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Fault detection and isolation on a three tank system using differential flatness

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***Abstract*— This paper addresses a fault detection and isolation technique for differential flat systems. For such nonlinear systems, it is possible to find a set of variables, named flat outputs such that states and control inputs can be expressed as functions of flat outputs and their derivatives. Flat systems properties are used to detect and isolate faults and the non-uniqueness property of the set of flat outputs is used for increase the number of residues and improve the fault isolation, the proposed approach will be applied on a classical three tank system.**

***Index terms*— Fault detection and isolation, Differential flatness, Three tank system.**

I. INTRODUCTION

In the past decade fault detection and isolation (FDI) has experienced increasing attention because early detection of faults, while the plant still operating in a controllable region, can help to avoid abnormal event progression and by consequence reduce productivity loss.

According with [1], the different fault detection and isolation approaches can be organized in three main groups of methods: quantitative model-based methods, qualitative model-based methods (e.g.,causal models), and process history based methods (e.g.,neural networks).

This work is focused in quantitative model-based method [2], specifically in analytical redundancy [3] and [4], meaning that fault indicators (residues) are computed using the actual and the expected (nominal, calculated based on the fault free model) monitored signal. Fault identification is performed by a decision algorithm.

Different methods have been used to compute this residual signals, state estimation [5], parameter estimation [6] , and parity space [7], among the many approaches reported in the literature [8], only a few consider to take advantages of the differential flatness [9]-[13], in most of the cases the approach is applied to a linear system.

In [9] - [11] only fault detection is carried out, however in [12] and [13] fault detection and identification is developed, these approaches differ from ours in two principal points:

- In the proposed approach, at least two algebraically independent flat vectors must be found in order to increase the number of residues.
- If the number of flat outputs is equal to the number of algebraically independent flat vectors, faults affecting flat outputs will always be isolated, regardless of the plant.

By consequence of the first point, detection and isolation will be more accurate and control reconfiguration will be faster and easier.

This paper is organized as follows: differential flatness and the trajectory generation method are presented in section two, fault detection and isolation proposed approach is explained in section three, section four presents the simulation results in a classical three tank system obtained with our approach, and, finally, the conclusion is presented in section five.

II. DIFFERENTIAL FLATNESS

Flatness concept initially introduced by Fliess et al. [14], concern all linear controllable systems and a class of nonlinear systems for whose dynamic behavior can be parametrized by a set of variables, called flat outputs and a finite number of its derivatives.

Let us consider the nonlinear system $\dot{x} = f(x,u)$, $x \in \Re^n$ the state vector, $u \in \Re^m$ the control vector and f a c^∞ function of x and u. The system is differentially flat if, and only if, it exists a flat output vector $z \in \Re^m$ such as:

- The flat output vector its expressed as function of the state x and the control input u and a finite number of its time derivatives.

$$z = \phi_z(x,u,\dot{u},...,u^{(\gamma)}) \tag{1}$$

- The state x and the control input u are expressed as functions of the vector z and a finite number of its time derivatives.

$$x = \phi_x(z,\dot{z},...,z^{(a)}) \tag{2}$$
$$u = \phi_u(z,\dot{z},...,z^{(a+1)}) \tag{3}$$

Where $z^{(a)}$ denotes the a^{th} time derivative of z.

A. *Trajectory generation by flatness*

Trajectory generation for nonlinear systems is simplified by using differential flatness property, because, it exists a one-to-one correspondence between trajectories of flat outputs and full state space and input trajectories [15]. Initial and final conditions and path planning constraints can be easily translate in term of flat outputs requirements, and then

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plan only the flat output trajectories (z_{ref}), in this way the full state and control inputs path can be calculated using the equations (2) and (3), in this work the reference flat output vector is designed in a simple way by using a polynomial interpolation approach.

Other efficient approaches can be found in [16] and [17].

III. FAULT DETECTION AND ISOLATION

The goal is to use differential flatness properties to develop an FDI methodology, especially the non-uniqueness property of the flat vector, in order to compute a larger number of residues facilitating the fault detection and isolation task.

