream processes and our outside of the analysis. The pressure in each column is controlled by the coolant flowrate (see figure).

Purity constraints are imposed on the four products (distillate and bottom streams of column 2 and 3) as follows

$$x_{A,D2} \ge 97\% A$$
  
 $x_{B,B2} \ge 97\% B$   
 $x_{C,D3} \ge 97\% C$   
 $x_{C,B3} \ge 97\% D$ 

To prevent flooding of the columns, the maximum vapor flowrate in the stripping sections of the columns are given by:

$$V_1 \le 5 \text{ mol/s}$$
  
 $V_2 \le 3 \text{ mol/s}$   
 $V_3 \le 4 \text{ mol/s}$ 

Your task is the minimization of the

costs given by

$$J = - \text{Profit} = \rho_F F + \rho_V \left( V_1 + V_2 + V_3 \right) - \rho_{D2} D_2 - \rho_{B2} B_2 - \rho_{D3} D_3 - \rho_{B3} B_3$$

In which the product prices are given by

$$p_{\rm A} = 5 \,\text{mol}$$
  
 $p_{\rm B} = 1 \,\text{mol}$   
 $p_{\rm C} = 5 \,\text{mol}$   
 $p_{\rm D} = 1 \,\text{mol}$ 

As the process is operated in Iceland with cheap industrial energy prices, the energy price is compared to the general industrial energy price very low and given by:

 $p_{\rm v} = 0.0001 \, \text{mol}$ 

Based on the above information, answer the following questions.

- a) How many dynamic and steady-state degree of freedoms does this system have? (NB! do not count the pressure controllers)
- b) Based on your experience and engineering know-how, which constraints will be active for the above mentioned system. Justify your answer.
- c) Propose a control structure for this case and draw it in the figure on the next page. Explain your choice.
- d) In the same figure, mark clearly where the TPM is located, and also mark clearly any remaining unconstrained degrees of freedom. Suggest how you will use any remaining unconstrained DOF.
- e) The product streams of A and C shall be sold directly to the customers, hence the product composition constraints are hard constraint and may not be violated at any point. Can the idea of squeeze and shift be used? How would you apply it?
- f) What happens to the active constraints if the feed rate increases?