Our methodology is divided in two stages, first, n (n =state dimension) residues are computed using one flat vector, denoted z_α. In a second stage another set of n residues is computed by using a second flat vector, which at least one element is not a simple algebraic combination of the previous one, denoted z_β.

A. Residues set for a couple of flat output vectors

Let us consider a nonlinear flat model of dimension n, and m control inputs, with z_α as flat outputs, which corresponds to m components of the state vector, also suppose that the full state is measured, using the states and inputs calculated from (2) and (3), it is always possible to compute n residues:

- n - m state residues, because the full state is supposed to be measured.
- m control inputs residues.

The residual signals are computed by using

$$r_{ix} = x_{mi} - x_{ci} \tag{4}$$

$$r_{iu} = u_{mi} - u_{ci} \tag{5}$$

where x_{mi} and u_{mi} are the i_{th} measured state and control input respectively and x_{ci} and u_{ci} are the i_{th} state and control input calculated using the differentially flat equations.

Assuming that there is a second flat output vector z_β which corresponds to m components of the state vector which at least one element is not just an algebraic combination of z_α components, by consequence the number of residual signals will be increased in n. A new set of $n-m$ state residues and m control input residues can be calculated in the same manner as in (4) and (5), as a result $n+n$ residues are available.

For instance for a nonlinear system composed by four states $[x_1\ x_2\ x_3\ x_4]^T \in n$ and two control inputs $[u_1\ u_2]^T \in m$ and by consequence two flat outputs, for example $[z_{\alpha 1}\ z_{\alpha 2}]^T = [x_1\ x_2]^T \in m$, four residuals can be obtained by using the set of differentially flat equations $\phi_{\alpha x}$ and $\phi_{\alpha u}$, using these residues, faults affecting flat outputs can be detected but cannot be isolated, faults appearing on input or state sensors are or are not isolated depending on the specific system.

Assuming that an algebraically independent flat vector exists, for instance $[z_{\beta 1}\ z_{\beta 2}]^T = [x_3\ x_4]^T \in m$, then it is possible to generate eight residues as depicted in Fig.1.

By this way the fault isolation procedure is simplified, the residual signals are obtained as follows.

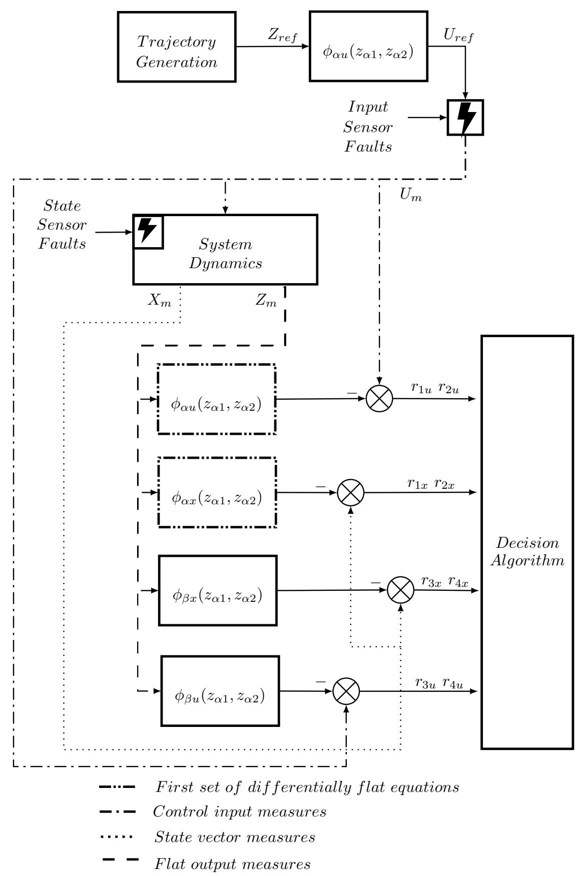

Fig. 1. Fault detection scheme

$$\begin{bmatrix} r_{1x} \\ r_{2x} \\ r_{1u} \\ r_{2u} \\ r_{3x} \\ r_{4x} \\ r_{3u} \\ r_{4u} \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ u_1 \\ u_2 \\ x_1 \\ x_2 \\ u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} \phi_{\alpha x}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T\ (e_3) \\ \phi_{\alpha x}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T\ (e_4) \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T\ (c_1) \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T\ (c_2) \\ \phi_{\beta x}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T\ (e_1) \\ \phi_{\beta x}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T\ (e_2) \\ \phi_{\beta u}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T\ (c_1) \\ \phi_{\beta u}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T\ (c_2) \end{bmatrix} \tag{6}$$

Where $e_k \in \mathbb{R}^4$, $e_k(i) = 0$, $\forall i \neq k$,$e_k(i) = 1 \Leftrightarrow i = k$ and $c_k \in \mathbb{R}^2$, $c_k(j) = 0$, $\forall j \neq k$,$e_k(j) = 1 \Leftrightarrow j = k$.

If all the elements of z_α and z_β are algebraically independent every single sensor fault (input or state) can be detected and isolated, if one or more flat output is not algebraically independent, fault isolation depends on the system equations, see section IV.

B. Derivatives estimation

Our approach needs fast and accurate time derivative estimation in order to compute all the ϕ functions. In this work a high-gain observer [18] is used to evaluate the time derivative of noisy signals.

In order to improve the performance of the high-gain observer, a low-pass filter is synthesized, the filter order is fixed regarding the maximal derivative used in the differentially flat equations, hence a better noise filtering is obtained. Let us define the equation of the high-gain observer

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u \tag{7}$$

Where

$$\hat{A} = \begin{bmatrix} -\zeta_1/\varepsilon & 1 & ... & ... & 0 \\ -\zeta_2/\varepsilon^2 & 0 & 1 & ... & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -\zeta_{n-1}/\varepsilon^{n-1} & ... & ... & 0 & 1 \\ -\zeta_n/\varepsilon^n & ... & ... & ... & 0 \end{bmatrix} \tag{8}$$

and

$$\hat{B} = \begin{bmatrix} -\zeta_1/\varepsilon & -\zeta_2/\varepsilon^2 & \dots & -\zeta_{n-1}/\varepsilon^{n-1} & -\zeta_n/\varepsilon^n \end{bmatrix}^T \tag{9}$$

The polynomial $S^n + \zeta_1 S^{n-1} + ... + \zeta_{n-1}S + \zeta_n$ is Hurwitz and $\varepsilon << 0$. The transfer function from u to $\hat{x}$ when $\varepsilon \Rightarrow 0$ is $T(s) = [1\ S\ ...\ S^{n-2}\ S^{n-1}]^T$, the system acts as a differentiator under the consideration that the input u is continuous and derivable. In this case the $n-1$ derivatives are obtained directly from the state vector.

A possible selection of the coefficients $\zeta_i (i = 1, \cdots n)$ is in such a way that the frequency bandwidth of the signal to be derivated is in the frequency bandwidth of the filter $1/(S^n + \zeta_1 S^{n-1} + \cdots + \zeta_{n-1}S + \zeta_n)$ and the ε small enough.

IV. EXAMPLE: THREE TANK SYSTEM

A. Nonlinear state space model

The detection and isolation technique is applied in a classical three tank system, see Fig. 2 , the system equations are expressed as follows:

$$\begin{aligned} S\dot{x}_1 &= -Q_{10}(x_1) - Q_{13}(x_1,x_3) + Q_1 & (10)\\ S\dot{x}_2 &= -Q_{20}(x_2) + Q_{32}(x_2,x_3) + Q_2 & (11)\\ S\dot{x}_3 &= Q_{13}(x_1,x_3) - Q_{32}(x_2,x_3) - Q_{30}(x_3) & (12) \end{aligned}$$

Where S is the transverse section of the tanks, x_i, $i = 1,2,3$, Q_{i0}, $i = 1,2,3$ the outflow between each tank and the central reservoir, Q_{13} and Q_{32} are the outflow between tank 1 and tank 3 and the outflow between tanks 3 and 2 respectively, Q_1 and Q_2 are the incoming flows of each pump.

The valves connecting tanks one and three with the central reservoir are considered closed, so Q_{10} and Q_{30} are always equals to zero. The flows Q_{13}, Q_{32} and Q_{20} can be expressed as follows: [19]

$$\begin{aligned} Q_{13}(x_1,x_3) &= a_{z1}S_n\sqrt{2g(x_1 - x_3)} & (13)\\ Q_{20}(x_2) &= a_{z2}S_n\sqrt{2g(x_2)} & (14)\\ Q_{32}(x_2,x_3) &= a_{z3}S_n\sqrt{2g(x_3 - x_2)} & (15) \end{aligned}$$

where S_n represents the transverse section of the pipes connecting the tanks and a_{zj}, $j = 1,2,3$ represents the flow coefficients.

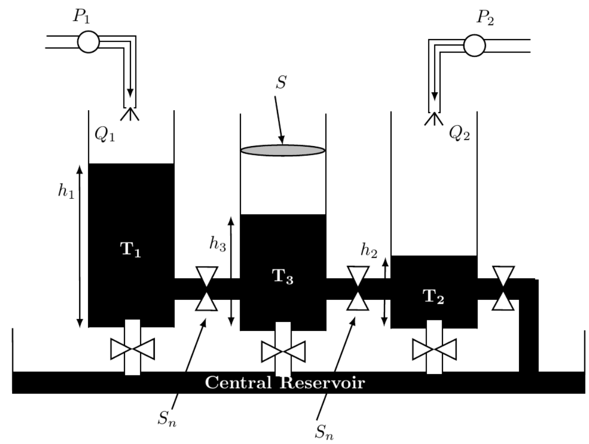

Fig. 2. Three Tank schema

B. Flat model

The flat model is computed by defining x_1 and x_3 as flat outputs, $z_\alpha = [x_1\ x_3]^T$, so the differentially flat equations can be writen as follows:

$$\begin{aligned} x_1^\alpha &= z_{\alpha 1} & (16)\\ x_2^\alpha &= z_{\alpha 2} - \frac{1}{2g}\left(\frac{a_{z1}S_n\sqrt{2g(z_{\alpha 1} - z_{\alpha 2})} - S\dot{z}_{\alpha 2}}{a_{z3}S_n}\right)^2 & (17)\\ x_3^\alpha &= z_{\alpha 2} & (18)\\ Q_1^\alpha &= S\dot{z}_{\alpha 1} + a_{z1}S_n\sqrt{2g(z_{\alpha 1} - z_{\alpha 2})} & (19)\\ Q_2^\alpha &= S\dot{x}_2^\alpha - a_{z3}S_n\sqrt{2g(z_{\alpha 2} - x_2^\alpha)} + a_{z2}S_n\sqrt{2gx_2^\alpha} & (20) \end{aligned}$$

$$\phi_{\alpha x}(z_{\alpha 1}, z_{\alpha 2}) = \begin{bmatrix} x_1^\alpha & x_2^\alpha & x_3^\alpha \end{bmatrix}^T \tag{21}$$

$$\phi_{\alpha u}(z_{\alpha 1}, z_{\alpha 2}) = \begin{bmatrix} Q_1^\alpha & Q_2^\alpha \end{bmatrix}^T \tag{22}$$

As mentioned above the flat vector for this system, it's not unique, so, it is possible to use $z_\beta = [x_2\ x_3]^T$ in order to compute another set of differentially flat equations.

$$\begin{aligned} x_1^\beta &= z_{\beta 2} + \frac{1}{2g}\left(\frac{a_{z3}S_n\sqrt{2g(z_{\beta 2} - z_{\beta 1})} + S\dot{z}_{\beta 2}}{a_{z1}S_n}\right)^2 & (23)\\ x_2^\beta &= z_{\beta 1} & (24)\\ x_3^\beta &= z_{\beta 2} & (25)\\ Q_1^\beta &= S\dot{x}_1^\beta + a_{z1}S_n\sqrt{2g(x_1^\beta - z_{\beta 2})} & (26)\\ Q_2^\beta &= S\dot{z}_{\beta 1} - a_{z3}S_n\sqrt{2g(z_{\beta 2} - z_{\beta 1})} + a_{z2}S_n\sqrt{2gz_{\beta 1}} & (27) \end{aligned}$$

$$\phi_{\beta x}(z_{\beta 1}, z_{\beta 2}) = \begin{bmatrix} x_1^\beta & x_2^\beta & x_3^\beta \end{bmatrix}^T \tag{28}$$

$$\phi_{\beta u}(z_{\beta 1}, z_{\beta 2}) = \begin{bmatrix} Q_1^\beta & Q_2^\beta \end{bmatrix}^T \tag{29}$$

C. Simulation results

The reference trajectories of flat outputs were calculated using a fifth order polynomial, this generation method facilitate the tuning of initial and final conditions, white noise is added to the measured outputs with a relevant level to the real process measure level.

Flow coefficients a_{z1} and a_{z3} are equal to 0.75, a_{z2} value is 0.76, the transverse section of the tanks and the transverse section of the connecting pipes are $15.4*10^{-3}$ and $5*10^{-5}$ respectively.

Five different multiplicative sensor faults are analyzed, three state faults and two input sensor faults. Once the failure occurs (at 250s), it is persistent until the end of the simulation, for simplicity sake only one single fault is presented at a time, actuators are considered fault-free at any time.

The detection threshold was defined by changing the flow parameters in the range of $+/-$ 10%, then, the maximal value for each residue (positive and negative) in addition with an error margin is used as the final amplitude of the detection threshold. This margin adds robustness and avoids false alarms.

Once the threshold is passed the fault is considered detected.

D. Fault detection and isolation

1) Case A: n residues: Using equations (21) and (22) n residues are computed as follows:

$$\begin{bmatrix} r_{1x} \\ r_{1u} \\ r_{2u} \end{bmatrix} = \begin{bmatrix} h_2 \\ Q_1 \\ Q_2 \end{bmatrix} - \begin{bmatrix} \phi_{\alpha x}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [0\ 1\ 0]^T \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [1\ 0]^T \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [0\ 1]^T \end{bmatrix} \quad (30)$$

Faults non-affecting flat outputs are detected and isolated by simply testing the amplitude of residues, (see Fig.3 to 5). If the fault appears in a flat output (x_1 or x_3), the fault is detected, but it is not possible to isolate it by a simple comparison between the residual signal and the respective threshold, because all the residues are triggered. Fig. 6 and 7.

2) Case B: $n+n$ residues: Using a second set of differentially flat equations $n+n$ residues could be computed as follows:

$$\begin{bmatrix} r_{1x} \\ r_{1u} \\ r_{2u} \\ r_{2x} \\ r_{3u} \\ r_{4u} \end{bmatrix} = \begin{bmatrix} h_2 \\ Q_1 \\ Q_2 \\ h_1 \\ Q_1 \\ Q_2 \end{bmatrix} - \begin{bmatrix} \phi_{\alpha x}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [0\ 1\ 0]^T \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [1\ 0]^T \\ \phi_{\alpha u}(z_{\alpha 1}, \dot{z}_{\alpha 1}, z_{\alpha 2}, \dot{z}_{\alpha 2})^T [0\ 1]^T \\ \phi_{\beta x}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T [1\ 0\ 0]^T \\ \phi_{\beta u}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T [1\ 0]^T \\ \phi_{\beta u}(z_{\beta 1}, \dot{z}_{\beta 1}, z_{\beta 2}, \dot{z}_{\beta 2})^T [0\ 1]^T \end{bmatrix} \quad (31)$$

In this case every single fault is detected and isolated, because each fault generates a different residual pattern, see Fig. 8 to 12 and table I.

E. Results discussion

As expected, a fault appearing in water level sensor x_2, is detected and isolated in both cases, this behavior is explained by the dependence between residues exceeding the detection threshold and the measure of h_2. See (31).

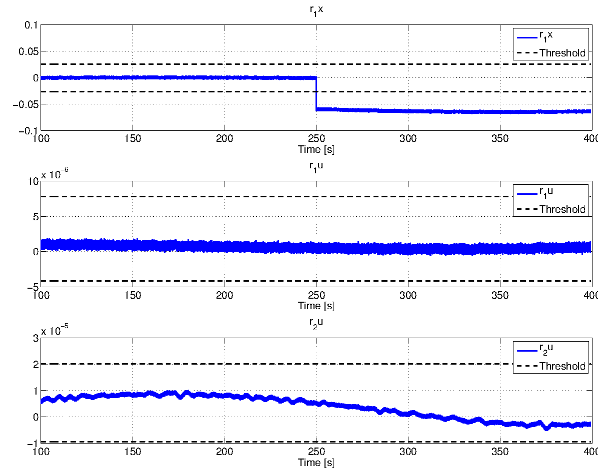

Fig. 3. Case A: 20% loss h_2 water level sensor

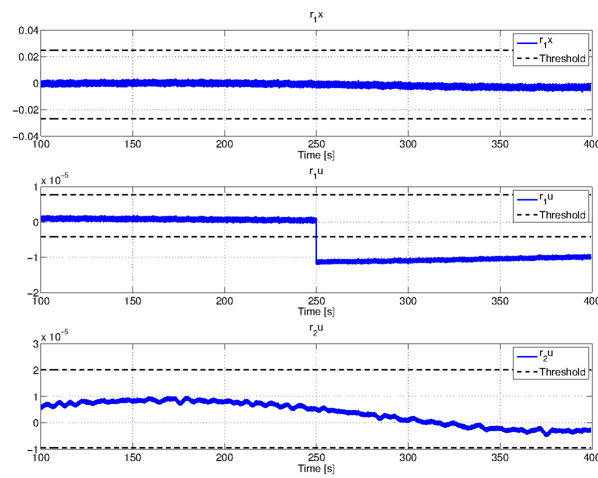

Fig. 4. Case A: 20% loss Q_1 input sensor

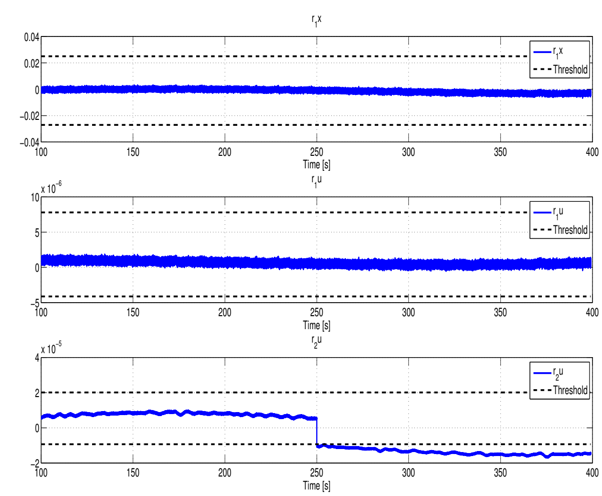

Fig. 5. Case A: 20% loss Q_2 input sensor

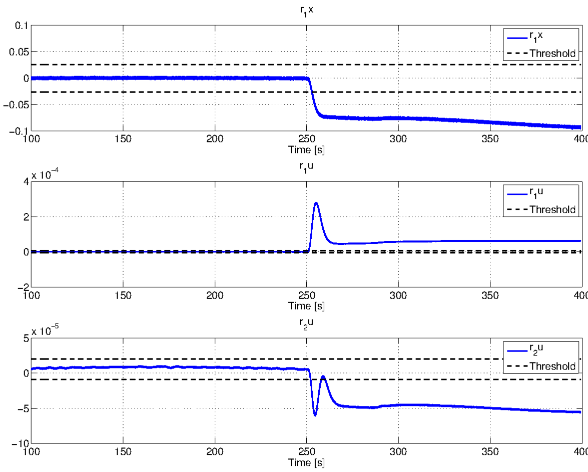

Fig. 6. Case A: 20% loss h_1 water level sensor

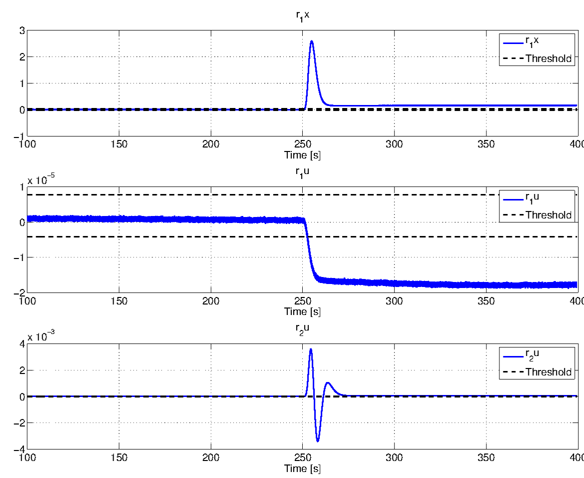

Fig. 7. Case A: 20% loss h_3 water level sensor

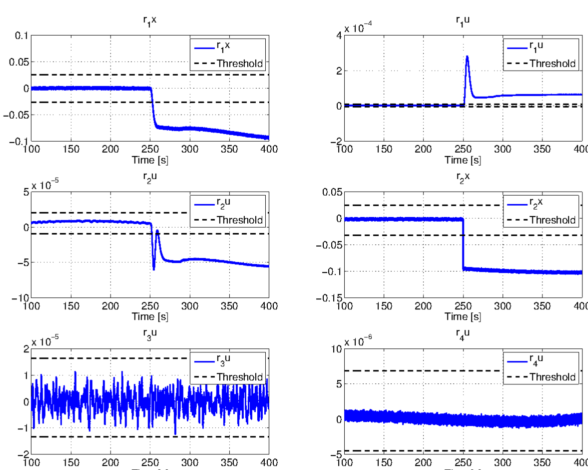

Fig. 8. Case B 20% loss h_1 water level sensor

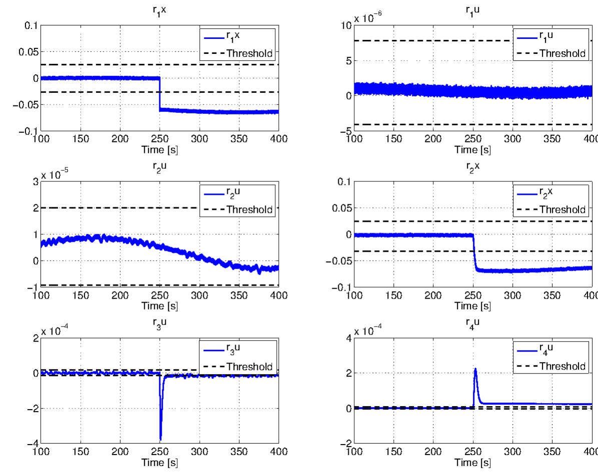

Fig. 9. Case B 20% loss h_2 water level sensor

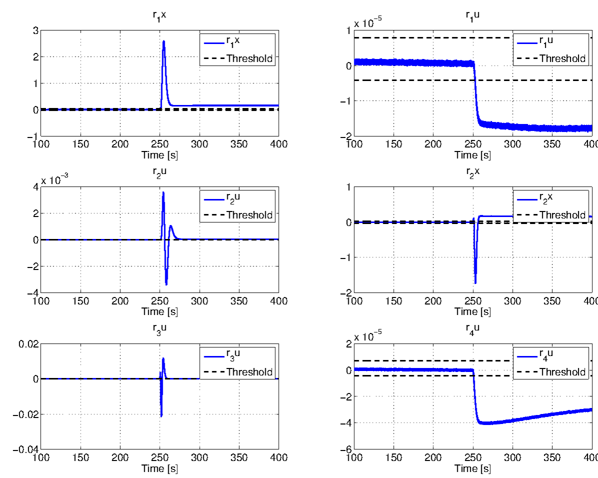

Fig. 10. Case B 20% loss h_3 water level sensor

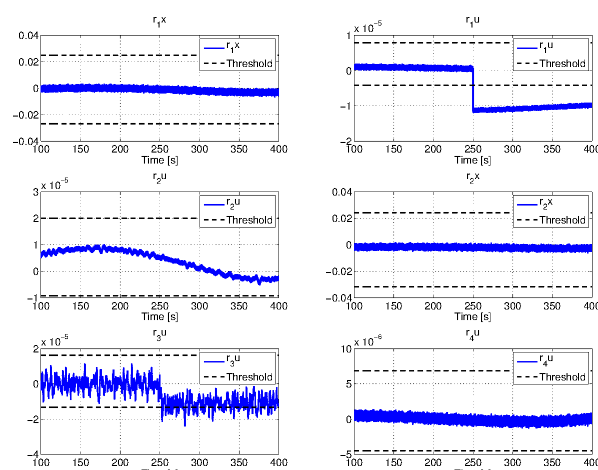

Fig. 11. Case B 20% loss Q_1 input sensor

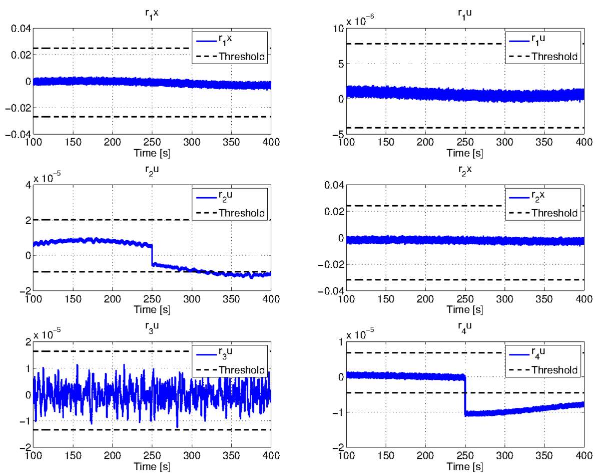

Fig. 12. Case B 20% loss Q_2 input sensor

When the fault affects a flat output, Fig. 6, 7, 8 and 10, it is isolated only in the case B, this is explained by the fact that in the case A, all the residues are constructed using h_1 and h_3 as flat outputs and by consequence if one of the measures is defective it will be reflected in every residual signal, nonetheless in the case B, two residues does not depend of h_1, by consequence they are not affected by this fault, if the fault is presented in h_3 all residues will be affected in both cases, however in the second case this is the only framework where all residues are triggered.

Faults appearing on inputs Q_1 and Q_2 have the same comportment of h_2, see table I

TABLE I
RESIDUES MATRIX

	Fault	r_{1x}	r_{1u}	r_{2u}	r_{2x}	r_{3u}	r_{4u}
Case A	F_{h1}	1	1	1	-	-	-
	F_{h2}	1	0	0	-	-	-
	F_{h3}	1	1	1	-	-	-
	F_{Q1}	0	1	0	-	-	-
	F_{Q2}	0	0	1	-	-	-
Case B	F_{h1}	1	1	1	1	0	0
	F_{h2}	1	0	0	1	1	1
	F_{h3}	1	1	1	1	1	1
	F_{Q1}	0	1	0	0	1	0
	F_{Q2}	0	0	1	0	0	1

V. CONCLUSIONS

This work presents a new approach to detect and isolate faults, by using differential flatness. The method was successfully applied in a classical three tank system, since, even in the presence of noise, every single fault is detected and isolated.

This method was tested in the specific case where the flat outputs are simply elements inside the state vector. Even if this characteristic is shown by many systems, this is not the only possibility, as explained in section two, flat outputs could be functions of inputs and states, so future work will be focus on extending our fault detection approach to a more complex case.

Regarding control reconfiguration, redundant signals can be used on reconfiguration purpose.

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